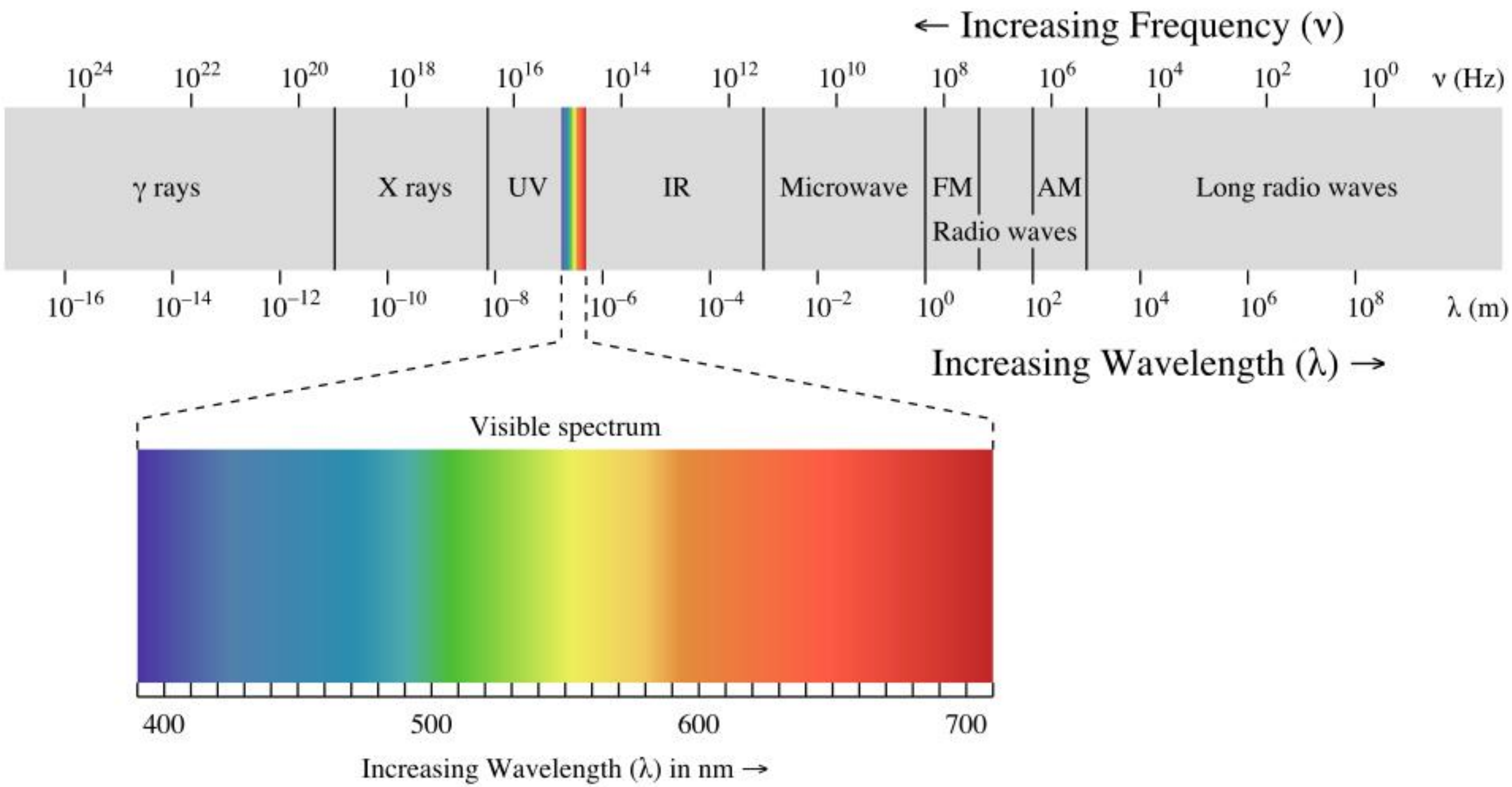
Chapter 32 – Electromagnetic Waves

Visible light, radio waves, cell communications, UV tanning, HDTV – these are all examples of EM waves



