Chapter 24
Capacitors and Dielectrics
What is Capacitance?

• Capacitance (C) is equal to the Charge (Q) between two charges or charged “regions” divided by the Voltage (V) in those regions.
• Here we assume equal and opposite charges (Q)
• Thus C = Q/V or Q = CV or V=Q/C
• The units of Capacitance are “Farads” after Faraday denoted F or f
• One Farad is one Volt per Coulomb
• One Farad is a large capacitance in the world of electronics
• “Capacitors” are electronic elements capable of storing charge
• Capacitors are very common in electronic devices
• All cell phones, PDA’s, computers, radio, TV’s … have them
• More common units for practical capacitors are micro-farad (10^{-6} f = μf), nano-farad (10^{-9} f = nf) and pico-farads (10^{-12} f = pf)
A classic parallel plate capacitor

(a) Arrangement of the capacitor plates

(b) Side view of the electric field $\vec{E}$

When the separation of the plates is small compared to their size, the fringing of the field is slight.
General Surfaces as Capacitors
The Surfaces do not have to be the same
Cylindrical – “Coaxial” Capacitor
Spherical Shell Capacitor

Inner shell, charge $+Q$

Gaussian surface

Outer shell, charge $-Q$
How most practical cylindrical capacitors are constructed:

- **Conductor** (metal foil)
- **Dielectric** (plastic sheet)
Dielectrics – Insulators – Induced and Aligned Dipole Moments

In the absence of an electric field, nonpolar molecules are not electric dipoles.

In the absence of an electric field, polar molecules orient randomly.
Aligning Random Dipole Moments

When an electric field is applied, the molecules tend to align with it.
Creating Dipole Moments – Induced Dipoles

(b)

An electric field causes the molecules’ positive and negative charges to separate slightly, making the molecule effectively polar.
Adding Dielectric to a Capacitor INCREASES its Capacitance since it DECREASES the Voltage for a GIVEN Charge.

For a given charge density $\sigma$, the induced charges on the dielectric’s surfaces reduce the electric field between the plates.
Induced Dipole Moments in a Normally Unpolarized Dielectric
Adding the dielectric reduces the potential difference across the capacitor.
Electrostatic Attraction – “Cling”
Forced on Induced Dipole Moments
Energy Stored in a Capacitor

- Capacitors store energy in their electric fields
- The force on a charge in E field $E$ is $F = qE$
- The work done moving a charge $q$ across a potential $V$ is $qV$
- Let's treat a capacitor as a storage device we are charging
- We start from the initial state with no charge and start adding charge until we reach the final state with charge $Q$ and Voltage $V$.
- Total work done $W = \int V(q) \, dq$ (we charge from zero to $Q$)
- BUT $V = q/C$
- We assume here the Capacitance is NOT a function of $Q$ and $V$ BUT only of Geometry
- Thus the energy stored is $W = 1/C \int q \, dq = \frac{1}{2} \frac{Q^2}{C} = \frac{1}{2} CV^2$
Calculating Parallel Capacitor Capacitance

- Assume two metal plates, area A each, distance d apart, Voltage V between them, Charge ±Q on Plates
- \( \sigma = \frac{Q}{A} \quad V = \int E \, dx = Ed \quad E = \frac{\sigma}{\varepsilon_0} \) (from Gauss)
- Therefore \( C = \frac{Q}{V} = \frac{\sigma A}{Ed} = \varepsilon_0 \frac{A}{d} \)
- **Note** – As d decreases C increases

(a) Arrangement of the capacitor plates  
(b) Side view of the electric field \( \vec{E} \)

When the separation of the plates is small compared to their size, the fringing of the field is slight.
(a) No dielectric

(b) Dielectric just inserted

(c) Induced charges create electric field

(d) Resultant field

Original electric field

Weaker field in dielectric due to induced (bound) charges
Force on a Dielectric inserted into a Capacitor
Force on Capacitor Plates

\[ F = QE \quad V = Ed \quad (d \text{ separation distance}) \]
\[ F = QV/d \]
\[ Q = CV \quad \rightarrow \quad F = CV^2/d \]
Recall \( W \) (Stored Energy) = \( \frac{1}{2} CV^2 \)
Hence \( F = 2W/d \) or \( W = \frac{1}{2} Fd \)
Capacitors in Series

(a) Two capacitors in series

**Capacitors in series:**
- The capacitors have the same charge $Q$.
- Their potential differences add:
  \[ V_{ac} + V_{cb} = V_{ab}. \]

![Diagram of two capacitors in series](image)

\[ V_{ab} = V \]

\[ V_{ac} = V_1 \]

\[ V_{cb} = V_2 \]

(b) The equivalent single capacitor

Charge is the same as for the individual capacitors.

