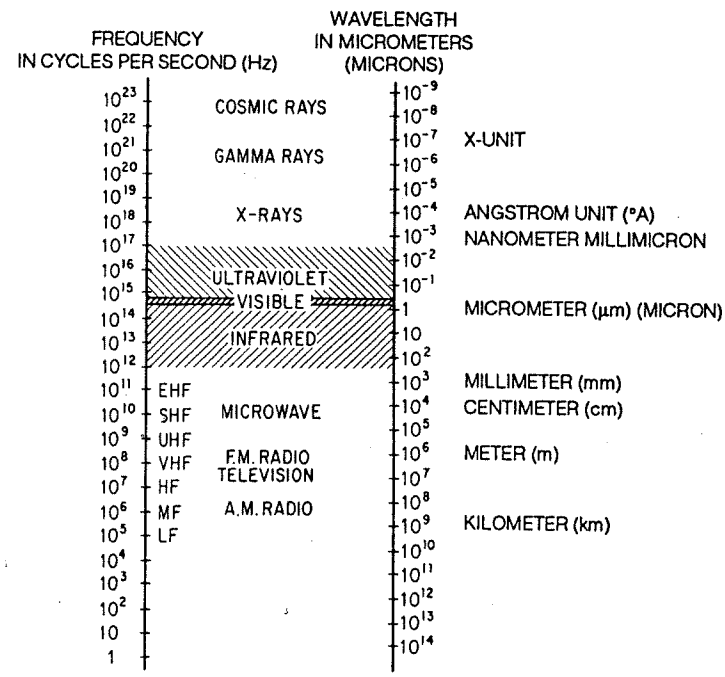


Figure 1.1 The electromagnetic spectrum.

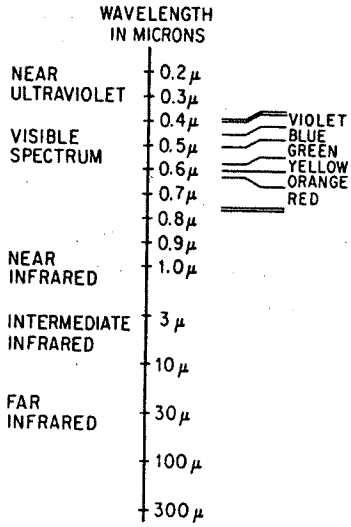


Figure 1.2 The "optical" portion of the electromagnetic spectrum.

Units

$\mu_0 = 4\pi \times 10^{-7}$ H/m

H – Henry
What is a Henry?

Recall $V = \underset{\text{Henry}}{L} \underset{\text{C/s}^2}{\frac{di}{dt}}$

Units Volts as J/C

Coulomb – Charge

$$H \equiv \frac{J/C}{C/s^2} = \frac{J\text{-}s^2}{C^2} = \frac{kg\ m^2}{C^2} = \text{Henry}$$

$\epsilon_0 \cong 8.854 \times 10^{-12}$ F/m

What's a farad?

$Q = \tilde{C} V$ $\quad \tilde{C}$ is cap in farad

Charge

$$\tilde{C} \equiv C/V = \frac{C}{J/C} = \frac{C^2}{J} = \frac{C^2 - s^2}{kg\text{-}m^2} = \text{Farr}$$

$\mu_0 \cong 4\pi \times 10^{-7}$ kg m C^{-2}
$\epsilon_0 \cong 8.854 \times 10^{-12}$ $C^2 s^2$ $kg^{-1} m^{-3}$

Waves

Read Supplementary topic VI

Coulomb/Gauss	$\nabla \cdot \bar{E} = \rho/\epsilon_0$	$\oint \bar{E} \cdot d\bar{\sigma} = Q/\epsilon_0$
"	$\nabla \cdot \bar{B} = 0$	$\oint \bar{B} \cdot d\bar{\sigma} = 0$
Faraday	$\nabla \times \bar{E} = -\partial \bar{B}/\partial t$	$\oint \bar{E} \cdot d\bar{l} = -\frac{d\Phi_B}{dt}$
Ampere	$\nabla \times \bar{B} = \mu_0(\bar{J} + \bar{J}_D)$	$\oint \bar{B} \cdot d\bar{l} = \mu_0(i + i_D)$
	$\bar{J}_D = \epsilon_0 \, \partial \bar{E}/\partial t$	$i_D = \epsilon_0 \frac{d}{dt} \Phi_E$