← Increasing Frequency (ν)
10^24 10^22 10^20 10^18 10^16 10^14 10^12 10^10 10^8 10^6 10^4 10^2 10^0 ν (Hz)
γ rays
X rays
UV
IR
Microwave
FM
AM
Radio waves
Long radio waves
10^−16 10^−14 10^−12 10^−10 10^−8 10^−6 10^−4 10^−2 10^0 10^2 10^4 10^6 10^8 λ (m)
Increasing Wavelength (λ) →
Visible spectrum
400 500 600 700
Increasing Wavelength (λ) in nm →

CLASS	FREQUENCY	WAVELENGTH	ENERGY
	300 EHz	1 pm	1.24 MeV
Y			
	30 EHz	10 pm	124 keV
HX			
	3 EHz	100 pm	12.4 keV
SX			
	300 PHz	1 nm	1.24 keV
	30 PHz	10 nm	124 eV
EUV			
	3 PHz	100 nm	12.4 eV
NUV			
	300 THz	1 µm	1.24 eV
NIR			
	30 THz	10 µm	124 meV
MIR			
	3 THz	100 µm	12.4 meV
FIR			
	300 GHz	1 mm	1.24 meV
EHF			
	30 GHz	1 cm	124 µeV
SHF			
	3 GHz	1 dm	12.4 µeV
UHF			
	300 MHz	1 m	1.24 µeV
VHF			
	30 MHz	10 m	124 neV
HF			
	3 MHz	100 m	12.4 neV
MF			
	300 kHz	1 km	1.24 neV
LF			
	30 kHz	10 km	124 peV
VLF			
	3 kHz	100 km	12.4 peV
VF/ULF			
	300 Hz	1 Mm	1.24 peV
SLF			
	30 Hz	10 Mm	124 feV
ELF			
	3 Hz	100 Mm	12.4 feV

Maxwells Equations in Vacuum
No charges or currents

$$\nabla \cdot \mathbf{E} = 0 \qquad (1)$$

$$\nabla \cdot \mathbf{B} = 0 \qquad (3)$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad (2)$$

$$\nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad (4)$$

.

Let there be Light – Vacuum Solution

A mathematical memory from vector calculus – no Physics

$$\nabla \times (\nabla \times \mathbf{A}) = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

Now add Physics

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t}\right) \qquad (5)$$

$$\nabla \cdot \mathbf{E} = 0 \qquad (1)$$

$$\nabla \times (\nabla \times \mathbf{E}) = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E} \qquad (6)$$

$$\nabla \times \left(-\frac{\partial \mathbf{B}}{\partial t}\right) = -\frac{\partial}{\partial t}(\nabla \times \mathbf{B}) = -\mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad (7)$$

Putting this all together we get the vacuum wave equations for E and B

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

Wave Equations

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \qquad \nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

This is similar to a scalar wave equation where c is the speed of the wave

$$\nabla^2 f = \frac{1}{{c_0}^2} \frac{\partial^2 f}{\partial t^2}$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Solutions to the Wave Equations

The following is a solution for ANY well behaved function f
K is called the wave vector – here it is a unit vector that points in the direction of wave propagation

$$\mathbf{E} = \mathbf{E}_0 f\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right)$$

$$\nabla \cdot \mathbf{E} = \hat{\mathbf{k}} \cdot \mathbf{E}_0 f'\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right) = 0 \qquad \mathbf{E} \cdot \hat{\mathbf{k}} = 0$$

