Chapter 25 Resistance and Current

Current in Wires

- We define the Ampere (amp) to be one Coulomb of charge flow per second
- A Coulomb is about 7×10^{18} electrons (or protons) of charge
- For reference a "mole" is about 6.02×10^{23} units
- Thus a "mole" of Copper 63.5 g/mole (z=29, A=63 (69.15% - 34 Neutrons, A=65 (30.85% - 36 Neutrons)
- Contains about 3×10^{6} Coulombs BUT only outer electrons are free to move ($4S^1$ state) – one electron per Cu atom in "valence band"
- Density of Copper is about 8.9 g/cm^3
- Density of free electrons in Cu ~ 1.4×10^{4} $Coul/cm^3$
- Or density of free electrons ~ 10^{23} e/cm^3

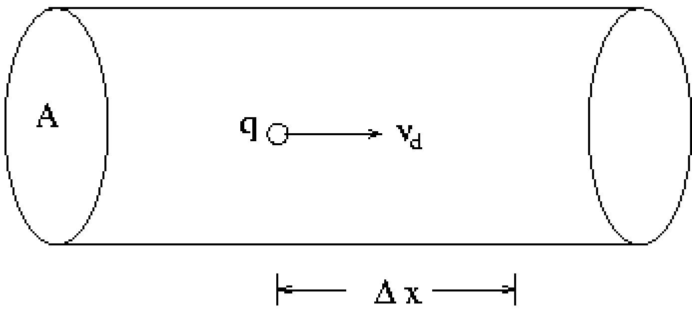
A bit of History

- *chalkos* (χαλκός) in Greek
- *Cyprium* in Roman times as it was found in Cyprus
- This was simplified to Cuprum in Latin and then
- Copper in English
- Copper mined in what is now Wisconsin 6000-3000 BCE
- Copper plumbing found in Egyptian pyramid 3000 BCE
- Small amount of Tin (Sn) helps in casting – Bronze (Cu-Sn)

Ancient mine in Timna Valley – Negev Israel

Current in wire

- Lets assume a metal wire has n free charges/ vol
- Assume the wire has cross sectional area A
- Assume the charges (electrons) move at "drift speed" v_d
- Lets follow a section of charge Δq in length Δx
- $\Delta q = n*A*\Delta x$ (n*volume)e
- Where e = electron charge
- This volume move (drifts) at speed v_d
- This charge moves thru Δx in time
- $\Delta t = \Delta x/v_d$
- The current is $I = \Delta q/\Delta t = n*A*\Delta x*e/(\Delta x/v_d) = nAv_de$



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Wire gauges
AWG – American Wire Gauge

- Larger wire gauge numbers are smaller size wire
- By definition 36 gauge = 0.005 inches diam
- By definition 0000 gauge “4 ot” = 0.46 inch diam
- The ratio of diameters is 92 = (0.46/0.005)
- There are 40 gauges size from 4 ot to 36 gauge
- Or 39 steps

$$d_n = 0.005 \text{ inch} \times 92^{\frac{36-n}{39}} = 0.127 \text{ mm} \times 92^{\frac{36-n}{39}}$$

$$d_n = e^{-1.12436-.11594\times n} \text{ inch} = e^{2.1104-.11594\times n} \text{ mm}$$

$$A_n = \frac{\pi}{4} d_n^2 = 0.000019635 \text{ inch}^2 \times 92^{\frac{36-n}{19.5}} = 0.012668 \text{ mm}^2 \times 92^{\frac{36-n}{19.5}}$$