(Displacement)

Changing $E \rightarrow B$ Faraday

Changing $B \rightarrow E$ Ampere + Maxwell

$\mu_0 = 4\pi \times 10^{-7}$ H/m $\quad \epsilon_0 \cong 8.854 \times 10^{-12}$ F/m

kg m C^{-2} $\quad$ C^2 s^2 kg^{-1} m^{-3}

Non linear phenomenon

Imagine an E field in space caused by some charges now remove charges $E \rightarrow 0$ but as $E \rightarrow 0$ B becomes non zero as $B \rightarrow E \neq 0$ so one beget the other in a sort of dance ~~out of sync~~.

Wave equation — optional

$\nabla \times E = -\partial B/\partial t$

$\nabla \times B = \mu_0 \epsilon_0 \, \partial E/\partial t$ in free space

$$\nabla \times (\nabla \times B) = \nabla(\underbrace{\nabla \cdot \bar{B}}_{=0}) - \nabla^2 B = \mu_0 \epsilon_0 \frac{\partial}{\partial t}(\nabla \times E)$$

$$= -\mu_0 \epsilon_0 \frac{\partial^2 \bar{B}}{\partial t^2} = -\nabla^2 B$$

$$\nabla^2 \bar{B} - \mu_0 \epsilon_0 \frac{\partial^2 \bar{B}}{\partial t^2} = 0$$

also $\nabla^2 \bar{E} - \mu_0 \epsilon_0 \frac{\partial^2 \bar{E}}{\partial t^2} = 0$

homogeneous wave eq

$\rightarrow$ propogating waves — speed $v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$

General solution $f(\bar{k}\cdot\bar{x} \pm \omega t)$

$$\bar{E}(x,t) = \bar{E}_0 \sin(\bar{k}\cdot\bar{x} - \omega t) = \bar{E}_0 \sin\left(2\pi\left(\frac{\bar{x}\cdot\hat{k}}{\lambda} - \right.\right.$$

$$\bar{B}(x,t) = \bar{B}_0 \sin(\bar{k}\cdot\bar{x} - \omega t)$$

$$k^2 = \mu_0 \epsilon_0 \omega^2 \qquad k \equiv \frac{2\pi}{\lambda} \qquad \omega = 2\pi\nu$$

1) Velocity of wave $= \frac{\omega}{k} = \sqrt{\frac{1}{\mu_0\epsilon_0}} \equiv \lambda\nu$

$$= 3.0 \times 10^8 \text{ m/sec} = \underline{\underline{c}} \checkmark$$

$\rightarrow$ 1 ft/nanosecond $\rightarrow$ 1 GHz $\sim$ 1 ft (30 cm)

$$\therefore \quad c = \frac{1}{\sqrt{\mu_0\epsilon_0}} = \underline{\underline{\lambda\nu}}$$

$\omega = ck$ – Dispersion relation

2) Direction of wave propagation is $\bar{k}$

$\nabla\cdot E = 0 \qquad \nabla\cdot B = 0$ in free space

$$\rightarrow \quad \bar{E}_0\cdot\bar{k} = 0 \qquad \bar{B}_0\cdot\bar{k} = 0$$

$$\rightarrow \quad \bar{k} \propto \bar{E}\times\bar{B}$$

$\rightarrow$ $\therefore$ Waves transverse $\bar{E}, \bar{B} \perp \bar{k}$

Can also show $\bar{E} \perp \bar{B}$

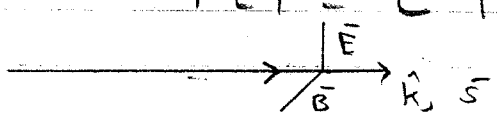
III $\rightarrow$ $\bar{k}\times\bar{E}_0 - \omega\bar{B}_0 = 0$

$$\rightarrow \quad \bar{B}_0 = \frac{k}{\omega}(\hat{k}\times\bar{E}_0) = \frac{1}{c}(\hat{k}\times\bar{E}_0)$$