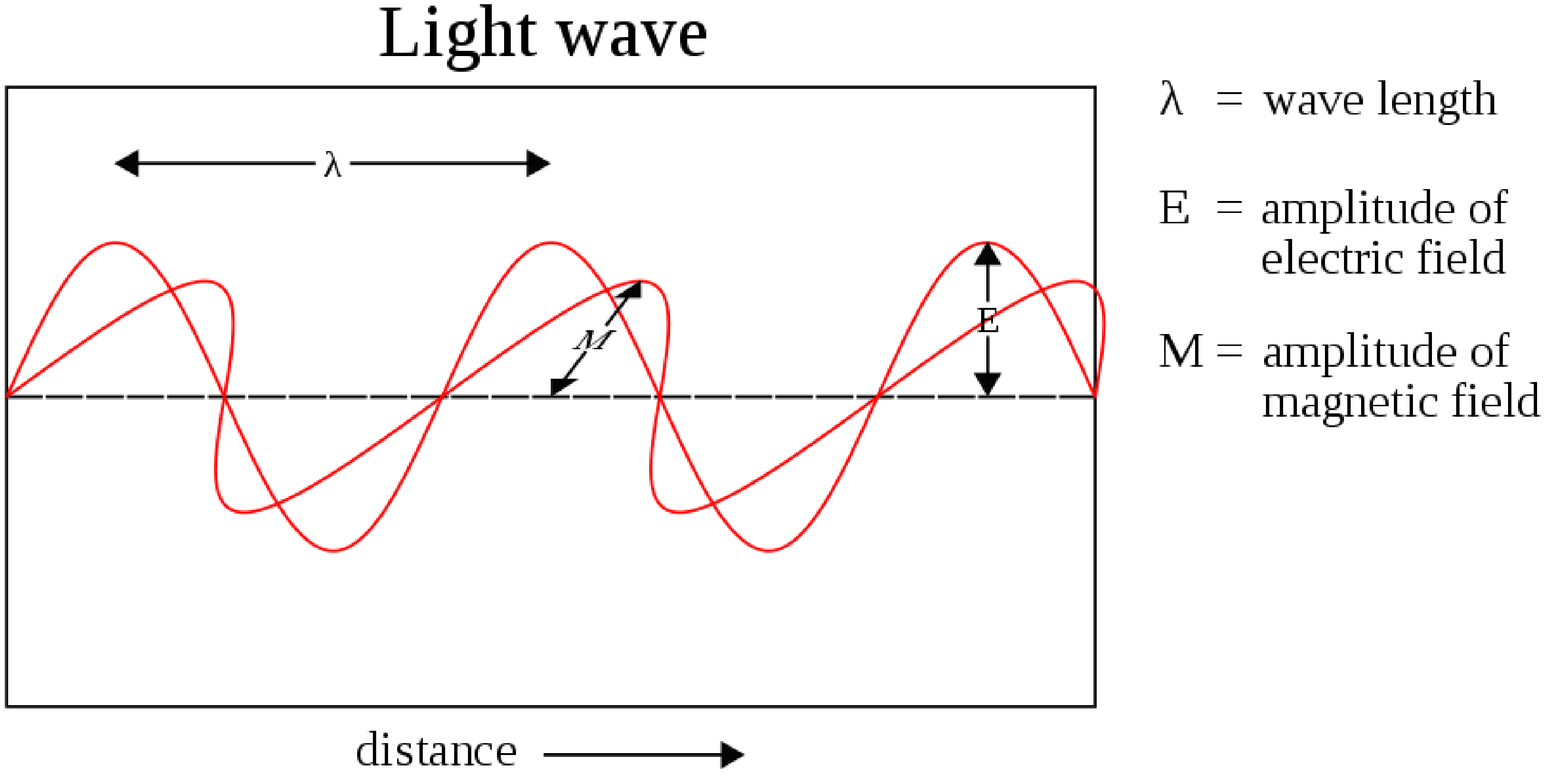
$$\nabla \times \mathbf{E} = \hat{\mathbf{k}} \times \mathbf{E}_0 f'\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right) = -\frac{\partial \mathrm{B}}{\partial t} \qquad \mathbf{B} = \frac{1}{c_0}\hat{\mathbf{k}} \times \mathbf{E}$$