AWG	Diameter		Turns of wire		Area		Copper resistance[6]		NEC copper wire ampacity with 60/75/ 90°C insulation (A)[7]
	(inch)	(mm)	(per inch)	(per cm)	(kcmil)	(mm²)	(Ω/km)	(Ω/kFT)	
0000 (4/0)	0.46	11.684	2.17	0.856	212	107	0.1608	0.04901	195 / 230 / 260
000 (3/0)	0.4096	10.404	2.44	0.961	168	85	0.2028	0.0618	165 / 200 / 225
00 (2/0)	0.3648	9.266	2.74	1.08	133	67.4	0.2557	0.07793	145 / 175 / 195
0 (1/0)	0.3249	8.252	3.08	1.21	106	53.5	0.3224	0.09827	125 / 150 / 170
1	0.2893	7.348	3.46	1.36	83.7	42.4	0.4066	0.1239	110 / 130 / 150
2	0.2576	6.544	3.88	1.53	66.4	33.6	0.5127	0.1563	95 / 115 / 130
3	0.2294	5.827	4.36	1.72	52.6	26.7	0.6465	0.197	85 / 100 / 110
4	0.2043	5.189	4.89	1.93	41.7	21.2	0.8152	0.2485	70 / 85 / 95
5	0.1819	4.621	5.5	2.16	33.1	16.8	1.028	0.3133	
6	0.162	4.115	6.17	2.43	26.3	13.3	1.296	0.3951	55 / 65 / 75
7	0.1443	3.665	6.93	2.73	20.8	10.5	1.634	0.4982	
8	0.1285	3.264	7.78	3.06	16.5	8.37	2.061	0.6282	40 / 50 / 55
9	0.1144	2.906	8.74	3.44	13.1	6.63	2.599	0.7921	
10	0.1019	2.588	9.81	3.86	10.4	5.26	3.277	0.9989	30 / 35 / 40
11	0.0907	2.305	11	4.34	8.23	4.17	4.132	1.26	
12	0.0808	2.053	12.4	4.87	6.53	3.31	5.211	1.588	25 / 25 / 30 (20)
13	0.072	1.828	13.9	5.47	5.18	2.62	6.571	2.003	
14	0.0641	1.628	15.6	6.14	4.11	2.08	8.286	2.525	20 / 20 / 25 (15)
15	0.0571	1.45	17.5	6.9	3.26	1.65	10.45	3.184	
16	0.0508	1.291	19.7	7.75	2.58	1.31	13.17	4.016	— / — / 18 (10)
17	0.0453	1.15	22.1	8.7	2.05	1.04	16.61	5.064	
18	0.0403	1.024	24.8	9.77	1.62	0.823	20.95	6.385	— / — / 14 (7)
19	0.0359	0.912	27.9	11	1.29	0.653	26.42	8.051	
20	0.032	0.812	31.3	12.3	1.02	0.518	33.31	10.15	
21	0.0285	0.723	35.1	13.8	0.81	0.41	42	12.8	
22	0.0253	0.644	39.5	15.5	0.642	0.326	52.96	16.14	

How fast do the electrons move in Cu wire?

- Lets assume we have a current of ten amp
- In a 12 gauge wire (common in 20 amp wall outlet
- Area of 12 gauge wire ~ 3 mm^2 = 0.03 cm^2
- n ~ 10^{23} e/cm^3
- Charge of the electron e ~ 1.6×10^{-19} Coulomb
- $v_d = I/nAe = 10/(\ 10^{23} \times 0.03 \times 1.6 \times 10^{-19}\ ) \sim 1/30$ cm/s
- You walk MUCH faster than this!
- Why is the drift speed of electrons sooo slow?
- **Answer – the electron density is so high**
- Compare this speed to the speed of molecules in air at room temp ~ 300 m/s

Resistivity

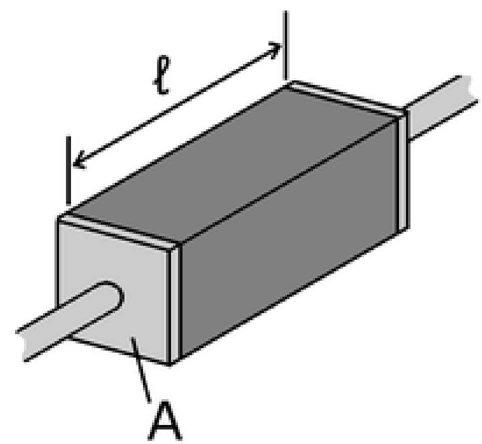
- Perfect metals have NO electric field inside them
- Real metal have a small E field when current flows
- Why?
- The reason is that real metal have a friction term then dissipates the kinetic energy (motion) of the electrons into heat
- This translates into a drag force
- To overcome this drag force an electric field is needed to keep the electrons moving
- **This effect is quantified by the resistivity of the metal**