$(\omega = ck)$

$\therefore$ $B \perp E$ and

$|\bar{E}| = c\,|\bar{B}|$ also see Book pg 907-909



E, B : <u>in phase</u>

1 T $\rightarrow$ 3×10^8 V/m $\qquad$ 1 g $\rightarrow$ 3×10^4 V/m

Examples of E-M radiation

1)	10 - 20 KHz	submarine communication
2)	540 - 1600 KHz	AM
3)	2 - 20 MHz	SW radio
4)	27 - 28 MHz	CB radio
5)	49 MHz	New cordless telephones
6)	54 MHz —	TV Ch 2
7)	88 - 108 MHz	FM
8)	900 MHz	top of TV UHF
9)	1420 MHz	21 cm neutral H
10)	2 GHz	Microwave ovens
11)	2.154 GHz	HBO
12)	3.7 - 4.2	Direct broadcast satellite T
13)	30 - 300 GHz	CBR
14)	115 GHz	CO emission
15)	$10^{14} - 10^{15}$ Hz	Visible light

Poynting Vector (Vacuum - free space

Let "Poynts" along the direction of propogation

$$\nabla \cdot (\bar{E} \times \bar{B}) = \bar{B} \cdot (\nabla \times \bar{E}) - \bar{E} \cdot (\nabla \times \bar{B})$$

$$= \bar{B} \cdot \left(- \frac{\partial \bar{B}}{\partial t} \right) - \bar{E} \cdot \left(\mu_0 \epsilon_0 \frac{\partial \bar{E}}{\partial t} \right)$$ free space $J = 0$

$$= -\mu_0 \frac{\partial}{\partial t} \left(\frac{1}{2\mu_0} B^2 + \frac{1}{2} \epsilon_0 E^2 \right)$$

magnetic field energy density	electric field energy density
Pg 804	Pg 804
ρ_m	ρ_E

Let $\bar{S} = \frac{1}{\mu_0} (\bar{E} \times \bar{B})$ = Poynting Vector

$$\oint \bar{S} \cdot d\bar{\sigma} = -\frac{\partial}{\partial t} \int (\rho_m + \rho_E) \, dv = \int \nabla \cdot \bar{S} \, dv$$

Then $\nabla \cdot \bar{S} + \frac{\partial}{\partial t} (\rho_m + \rho_E) = 0$

Conservation law of energy

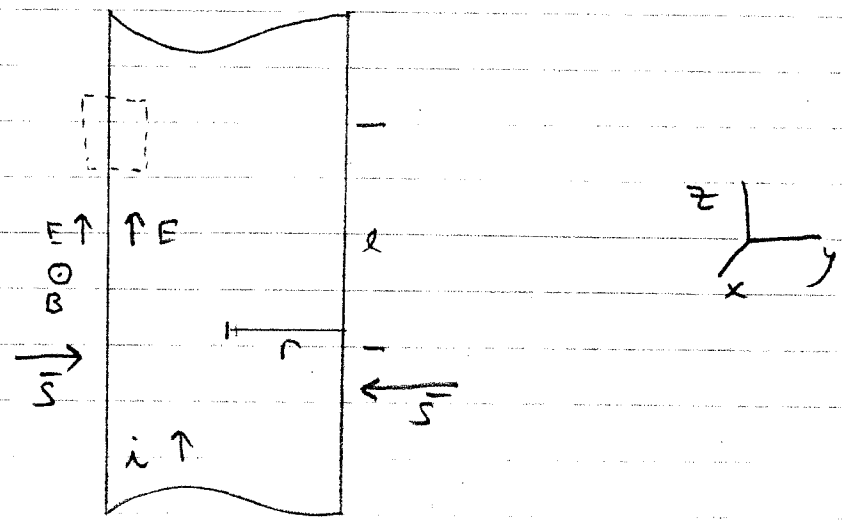
Recall Conservation of charge
$\nabla \cdot \bar{J} + \partial \rho / \partial t = 0$

