Figure 1.1 The electromagnetic spectrum.

Figure 1.2 The "optical" portion of the electromagnetic spectrum.
Units

\( M_0 = 4\pi \times 10^{-7} \text{ H/m} \)

**Henry**

What is a Henry?

Recall \( V = L \frac{di}{dt} \)

\( \uparrow \quad \rightarrow \quad d/s^2 \)

\( \text{Henry} \)

Units: Volts is \( J/C \)

\[ H = \frac{J/C}{d/s^2} = \frac{J-s^2}{d^2} = \frac{\text{kg m}^2}{\text{C}^2} = \text{Henry} \]

\( C_0 = 8.854 \times 10^{-12} \text{ F/m} \)

What's a farad?\(^7\)

\( Q = \bar{C}V \quad \bar{C} \) is cap in farad

\( \bar{C} = \frac{d}{V} = \frac{d}{J/C} = \frac{d^2}{J} = \frac{d^2 - s^2}{\text{kg} \cdot \text{m}^2} = \text{Far} \)

\( M_0 = 4\pi \times 10^{-7} \text{ kg m} \cdot \text{d}^{-2} \)

\( \mu_0 = 8.854 \times 10^{-12} \text{ C}^2 \cdot \text{m} \cdot \text{kg}^{-1} \cdot \text{d}^{-2} \)
Waves

Free Supplementary Topic

Coulomb/Gauss
\[ \nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0} \]
\[ \nabla \cdot \mathbf{B} = 0 \]
\[ \mathbf{E} \cdot \mathbf{d} \mathbf{a} = \frac{\rho(\mathbf{r})}{\varepsilon_0} \]

Faraday
\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \]
\[ \mathbf{E} \cdot \mathbf{d} \mathbf{a} = -\frac{\partial \mathbf{B}}{\partial t} \]

Ampère
\[ \nabla \times \mathbf{B} = \mu_0 (\mathbf{J} + \mathbf{J}_d) \]
\[ \mathbf{B} \cdot \mathbf{d} \mathbf{a} = \mu_0 (\mathbf{J} + \mathbf{J}_d) \]
\[ \mathbf{J}_d = \mathbf{e}_0 \frac{\partial \mathbf{E}}{\partial t} \]

(Dispacement)

\[ \mathbf{J}_d = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \]

Charging \( E \to B \) — Faraday
Charging \( B \to E \) — Ampère + Maxwell

\[ \mu_0 = 4 \pi \times 10^{-7} \ \text{H/m} \]
\[ \varepsilon_0 = 8.85 \times 10^{-12} \ \text{F/m} \]

Non-linear phenomenon

Imagine an \( E \) field in space caused by some charge now remove charge \( E \to 0 \) but as \( E \to 0 \) \( B \) becomes zero as \( B \to E \to 0 \) so are neglect the other in a sort of dance out of phase.

Wave equation — optional
\[ \nabla \times \mathbf{E} = \frac{\partial \mathbf{B}}{\partial t} \]
\[ \nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \quad \text{in free space} \]
\[ \nabla \times (\nabla \times \mathbf{B}) = \nabla (\nabla \cdot \mathbf{B}) - \nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial^2}{\partial t^2} (\nabla \times \mathbf{B}) \]
\[ \Delta \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} = -\nabla^2 \mathbf{B} \]
\[ \nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0 \]

Also \( \nabla^2 \mathbf{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0 \)

Homogeneous wave eq

\[ \rightarrow \text{propagating waves} \quad \text{speed} \quad \nu = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \]
General solution \( J(K \cdot x + \omega t) \)

\[
E(x, t) = E_0 \sin (\frac{K \cdot x - \omega t}{\lambda}) = \overline{E_0} \sin (\frac{2\pi K \cdot x}{\lambda})
\]

\[
B(x, t) = B_0 \sin (\frac{K \cdot x - \omega t}{\lambda})
\]

\[
h^2 = \mu_0 \varepsilon_0 \omega^2 \quad K = \frac{2\pi}{\lambda} \quad \omega = 2\pi f
\]

1) Velocity of wave \( \frac{\omega}{K} = \sqrt{\frac{\mu_0 \varepsilon_0}{\varepsilon_0}} \equiv \lambda \)

\[
\lambda = 3.0 \times 10^8 \text{ m/s} = \frac{c}{\sqrt{\varepsilon_0}} \rightarrow 1 \text{ ft/second} \approx 1 \text{ GHz} \approx \frac{1}{\text{ft}} (30 \text{ cm})
\]

\[
\lambda = \sqrt{\frac{\mu_0 \varepsilon_0}{\varepsilon_0}} = \lambda \sqrt{\varepsilon_0}
\]