Resistivity and Resistance

- A perfect metal has ZERO resistivity
- A perfect insulator has INFINITE resistivity
- ρ = resistivity (units are V-m/A = Ω-m)
- J = current density (Amps/m^2)
- R = resistance
- R = ρ L/A L= length

$$\rho = \frac{E}{J}$$

- Resistance is futile



Conductivity and Resistivity

- Conductivity σ is defined as 1/resistivity
- $\sigma = 1/\rho$
- Units of resistivity are ohm-meter (Ω-m)
- Units of conductivity are 1/ (Ω-m)
- Units of Conductivity are often given in SI units of siemens per metre ($S \cdot m^{-1}$)
- It is the same as 1/ (Ω-m)

Material	Resistivity (Ω·m) at 20 °C	Temperature coefficient* [K^{-1}]	
Silver	1.59×10^{-8}	0.0038	
Copper	1.72×10^{-8}	0.0039	
Gold	2.44×10^{-8}	0.0034	
Aluminium	2.82×10^{-8}	0.0039	
Calcium	3.36×10^{-8}		
Tungsten	5.60×10^{-8}	0.0045	
Zinc	5.90×10^{-8}	0.0037	
Nickel	6.99×10^{-8}		
Iron	1.0×10^{-7}	0.005	
Platinum	1.06×10^{-7}	0.00392	
Tin	1.09×10^{-7}	0.0045	
Lead	2.2×10^{-7}	0.0039	
Manganin	4.82×10^{-7}	0.000002	
Constantan	4.9×10^{-7}	0.000008	
Mercury	9.8×10^{-7}	0.0009	
Nichrome[6]	1.10×10^{-6}	0.0004	
Carbon[7]	3.5×10^{-5}	−0.0005	**Note these are negative**
Germanium[7]	4.6×10^{-1}	−0.048	“”
Silicon[7]	6.40×10^{2}	−0.075	“”
Glass	10^{10} to 10^{14}		
Hard rubber	approx. 10^{13}		
Sulfur	10^{15}		
Paraffin	10^{17}		
Quartz (fused)	7.5×10^{17}		
PET	10^{20}		
Teflon	10^{22} to 10^{24}		

Material	Electrical Conductivity (S·m-1)
Silver	63.0 × 10^6
Copper	59.6 × 10^6
Annealed Copper	58.0 × 10^6
Gold	45.2 × 10^6
Aluminium	37.8 × 10^6
Sea water	4.8
Drinking water	0.0005 to 0.05
Deionized water	5.5 × 10^-6
Jet A-1 Kerosene	50 to 450 × 10^-12
n-hexane	100 × 10^-12
Air	0.3 to 0.8 × 10^-14

Resistivity Temperature Dependence near Room Temp

- In the table above there is a temperature coefficient α
- Resistivity is often specified at 20 C
- 20 C = 293.15 K (near room temp)
- $\rho(T) = \rho\,(293.15\text{ K}) + \alpha\,(T-293.15)$
- where T is in Kelvin
- Metals have a POSITIVE α (positive temp coef)
- Semiconductors have a NEGATIVE α (neg temp coef)
- This is only valid near room temp as α is itself temperature dependent

Temperature Dependence

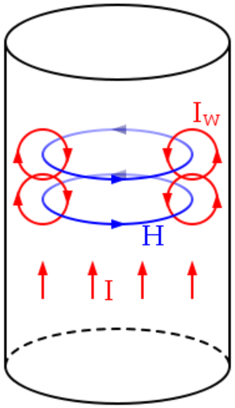
- In general metal resistances INCREASE with temperature
- In general semiconductor resistance DECREASE with T
- The effect is basically an interaction between the electrons and the phonons
- Phonons are mechanical vibration quanta
- Materials are stiff and hence vibrate
- Well described by Bloch–Grüneisen formula
- $\rho(0)$ is resistivity due to defect scattering, T = temp (K)
- Θ_R = Debye Temperature
- n=5 implies electrons are scattered by phonons (simple metal)
- n=3 implies s-d electron scattering – transition metals
- n=2 implies electron-electron scattering

$$\rho(T) = \rho(0) + A\left(\frac{T}{\Theta_R}\right)^n \int_0^{\frac{\Theta_R}{T}} \frac{x^n}{(e^x - 1)(1 - e^{-x})} dx$$