$\bar{S}$ is flux of power from electromagnetic wave

$\bar{S}$ – watts / m^2

Ex Prob 31 Chap 41

Show Poynting vector S points into wire carrying current i and equals rate of joule heating.
Radius r, resistivity ρ



1) In length l $V = iR$ $R = \rho l / A$

$A = \pi r^2$

$E = V/l = \frac{iR}{l}$

$\oint E \cdot dl = -\frac{d\Phi_B}{dt} = 0$ — Static case

$\rightarrow$ E tangential is constant

By symmetry $\bar{E}$ is along direction of current flow ie along wire $\hat{z}$

E just inside wire = E just outside

$\bar{S} = \frac{1}{\mu_0} \bar{E} \times \bar{B}$ points radially into wire

$\oint \bar{B} \cdot d\bar{l} = \mu_0 i \rightarrow B = \frac{\mu_0 i}{2\pi r}$

$|\bar{S}| = \frac{1}{\mu_0} \underbrace{\left(\frac{iR}{l}\right)}_{E} \underbrace{\left(\frac{\mu_0 i}{2\pi r}\right)}_{B} = \frac{i^2 R}{2\pi r l}$

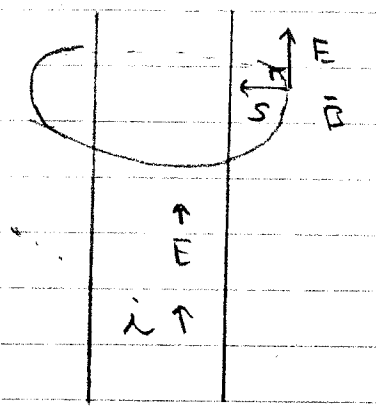
$\therefore$ Net flux in length l $= \int_{\Sigma} \bar{S} \cdot d\bar{\sigma}$

$= S\, 2\pi r l = i^2 R$

$\therefore$ Net flux due to Poynting vector = energy loss due to joule heating

Some Examples of Strange Poynting Vectors

a) Conductor with non zero resistance



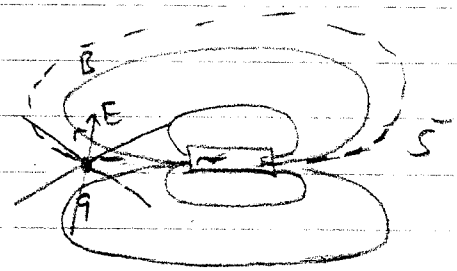
$E_{\parallel}$ is continuous if no $\partial B/\partial t$

$\therefore$ E outside = E inside

$\therefore$ $\bar{S}$ radially into wire

Can show $\int \bar{S} \cdot d\bar{a}$ over wire gives power = joule heating

b) Stationary charge & magnet



What does $\bar{S}$ mean?

It does <u>not</u> necessarily mean a net flow of useable energy

Examples

a) Fields from Sun
b) Fields from Room light

a) Sun

Solar constant $= 2.0$ cal/sec-cm^2
$= 1400$ W/m^2 at top of atmosphere

$$\bar{S} = \frac{1}{\mu_0}(\bar{E} \times \bar{B})$$

$$E = Bc \qquad \rho_E = \frac{1}{2}\epsilon_0 E^2 = \frac{1}{2}\epsilon_0 c^2 B^2 = \frac{1}{2}\frac{\epsilon_0}{c\mu_0}B^2$$

$$E = E_0 \cos \omega t \qquad = \frac{1}{2c\mu_0}B^2 = \rho_m \qquad \langle \rho_E \rangle = \langle \rho_m \rangle$$

$$S = \frac{1}{c\mu_0}E^2 \qquad S(t) = \frac{1}{c\mu_0}(E_0 \cos \omega t)^2$$

$$\langle \rho_E \rangle = \langle \tfrac{1}{2}\epsilon_0 E_0^2 \cos^2 \omega t \rangle = \tfrac{1}{4}\epsilon_0 E_0^2$$

$$\langle S \rangle = \frac{E_0^2}{2c\mu_0} = c\,\frac{\epsilon_0 E_0^2}{2} = c\,2\langle \rho_E \rangle = c\,\rho_E \checkmark$$

recall $c^2 = \dfrac{1}{\epsilon_0 \mu_0}$ $\qquad = c\langle \rho_E + \rho_m \rangle$