E, B and K form a mutual orthogonal system

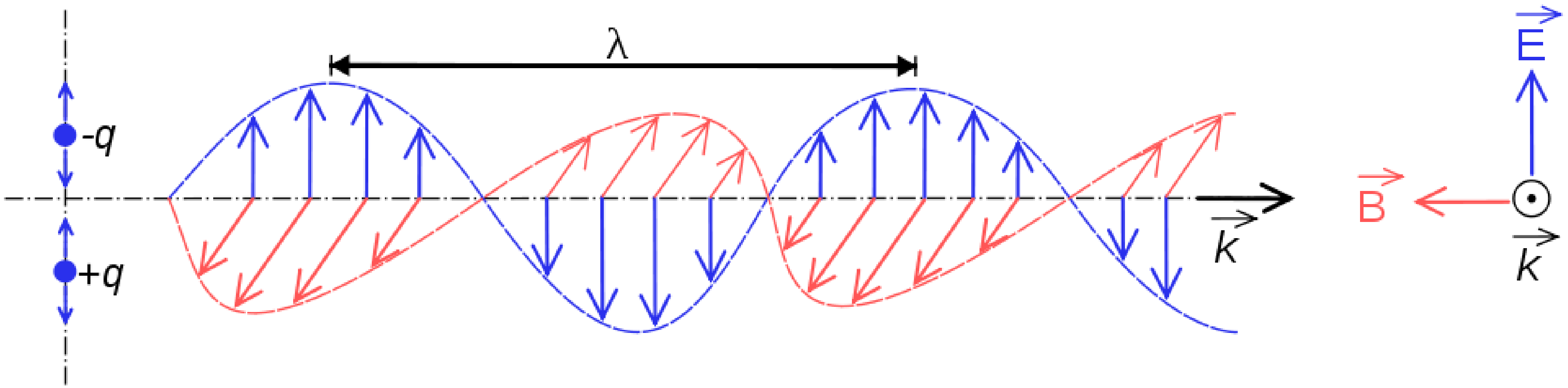
E, B (M) and wavelength

K = 2π/λ

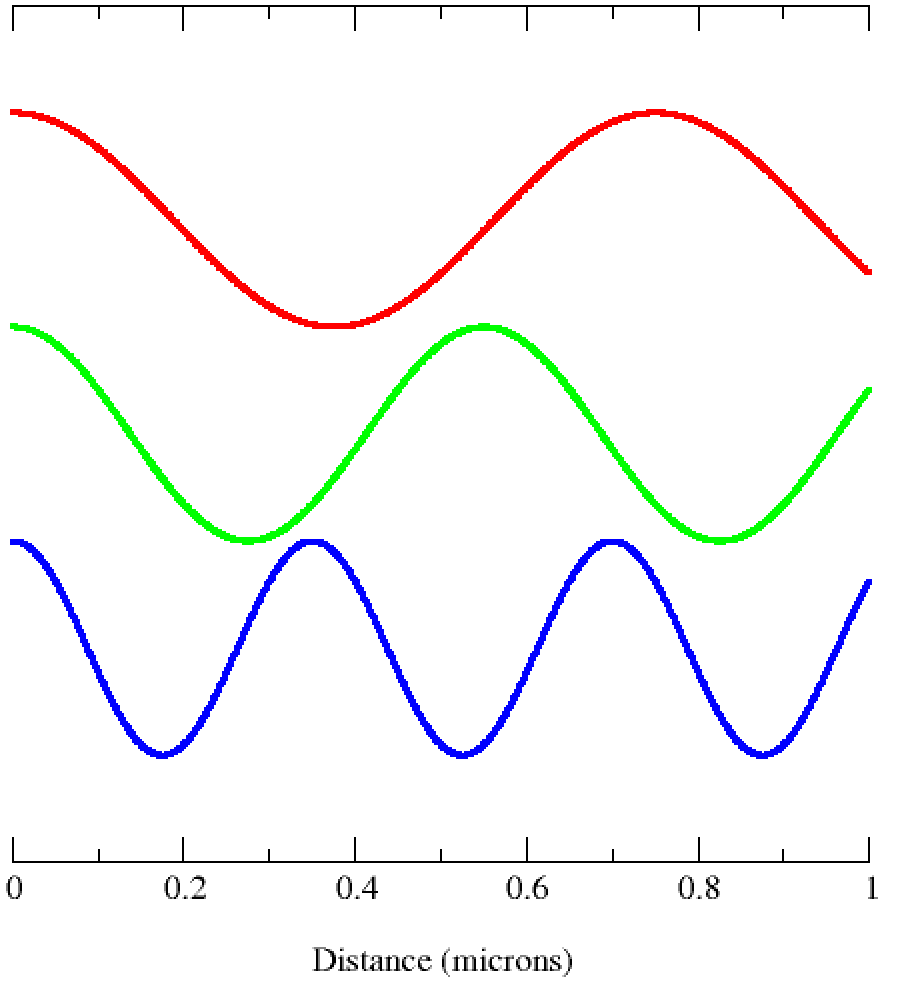
$$v = f\lambda$$



E, B and K – Right handed coordinate system



Visible light



Impedance of the Vacuum

Our modern view of the vacuum is that it is a sea of all things at negative energy.

Thus it is NOT NOTHING.

It has an impedance

$$Z_0 \stackrel{\text{def}}{=} \mu_0 c_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{\varepsilon_0 c_0}$$

$$\varepsilon_0 \stackrel{\text{def}}{=} \frac{1}{\mu_0 {c_0}^2} \approx 8.854\ 187\ 817\ldots \times 10^{-12}$$

