PHYS 141 OPTICS HW 2

2.14 According to equation $\theta_c = \sin^{-1}\frac{n_2}{n_1}$

we can obtain the critical angle for internal reflection in

water: $\theta_c = \sin^{-1}\left(\frac{1}{1.33}\right) = 48.75°$

and diamond: $\theta_c = \sin^{-1}\left(\frac{1}{2.42}\right) = 24.41°$

2.15 According to equation $\theta_B = \tan^{-1}\frac{n_2}{n_1}$

we can obtain the Brewster angle for external reflection in

water: $\theta_B = \tan^{-1}\frac{1.33}{1} =$

and diamond: $\theta_B = \tan^{-1}\frac{2.42}{1} =$

2.16 water: $R_s = |r_s|^2 \quad R_p = |r_p|^2$

$$r_s = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{\frac{1}{\sqrt{2}} - \sqrt{1.33^2 - \left(\frac{1}{\sqrt{2}}\right)^2}}{\frac{1}{\sqrt{2}} + \sqrt{1.33^2 - \left(\frac{1}{\sqrt{2}}\right)^2}} = -0.23$$

$R_s = |r_s|^2 = 0.053$

$$r_p = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{-1.33^2 \cdot \frac{1}{\sqrt{2}} + \sqrt{1.33^2 - \frac{1}{2}}}{1.33^2 \cdot \frac{1}{\sqrt{2}} + \sqrt{1.33^2 - \frac{1}{2}}} = -0.05$$

$R_p = |r_p|^2 = 0.0027$

Diamond: $R_s = |r_s|^2 \quad R_p = |r_p|^2$

$$r_s = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{\frac{1}{\sqrt{2}} - \sqrt{2.42^2 - \left(\frac{1}{\sqrt{2}}\right)^2}}{\frac{1}{\sqrt{2}} + \sqrt{2.42^2 - \left(\frac{1}{\sqrt{2}}\right)^2}} = -0.53$$

$R_s = |r_s|^2 = 0.28$

$$r_p = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{-2.42^2 \cdot \frac{1}{\sqrt{2}} + \sqrt{2.42^2 - \frac{1}{2}}}{2.42^2 \cdot \frac{1}{\sqrt{2}} + \sqrt{2.42^2 - \frac{1}{2}}} = -0.28$$

$R_p = |r_p|^2 = 0.08$

2.18 Since the Mooney rhomb is used to produce circularly polarized light. and there are two TIRs within the rhomb, So the phase difference between TE and TM polarization in one TIR is $\Delta = \frac{\pi}{4}$.

It's easy to obtain that the apex angle A is equal to the incident angle of TIR, θ.

According to equation (2.72) in the book,

$$\tan\frac{\Delta}{2} = \frac{\cos\theta\sqrt{\sin^2\theta - n^2}}{\sin^2\theta}$$

where $\Delta = \frac{\pi}{4}$, $n = \frac{1}{1.65}$

we can get $\cos\theta = 0.5 \Rightarrow \theta = 60°$

2.19 (1) According to the equation (2.67) in the book, the amplitude of ~~an~~ evanescent wave is

$$E'' e^{-\alpha|y|}$$

Therefore, $\alpha|y| = 1 \Rightarrow |y| = \frac{1}{\alpha}$

since, $\alpha = k''\sqrt{\frac{\sin^2\theta}{n^2} - 1}$, and $\theta = 45°$ $n = \frac{1}{1.5}$

& $k'' = \frac{2\pi}{\lambda_0} = \frac{2\pi}{n\lambda}$, $\lambda = 500$ nm.

So, $\alpha = \frac{2\pi \times 1.5}{500} \times \sqrt{\frac{(\frac{1}{\sqrt{2}})^2}{(\frac{1}{1.5})^2} - 1} = 2.96 \times 10^{-3} (nm^{-1})$

$|y| = \frac{1}{\alpha} = 337.6$ nm

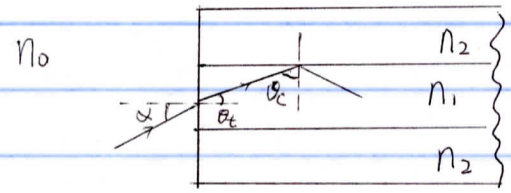
(2) $|y| = 1$ mm

$E_2 = E'' e^{-\alpha|y|} = E'' e^{-2.96\times10^{-3} \times 10^6} = E'' e^{-2960}$

$I_2 = \cancel{E''} |E_2|^2 = E''^2 e^{-5920}$

$$\frac{I_2}{I_1} = \frac{|E_2|^2}{|E_1|^2} = \frac{E''^2 e^{-5920}}{E''^2} = e^{-5920} \approx 10^{-2570}$$

2.20



When the incident angle on air-fiber boundary is equal to the acceptance angle α, the incident angle on core-cladding boundary equals to critical angle.