Equivalent capacitance is less than the individual capacitances:

\[ C_{eq} = \frac{Q}{V} \]

\[ \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} \]
Capacitors in Parallel

(a) Two capacitors in parallel

Capacitors in parallel:
- The capacitors have the same potential $V$.
- The charge on each capacitor depends on its capacitance: $Q_1 = C_1 V$, $Q_2 = C_2 V$.

(b) The equivalent single capacitor

Charge is the sum of the individual charges:
$Q = Q_1 + Q_2$

Equivalent capacitance:
$C_{eq} = C_1 + C_2$
Series and Parallel Capacitors

Replace these series capacitors by an equivalent capacitor ...

Replace these parallel capacitors by an equivalent capacitor ...
### Dielectric Constants of Some Common Materials

**Table 24.1** Values of Dielectric Constant $K$ at 20°C

<table>
<thead>
<tr>
<th>Material</th>
<th>$K$</th>
<th>Material</th>
<th>$K$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1</td>
<td>Polyvinyl chloride</td>
<td>3.18</td>
</tr>
<tr>
<td>Air (1 atm)</td>
<td>1.00059</td>
<td>Plexiglas</td>
<td>3.40</td>
</tr>
<tr>
<td>Air (100 atm)</td>
<td>1.0548</td>
<td>Glass</td>
<td>5–10</td>
</tr>
<tr>
<td>Teflon</td>
<td>2.1</td>
<td>Neoprene</td>
<td>6.70</td>
</tr>
<tr>
<td>Polyethylene</td>
<td>2.25</td>
<td>Germanium</td>
<td>16</td>
</tr>
<tr>
<td>Benzene</td>
<td>2.28</td>
<td>Glycerin</td>
<td>42.5</td>
</tr>
<tr>
<td>Mica</td>
<td>3–6</td>
<td>Water</td>
<td>80.4</td>
</tr>
<tr>
<td>Mylar</td>
<td>3.1</td>
<td>Strontium titanate</td>
<td>310</td>
</tr>
</tbody>
</table>
Two Dielectric Constants – As if capacitors in Series
Two Dielectric Constants – As if two capacitors in Parallel
Dielectric Plus Vacuum (Air)
Treat is if three capacitors in Series
### Capacitance of Simple Systems

<table>
<thead>
<tr>
<th>Type</th>
<th>Capacitance</th>
<th>Comment</th>
</tr>
</thead>
</table>
| **Parallel-plate capacitor**        | $\frac{\varepsilon A}{d}$                                                   | $A$: Area  
$d$: Distance                                                   |
| **Coaxial cable**                   | $\frac{2\pi \varepsilon l}{\ln (a_2/a_1)}$                                | $a_1$: Inner radius  
$a_2$: Outer radius  
$l$: Length                                                |
| **Pair of parallel wires**          | $\frac{2\pi \varepsilon l}{\text{arcosh} \left( \frac{d^2}{4a^2} - 1 \right)} = \frac{\pi l}{\text{arcosh} \left( \frac{d}{2a} \right)}$ | $l$: Length  
$d$: Distance, $d > 2a$                                      |
| **Wire parallel to wall**           | $\frac{4\pi \varepsilon l}{\text{arcosh} \left( \frac{d^2}{4a^2} - 1 \right)} = \frac{2\pi \varepsilon l}{\text{arcosh} \left( \frac{d}{a} \right)}$ | $l$: Wire length  
$d$: Distance, $d > a$                                      |
| **Concentric spheres**              | $\frac{2\pi a}{\ln \left( \frac{d}{2a} + \sqrt{\frac{d^2}{4a^2} - 1} \right)}$ | $a_1$: Inner radius  
$a_2$: Outer radius                                                  |
| **Two spheres, equal radius**       | $\sum_{n=1}^{\infty} \frac{\sinh \left( \ln \left( D + \sqrt{D^2 - 1} \right) \right)}{\sinh \left( n \ln \left( D + \sqrt{D^2 - 1} \right) \right)}$ | $D = d/2a$  
$\gamma$: Euler's constant                                             |
| **Sphere in front of wall**         | $\frac{4\pi a}{\text{ln} \left( \frac{D + \sqrt{D^2 - 1}}{a} \right)}$    | $D = d/a$  
$a$: Radius                                                   |
| **Sphere**                          | $\frac{4\pi \varepsilon a}{8\varepsilon a}$                              | $a$: Radius                                                             |
| **Circular disc**                   | $\frac{2\pi \varepsilon l}{\Lambda} \left( 1 + \frac{1}{\Lambda} (1 - \ln 2) + \frac{1}{\Lambda^2} \left[ 1 + (1 - \ln 2)^2 - \frac{\pi^2}{12} \right] + O \left( \frac{1}{\Lambda^3} \right) \right)$ | $l$: Length  
$\Lambda$: ln(l/a)                           |