$$Z_0 \approx 376.730\ 313\ 461\ 77\ldots \Omega$$

Electromagnetic Energy Density and Flux and Quanta

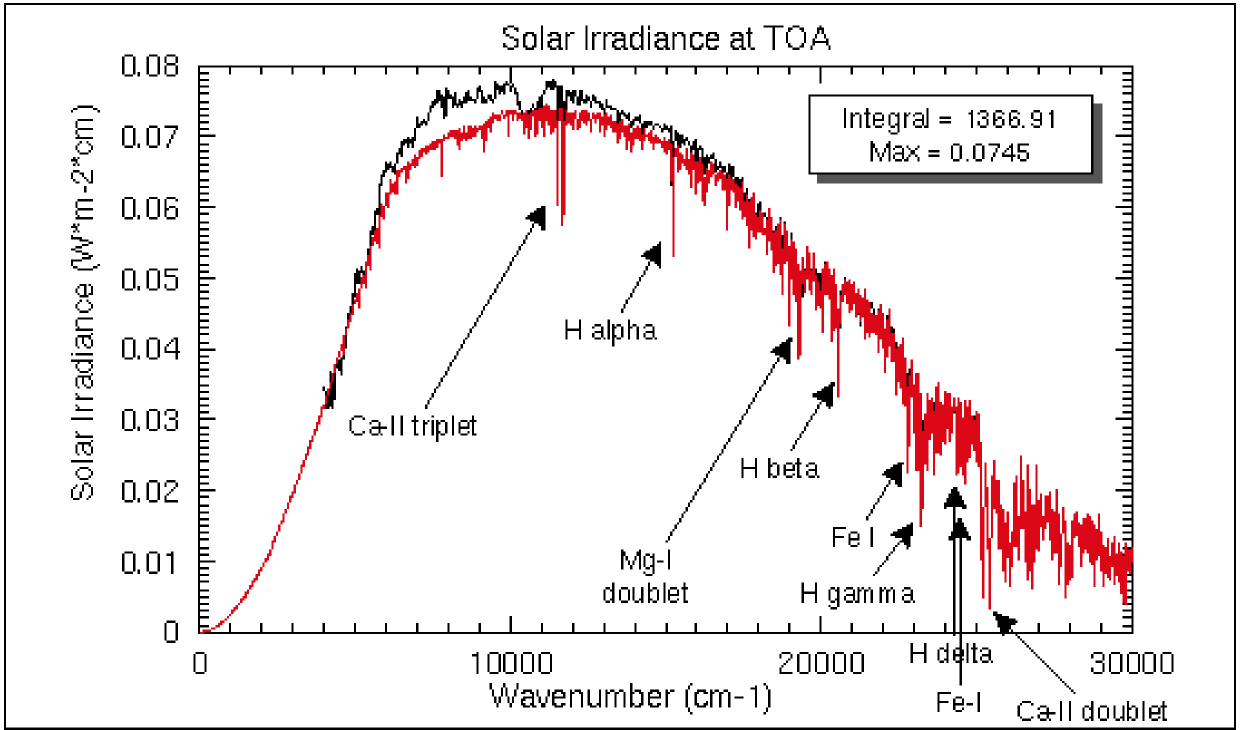
$$u_e = \frac{\epsilon_0}{2} E^2$$

$$u_m = \frac{1}{2\mu_0} B^2$$