Based on the Snell's law,

$$n_0 \sin\alpha = n_1 \sin\theta_t = n_1 \cos\theta_c = n_1\sqrt{1-\sin^2\theta_c}$$

Since $\sin\theta_c = \frac{n_2}{n_1}$

So $\sin\alpha = n_1\sqrt{1-(\frac{n_2}{n_1})^2} = \sqrt{n_1^2 - n_2^2}$

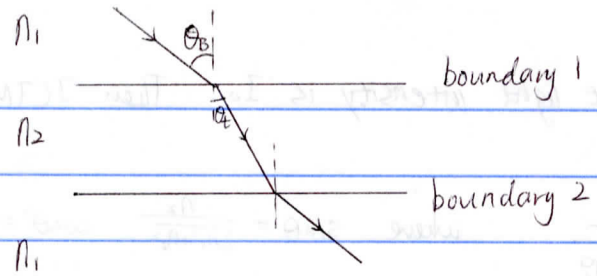
$$\alpha = \sin^{-1}\sqrt{n_1^2 - n_2^2}$$

2.21 $\tan\frac{\Delta}{2} = \tan[\frac{1}{2}(\delta_p - \delta_s)] = \tan(\beta - \alpha) = \frac{\tan\beta - \tan\alpha}{1 + \tan\beta \cdot \tan\alpha}$

$$\tan\beta = \frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}, \quad \tan\alpha = \frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}$$

$$\therefore \tan(\frac{\Delta}{2}) = \frac{\sqrt{\sin^2\theta - n^2}\left(\frac{1}{n^2\cos\theta} - \frac{1}{\cos\theta}\right)}{1 + \frac{\sin^2\theta - n^2}{n^2\cos^2\theta}} = \frac{\cos\theta\sqrt{\sin^2\theta - n^2}}{\sin^2\theta}$$

2.23 Before we calculate the degree of polarization, we need to prove that when a light beam is incident at Brewster's angle on a Brewster's window, the light beam on the second boundary is still incident at Brewster's angle.



Since $\tan\theta_B = n$, then $\sin\theta_B = \frac{n_2}{\sqrt{n_1^2+n_2^2}}$, $\cos\theta_B = \frac{1}{\sqrt{1+n^2}}$

where $n = \frac{n_2}{n_1}$

On boundary 1, according to Snell's Law: $n_1 \sin\theta_B = n_2 \sin\theta_t$

$$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_B = \frac{n_1}{n_2}\cdot\frac{n_2}{\sqrt{n_1^2+n_2^2}} = \frac{n_1}{\sqrt{n_1^2+n_2^2}}$$

and so $\tan\theta_t = \frac{n_1}{n_2}$, which means θ_t is Brewster's angle for $n_2 - n_1$ boundary. (boundary 2)

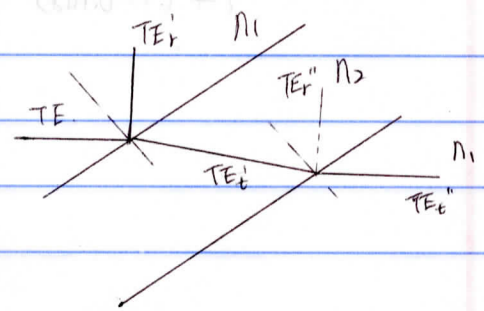
Now we can calculate the degree of polarization for Brewster Window.
According to the equation (2-27) in the book

$$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

The light is initially unpolarized, so it is 50% TE and 50% TM. When the light beam is incident at Brewster's angle, all TM light is transmitted, ~~but~~ however, part of TE light is reflected.

So, $P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$

$$= \frac{I(TM) - I(TE_t'')}{I(TM) + I(TE_t'')}$$



We assume the incident light intensity is I_0. Then $I(TM) = \frac{1}{2} I_0$.

$I(TE) = \frac{1}{2} I_0$.

$$r_{s1} = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} \quad \text{where} \quad \sin\theta = \frac{n_2}{\sqrt{n_1^2 + n_2^2}} \quad \cos\theta = \frac{1}{\sqrt{1+n^2}}$$

$$= \frac{\cos\theta - n\sqrt{1-\cos^2\theta}}{\cos\theta + n\sqrt{1-\cos^2\theta}} = \frac{\frac{1}{\sqrt{n^2+1}} - n\sqrt{\frac{n^2}{1+n^2}}}{\frac{1}{\sqrt{1+n^2}} + n\sqrt{\frac{n^2}{1+n^2}}} = \frac{1-n^2}{1+n^2}, \quad \text{where } n = \frac{n_2}{n_1}$$

$R_{s1} = |r_{s1}|^2$

$\therefore I(TE_t') = I(TE)(1 - R_{s1})$

Since on the second boundary, the incident angle is still Brewster's angle.

$$r_{s2} = \frac{1-n^2}{1+n^2} \quad \text{where} \quad n = \frac{n_1}{n_2}$$

$R_{s2} = |r_{s2}|^2 = R_{s1} = R$

$\therefore I(TE_t'') = I(TE_t')(1 - R_{s2}) = I(TE)(1-R_{s1})(1-R_{s2})$

$$\therefore P = \frac{I(TM) - I(TE_t'')}{I(TM) + I(TE_t'')} = \frac{1 - (1-R_{s1})(1-R_{s2})}{1 + (1-R_{s1})(1-R_{s2})} = \frac{1-(1-R)^2}{1+(1-R)^2}$$

For $n_2 = 1.5$ $\quad n_1 = 1.0$ (air)

$R_{s1} = 0.148 \quad R_{s2} = 0.148$

$$P = \frac{1-(1-0.148)^2}{1+(1-0.148)^2} = 0.159$$