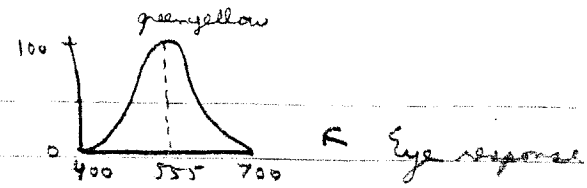
$$\langle S \rangle = 1400 \text{ W/m}^2 = c\,\frac{\epsilon_0 E^2}{2}$$

$$E_0 = \left(\frac{2\langle S \rangle}{c\,\epsilon_0}\right)^{1/2}$$

$$E_0 = 1.0 \times 10^3 \text{ V/m}$$
$$B_0 = E_0/c = 3.4 \times 10^{-6} \text{ T}$$
$$= 3.4 \times 10^{-2} \text{ g} = \underline{\underline{0.03 \text{ g}}}$$

Why aren't you shocked?
Why does metal get hot in sunlight

b) Room light



Fluorescent light tubes 8' each
~ 100 w

1 W @ 555 nm = 683 lumen

Fluorescent	70 lumens / W	20%
100% eff ave over eye	350 lumens / W	100%
Incandescent	17 "	5%

Fluorescent CW light

	53%	UV
	44%	heat
	3%	direct visible
of 53% UV phosphor {	18%	visible
	35%	heat

Typical lighting 20 – 50 w/m²
of electrical load → 4 – 10 w/m²
of light for fluorescent

say $\langle S \rangle = 4\ w/m^2$

$$E_0 = \left(\frac{2\langle S\rangle}{c\,\epsilon_0}\right)^{1/2} \sim 50\ V/m$$

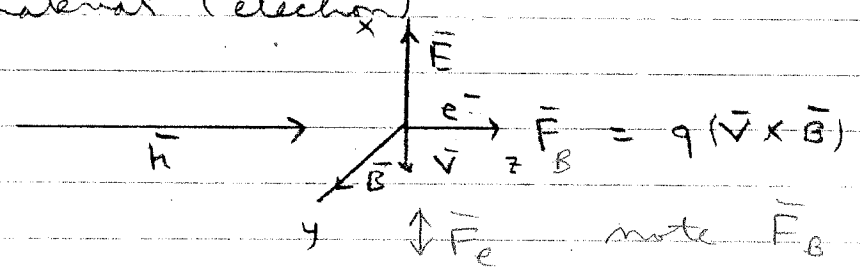
$$= \left(\frac{2 \times 4}{3\times10^8 \times 8.85\times10^{-12}}\right)^{1/2}$$

Radiation Pressure

Poynting vector

$$\bar{S} = \frac{1}{\mu_0}(\bar{E}\times\bar{B})$$ flux of radiation
watts / m^2

Why radiation pressure?
Look at interaction of light with material (electron)



note $\bar{F}_B$ always →

∴ $\bar{F}_B$ force on electron in direction of propogation $\bar{k}$
(also force on proton is in same direction as $\bar{k}$) → radiation pressure

electric $F_E = eE = \frac{m}{\tau}v_d = b\,v_d$ τ – damping time
$b = \frac{m}{\tau}$ in book

v_d – drift velocity

$F_e = b\,v_d$ ⇐ drag force

$$v_d = \frac{\tau}{m}eE$$

magnetic $F_B = q|(\bar{v}\times\bar{B})| = e\,v_d\,B = \frac{\tau}{m}e^2EB$

$$E = Bc$$

$$F_B = \frac{\tau e^2}{mc}E^2$$

Work done per unit time on electron $= F_E\,v_d = \frac{dV}{dt}$
(recall $F_B \perp v_d$ so work by $F_B \sim 0$)