Skin Depth for AC Currents

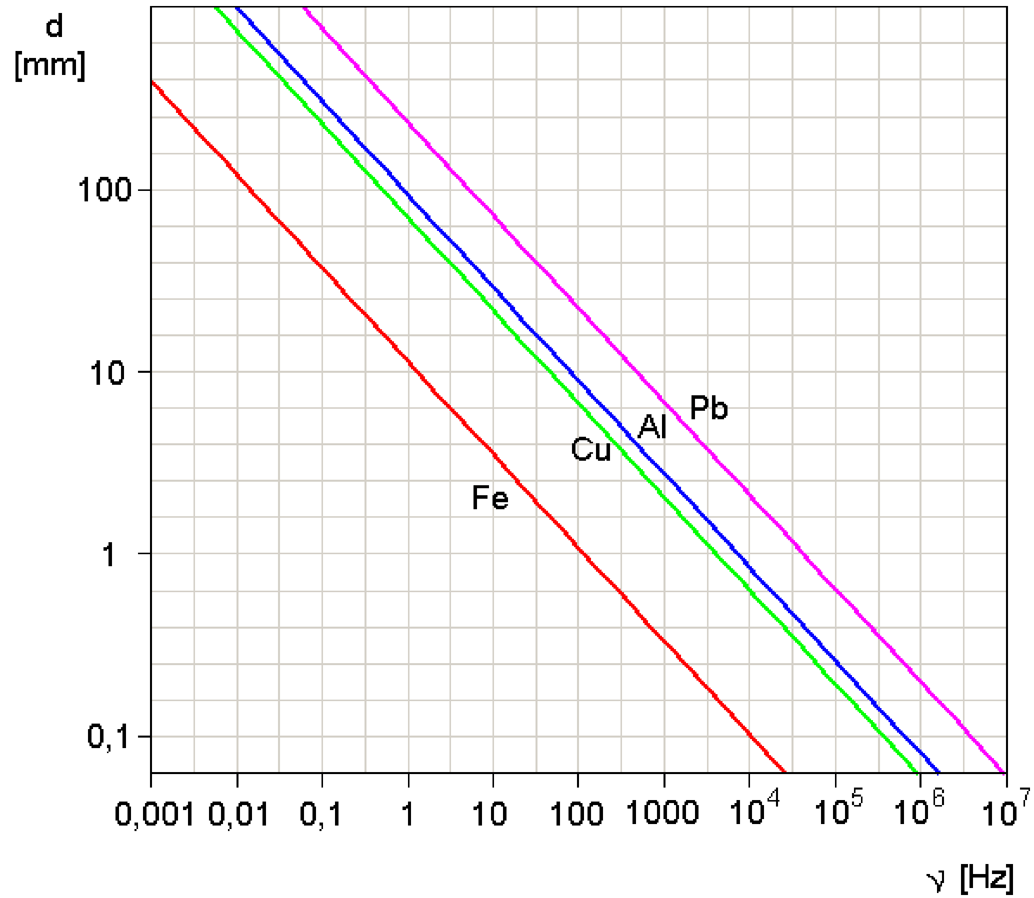
- For DC (direct current – constant current) there is no skin effect
- For AC (alternating current) the current exponetially decreases with depth into the metal
- The effect is called the "skin effect" as current stays on the "skin" of the conductor
- Horace Lamp 1883 first described it
- Eddy currents cancel E field in center of conductor
- For Copper at 60 Hz the "skin depth" is about 8.5 mm
- The current density J decreases exponentially $J = J_S e^{-d/\delta}$
- For "good conductors" like metals $\delta = \sqrt{\frac{2\rho}{\omega\mu}}$
- A wire of diameter D then really only is being used to a depth ~ δ
- The effective AC resistance of a wire of diameter D and length L is

$$R = \frac{\rho}{\delta}\left(\frac{L}{\pi(D-\delta)}\right) \approx \frac{\rho}{\delta}\left(\frac{L}{\pi D}\right)$$



Skin Depth Continued

- For Copper:
- 60 Hz δ ~ 8.5 mm
- 10 KHz δ ~ 0.66 mm
- 100 KHz δ ~ 0.22 mm
- 1 MHz δ ~ 0.066 mm
- 10 MHz δ ~ 0.021 mm