$$\mathbf{S} = \frac{1}{\mu} \mathbf{E} \times \mathbf{B},$$

$$E = hf$$

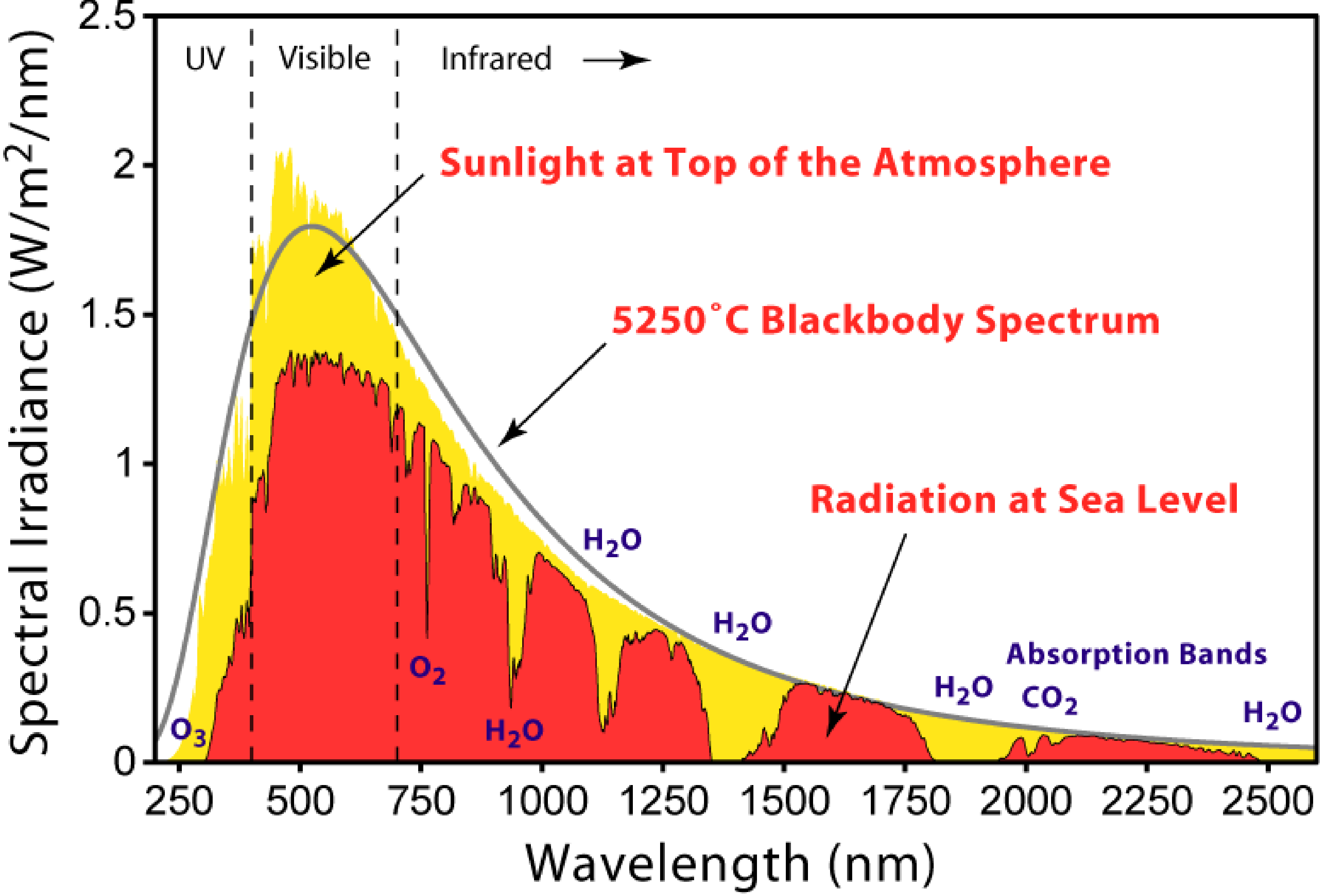
$$h = 6.626\ 068\ 96(33) \times 10^{-34}\ \mathrm{J\ s} = 4.135\ 667\ 33(10) \times 10^{-15}\ \mathrm{eV\ s}.$$