Radiation press cont

$$\frac{dU}{dt} = F_E V_d = (eE)\left(\frac{\tau}{m} e E\right) = \frac{\tau}{m} e^2 E^2$$

$\simeq$ U – energy absorbed by e^-

$$F_B = \frac{1}{c}\frac{\tau}{m} e^2 E^2 = \frac{dp}{dt}$$ p – momentum transferred to e^- along $\hat{k}$

Comparing $\rightarrow$ $p = \frac{U}{c}$ (p – momentum

$\rightarrow$ $p = \frac{U}{c}$ for complete abs

$p = 2\frac{U}{c}$ for complete refl

p – momentum U – energy incident

Factor of 2 from momentum conservation just like a tennis ball hitting a bowling ball.

Pressure $P = \frac{|\bar{S}|}{c}$ absorbed

$P = 2\frac{|\bar{S}|}{c}$ reflection

P – N/m^2 – pressure

U – Energy incident

momen $p = \frac{U}{c} = \frac{SA\tau}{c}$ $F = \frac{dp}{dt} = \frac{SA}{c}$

pressure $P = \frac{F}{A} = \frac{S}{c}$ ✓

Also $S = uc$ u – energy density

$\therefore$ $P = u = |\bar{S}|/c$

Note units of press $\equiv J/m^3$

Ex Prob 42-4

Prove Pressure = energy density outside surface (partial reflection)

let f be reflected fraction

1) Absorbed part $(1-f)$

$$P_a = \frac{(1-f)S}{c}$$

2) Reflected part (f)

$$P_r = \frac{2fS}{c}$$

$$P_T = P_a + P_r = \frac{(1+f)S}{c}$$

Energy density in incident wave $u_i = S/c$

Reflected energy density $u_r = fS/c$

$$u_T = u_i + u_r = \frac{(1+f)S}{c}$$

$\therefore \quad P_T = u_T$

Ex Radiation Pressure from sunlight

$$P = \frac{|S|}{c}$$

$$|S| = 1400 \ W/m^2$$

$$P = 4.7 \times 10^{-6} \ N/m^2 = \underline{\underline{6.9 \times 10^{-10} \ psi}}$$

recall 1 atm = 14.7 psi = 10^5 N/m^2
1 $N/m^2 \equiv$ 1 PASCAL

Force of sunlight over entire earth

$$F = PA = 4.7 \times 10^{-6} \left(\pi \times \left(6 \cancel{000}\ \times 10^9\right)^2 \right)$$

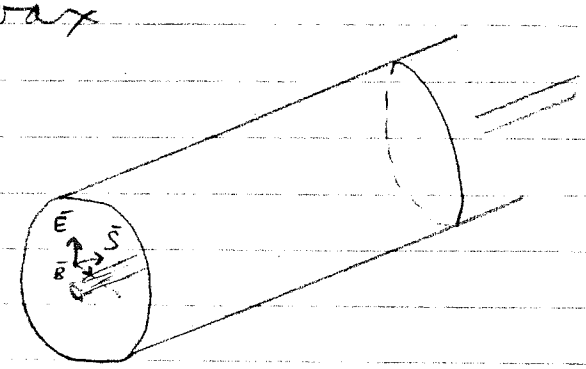
$r_e \sim 6000$ Km $= 6 \times 10^9$ m

$$F = 5.3 \times 10^{14} \ N = \cancel{118} \ \cancel{\text{pounds}} \ 1.2 \times 10^{14} \text{ lbs} = 6 \times 10^{10} \text{ T}$$

(recall $F = mg$ $g \cong 9.8 \ m/s^2$ 2.2 "lb" $\cong$ 1 Kg
2.2 "lb" = 9.8 N 1 "lb" = 4.5 N

Waveguides

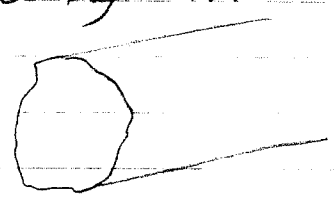
1) Coax



TEM wave $\bar{S}$ along conductor
"Transverse Electromagnetic"

2) Waveguide - hollow no inner conductor
TE or TM mode no TEM mode
limiting case of TEM mode
when wavelength << size of guide

Why no TEM ?