\( W = CK \) - Dispersion relation

2) Direction of wave propagation is \( \overrightarrow{K} \)

\( \nabla \cdot E = 0 \quad \nabla \cdot B = 0 \quad \text{in free space} \)

\[
\nabla \cdot E_0 = 0 \quad \nabla \cdot \overline{B_0} = 0
\]

\[
\rightarrow \quad E_0 \cdot K = 0 \quad \overline{B_0} \cdot K = 0
\]

\[
\rightarrow \quad K \times E_0 \times B_0
\]

Waves transverse \( E \parallel \overline{B} \) can also show \( E \perp \overline{B} \)

\[
\rightarrow \quad \overline{K} \times \overline{E_0} - \omega \overline{B_0} = 0
\]

\[
\rightarrow \quad \overline{B_0} = \frac{K}{\omega} (\overline{K} \times \overline{E_0}) = \frac{1}{c} (\overline{K} \times \overline{E_0})
\]

\( W = CK \)

\( \rightarrow \quad B \perp E \quad \text{and} \quad |E| = c |B| \quad \text{also see Book pg 703-709} \)

\[
\rightarrow \quad \overrightarrow{\frac{E}{B}} = \frac{1}{c} \quad \frac{E}{B} \quad \text{in phase}
\]

\( f \rightarrow 3 \times 10^8 \text{ V/m} \quad g \rightarrow 3 \times 10^4 \text{ V/m} \)
Examples of EM radiation:

1) 10 - 20 kHz  submarine communication
2) 540 - 1600 kHz  AM
3) 2 - 20 MHz  SW radio
4) 27 - 28 MHz  CB radio
5) 40 MHz  New cordless telephones
6) 54 MHz  TV CB?
7) 88 - 108 MHz  FM
8) 900 MHz  Top of TV UHF
9) 14.2 GHz  21 cm radiant H
10) 2 GHz  Microwave ovens
11) 2.15 GHz  HFB
12) 3.7 - 4.2 GHz  Direct broadcast satellite 7
13) 30 - 300 GHz  CBR
14) 115 GHz  CO emission
15) 10^{14} - 10^{15} Hz  Visible light
Poynting Vector (Vacuum case)

Let "Poynting" along the direction of propagation

\[ D \cdot (E \times B) = B \cdot (D \times E) - E \cdot (D \times B) \]

\[ = B \cdot (\frac{1}{c} \frac{\partial E}{\partial t}) - E \cdot (\mu_0 \varepsilon_0 \frac{\partial B}{\partial t}) \]

\[ = -\mu_0 \frac{1}{2} \varepsilon_0 \left( \frac{\partial B}{\partial t}^2 + \frac{1}{2} \varepsilon_0 \frac{\partial E}{\partial t}^2 \right) \]

magnetic field energy density
electric field energy density

\[ S = \frac{1}{\mu_0} \cdot (E \times B) \]

Conservation laws for energy

Recall Conservation of charge

\[ D \cdot \vec{S} + \frac{1}{c^2} \frac{\partial \rho}{\partial t} = 0 \]

\[ \vec{S} \] is flux of power for electromagnetic wave

\[ \vec{S} \text{ - watts/m}^2 \]
Problem 31, Chap. 41

Show vector \( \mathbf{E} \) points into wire carrying current \( i \) and equals rate of work done.
Radius \( r \), resistivity \( \rho \).

\[
\begin{align*}
L & \quad \text{length} \\
V & = i R \\
R & = \rho \frac{L}{A} \\
A & = \pi R^2 \\
E & = \frac{V}{L} = \frac{i R}{L}
\end{align*}
\]

\[
\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \left( \frac{i R}{L} \right) = 0 \quad \text{static case}
\]

\[
\rightarrow \quad \mathbf{E} \quad \text{tangential is constant}
\]

By symmetry \( \mathbf{E} \) is along direction of current flow i.e. along wire \( \frac{1}{r} \)

\[
\mathbf{E} \quad \text{just inside wire} \quad \mathbf{E} \quad \text{just outside}
\]

\[
\overline{S} = \frac{1}{\mu_0} \mathbf{E} \times \mathbf{B} \quad \text{points radially into wire}
\]

\[
\oint \overline{B} \cdot d\mathbf{s} = \mu_0 i 
\]

\[
\frac{1}{\mu_0} = \left( \frac{i R}{L} \right) \left( \frac{\mu_0 i}{2 \pi r} \right) = \frac{i^2 R}{2 \pi \mu_0}
\]
- Net flux on length $l = \int \int \int \cdot \cdot \cdot$
  
  $= \pi \cdot l \cdot e^2 = i^2 R$

- Net flux due to damping vector = energy loss due to joule heating
Some Examples of Strange Counting Vectors

a) Conductor with non-zero resistance

\[ \vec{E}_\parallel \text{ is continuous} \]
\[ \vec{E}_\parallel \text{ outside} = \vec{E}_\parallel \text{ inside} \]
\[ \vec{S} \text{ radially outwards} \]

Can show \( \vec{S} \cdot \vec{dS} \) gives power = Joule heating

b) Stationary charge + magnet

What does \( \vec{S} \) mean?