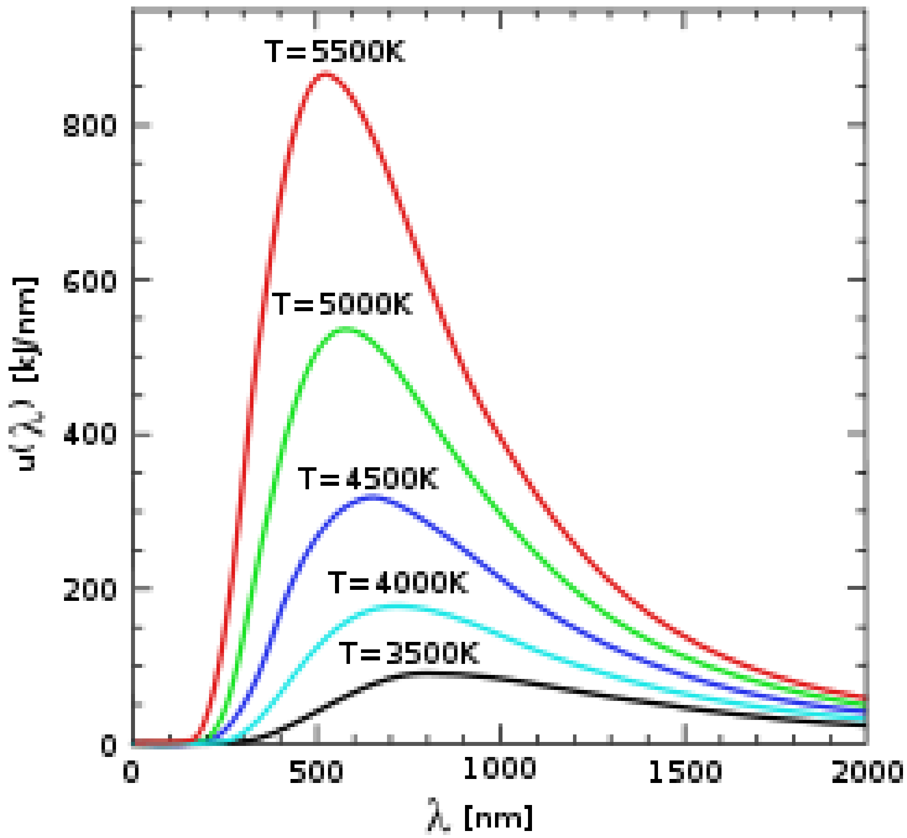


Ca-II triplet: 11545, 11707, 11767
H alpha: 15237
Mg-I doublet: 19292, 19332
H beta: 20571
Fe-I: 22812
H gamma: 23039
H delta: 24380
Fe-I: 24723
Ca-II doublet: 25202, 25426

Balmer Series, n = 2,3,4,5...
27427*(1 -- 4/n^2)
= 27430 * (5/9, 3/4, 21/25, 8/9)
= 15237,20570,23039,24380

Solar Radiation Spectrum





T=5500K
T=5000K
T=4500K
T=4000K
T=3500K
u(λ) [kJ/nm]
0
200
400
600
800
0
500
1000
1500
2000
λ [nm]

Dispersion in materials – polarization depends on frequency