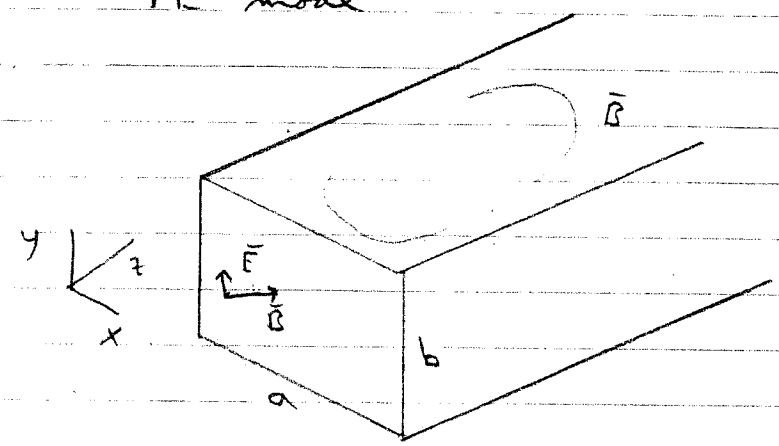


$E = 0$ in conductor $\rightarrow$ V = constant
in conductor $\rightarrow$ V = constant
inside (Since V is harmonic $\nabla^2 V = 0$
See Buck Adv. Calculus p. 441)
$\rightarrow$ $E = 0$ inside $\rightarrow$ $\frac{\partial \bar{B}}{\partial t} = 0$ $\left(\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}\right)$

$\rightarrow$ <u>no wave</u>

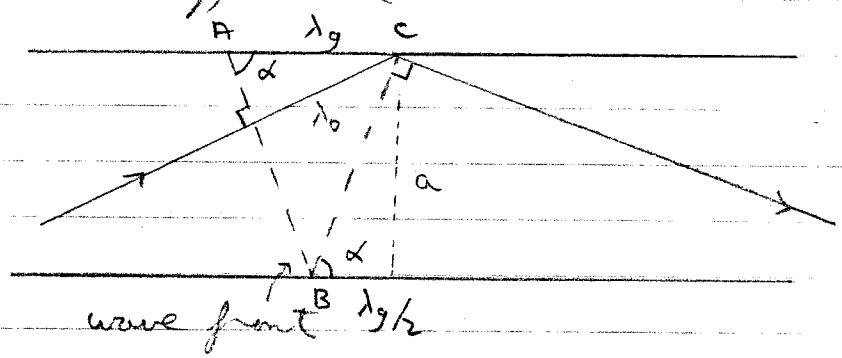
$\therefore$ No TEM wave in hollow waveguide

3) Rectangular waveguide
TE mode



$\bar{E}$ along y
$\bar{B}$ in plane in xz but <u>not</u> x or z
ie some component of $\bar{B}$ along z
$\rightarrow$ $\bar{S}$ not purely $\hat{z}$ but some $\hat{x}$

Reflections off wall



$$\tan\alpha = a/\lambda_g/2 = \frac{2a}{\lambda_g}$$

$$\sin\alpha = \frac{\lambda_0}{\lambda_g} \qquad \cos\alpha = \frac{\sin\alpha}{\tan\alpha} = \frac{\lambda_0}{2a}$$

$$\sin\alpha = \frac{\lambda_0}{\lambda_g} = \sqrt{1-\cos^2\alpha} = \sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}$$

$$\lambda_g = \frac{\lambda_0}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}} > \lambda_0$$

1) λ_0 must be $< 2a$ = high pass filter
2) $v_{phase} = c/\sin\alpha = \frac{c}{\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}} > c$
3) $v_{group} = c\sin\alpha = c\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2} < c$
as $\lambda_0 \rightarrow 2b$ $\lambda_g \rightarrow \infty$, $v_p \rightarrow \infty$ $v_g \rightarrow 0$

Cylindrical Cavity

Consider a circular capacitor at high freq.