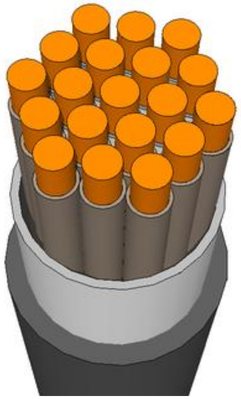
- Note – for Fe
- while the resistivity is low
- The magnetic permeabilty
- Is large and hence the skin depth
- Is smaller than Cu
- For high voltage 60 Hz power lines thin Aluminum over steel is used
- The Al for good conductivity – the center steel core for strength

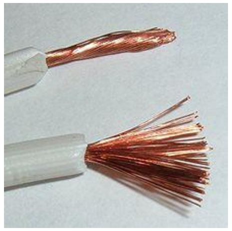
Skin Depth in your Life

- Your cell phone operates between 1 and 2 GHz depending on your service
- The skin depth for common metal in cell phones
- Is only a few microns – hence metalized plastic can work well
- For making mirrors for satellite TV (~ 10 GHz)
- The skin depth is less than 1 microns – hence metal coated plastic mirrors are fine
- Aluminum $\delta \sim$ 0.80 microns
- Copper $\delta \sim$ 0.65 microns
- Gold $\delta \sim$ 0.79microns
- Silver $\delta \sim$ 0.64 microns

Litz Wire and other Effects

- In AC cables the Skin effect and proximity effect can be severe problems that increase the effective resistance of the cable
- Solution is to use lots of small diam wires - insulated
- BUT due to interaction between wires (proximity effect) the wires are wound in patterns so that equal time is spent inside as outside the bundle
- Litz is one example – from German
- *Litzendraht* for wire bindle
- Not to be confused with normal stranded wire





Collision times in wires

- Lets try to calculate the time between collisions of the electrons in a wire
- How should we do this?
- Lets think about Linear Momentum P
- In a real wire there is a small E field
- Thus there is a force on the electrons F=eE
- If the mean time between collisions is τ the momentum gained between collisions is $P=F\tau$
- Or $P = eE\tau$
- But the momentum of the electrons is $P=mv_d$
- Equating we get $mv_d = eE\tau$
- Thus $\tau = mv_d / eE$ or $v_d = eE\tau/m = P/m$

Current Density and Mean Collision Time τ

- Recall we defined $\rho = E/J$
- Hence $J = E/\rho = \sigma E$
- Recall Current $I = nAv_d e$
- Thus $J = I/A = nv_d e$ but we just found
- Hence $J = n(eE\tau/m)e = e^2 nE\tau/m = (e^2 n\tau/m)E$
- But $J = \sigma E$
- Thus $\sigma = e^2 n\tau/m$
- And $\rho = m/n\tau e^2$

Power Dissipation in Wires and Resistors

- Real wires have a damping term
- This ultimately causes heat dissipation
- The power dissipated is Power = Force x speed
- Here F = eE and speed = v_d
- Hence P_e (power per electron) = eEv_d
- Total power = power per electron x number of electrons
- Number of electrons = nAL (L= length of wire/ resistor and A = cross sectional area)
- Hence $P_{T\,(total)} = nAL\,eEv_d$
- Recall $J = nev_d$
- $P_{T\,(total)}$ = J AL E = I L E but V (voltage) = LE and hence
- $P_{T\,(total)}$ = IV (current x voltage

"Ohms Law"

- "Ohms Law" is really a statement about the linear relationship between resistivity and E
- Recall we defined $\rho = E/J$
- In linear materials ρ does NOT depend on E or J
- In non linear material like semiconductors (diodes, transistors etc) this is NOT true
- In normal resistors linear is a good approx
- In linear materials:
- Recall for a resistor of length L and cross section A
- $R = \rho L/A = EL/AJ$ but $V = EL$ and $I = AJ$
- Hence $R = V/I$ or $V = IR$ (usually referred to as Ohms Law)
- Note from this $R = V/I$

Ohms Law and Power Dissipation

- For linear materials R does not depend on V or I
- $V = IR$
- $P_T = IV = I^2R = V^2/R$