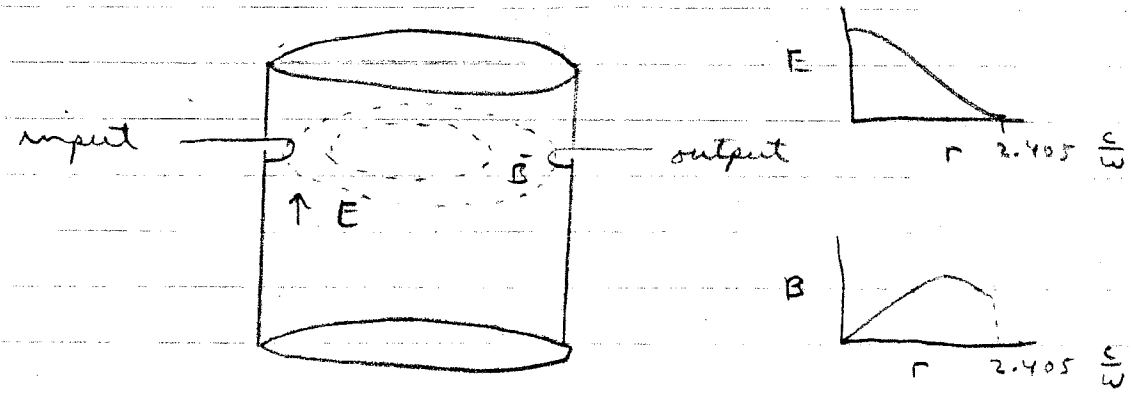
$$\oint \bar{E}\cdot d\bar{l} = -\frac{d\Phi_B}{dt}$$
$$\oint \bar{B}\cdot d\bar{l} = \mu_0 i + \mu_0\epsilon_0 \frac{d\Phi_E}{dt}$$

Changing $E \rightarrow$ Changing $B \rightarrow$ Changing E^*
$\rightarrow$ Changing B^* - - -
E^*, B^* opposite to E, B

Larger $\omega \rightarrow$ larger E^*, B^*

$E_{total} = E + E^* + \ldots$
$B_{total} = B + B^* + \ldots$

Maybe $E = 0$ at some r
In fact $E = E_0 \sin\omega t \; J_0\left(\frac{\omega r}{c}\right)$
J_0 – Bessel function
$J_0 = 0$ for $\frac{\omega r}{c} \sim 2.405$ first
$B = B_0 \cos\omega t \; J_1\left(\frac{\omega r}{c}\right)$
To enclose capacitor with side walls
at $\frac{\omega r}{c} = 2.405$



Resonant cavity
$B \rightarrow 0$ at walls because of currents in walls

Ex

$\nu = 500$ MHz resonant cavity

$\frac{\omega r}{c} = 2.405$ for resonance

$\omega = 2\pi\nu$

$\rightarrow r \sim 23$ cm (9 in)

Least action

1) $S = \int_{t_1}^{t_2} (T - U)\, dt$ Feynman Vol 1 Pg 19-5

$= \int_{t_1}^{t_2} L\, dt$ Theoretical Physics Pg 24

$\delta S = 0$

2) $\delta \int_a^b p \cdot dr = 0$ Theoretical Physics Pg 27

$\parallel$

$\delta \int_a^b \sqrt{\mathcal{E} - U(r)}\, dl = 0$ Pg 27

$\mathcal{E}$ - total energy

Also Powell & Crasemann QM Pg 79

Clarification of last lecture

$$p = \frac{U}{c}$$

p - momentum
U - energy absorbed

$$P = \frac{|S|}{c}$$

P - pressure
S - Poynting vector

go over radiation pressure

momentum $p = \frac{U}{c} = \frac{SAt}{c}$ $\quad F = \frac{dp}{dt} = \frac{SA}{c}$

$P = F/A = \frac{S}{c}$ pressure

$S = uc$ $\quad u$ - energy density

$$\underline{\underline{p = u}}$$