It does not necessarily mean a net flow of usable energy
Examples

a) Fields from Sun
   1) Fields from starlight

   Solar constant = 2.0 cal/sec-cm²
   = 1400 W/m² at top of atmosphere

   \[ S = \frac{1}{\mu_0} (E \times B) \]

   \[ E = Bc \]
   \[ E = E_0 \cos \omega t \]
   \[ S = c \mu_0 E^2 \]
   \[ S(t) = \frac{1}{c \mu_0} (E_0 \cos \omega t)^2 \]
   \[ <S> = < \frac{1}{2} \epsilon_0 E_0^2 \cos^2 \omega t > = \frac{1}{2} \epsilon_0 E_0^2 \]
   \[ <S> = \frac{1}{2} \epsilon_0 E_0^2 \]

   Recall \[ \epsilon_0 = \frac{c^2}{2 \mu_0} \]

   \[ <S> = 1400 \text{ W/m}^2 = c \frac{\epsilon_0 E_0^2}{c^2} \]

   \[ E_0 = \left( \frac{2 <S>}{\epsilon_0} \right)^{\frac{1}{2}} \]

   \[ E_0 = 1.0 \times 10^3 \text{ V/m} \]
   \[ B_0 = \frac{E_0}{c} = 3.4 \times 10^{-6} \text{ T} \]

   \[ g = 3.4 \times 10^{-2} \]

   Why aren't you shocked? Why does metal get hot in sunlight?
b) Room light

Fluorescent light tubes 8' each
~ 100 W

1 W @ 555 nm = 683 lumens

Fluorescent: 70 lumens/W 20%
Incandescent: 350 lumens/W 100%

Fluorescent CW light
53% UV
44% heat
3% direct visible

53% UV { 18% visible
35% heat

Typical lighting: 20-50 W/m²

light for fluorescent

sky <s> = 4 W/m²

\[ E_0 = \left( \frac{2 \times <s>}{e_0} \right)^{\frac{1}{2}} \sim 50 \text{ V/m} \]

\[ = \left( \frac{2 \times 4}{3 \times 10^8 \times 8.85 \times 10^{-12}} \right)^{\frac{1}{2}} \]
Radiation Pressure

Projecting vector

\[ \vec{S} = \frac{1}{\mu_0} (\vec{E} \times \vec{B}) \] flux of radiation

\[ \text{units: } \text{watts/}\text{m}^2 \]

Why radiation pressure?

Look at interaction of light with material (electron)

\[ \vec{F} = q (\vec{v} \times \vec{B}) \]

\[ \vec{E} \]

\[ \vec{F}_e \quad \text{note } \vec{F}_e \text{ always } \]

\[ \cdot \vec{F}_B \text{ force on electron in direction of propagation } \vec{B} \]

(also force on proton in same direction as \( \vec{F}_e \))

electric \[ \vec{F}_e = e \vec{E} = \frac{e}{m} \vec{V}_d = \frac{1}{\gamma} \vec{V}_d \]

\[ \gamma \text{ - damping time} \]

\[ \vec{V}_d = \frac{\gamma}{m} e \vec{E} \]

magnetic \[ \vec{F}_B = q |(\vec{v} \times \vec{B})| = e \vec{V}_d \vec{B} = \frac{\gamma e^2 E B}{m} \]

\[ \frac{E}{B} = \frac{c}{e} \]

\[ \vec{F}_B = \frac{\gamma e^2}{m} \vec{E} \]

Work done on electron \[ \vec{F}_e \vec{v}_d = \frac{d}{dt} (\text{recall } \vec{F}_e \perp \vec{v}_d \text{ so work } \gamma \vec{F}_B = 0) \]
Reduction press cont

\[ \frac{d\dot{v}}{dt} = F_E \dot{v}_o = (eE)(-eE) = \frac{T e^2}{m} \]

\[ = U \text{ - energy absorbed by } e^- \]

\[ F_B = \frac{1}{c^2} e^2 E^2 = \frac{dp}{dt} \text{ momentum transferred to } e^- \text{ along } \hat{n} \]

Comparing \[ p = \frac{U}{c} \] (p - momentum)

\[ = p = \frac{U}{c} \text{ for complete absorbtion} \]

\[ p = 2 \frac{U}{c} \text{ for complete reflection} \]

\[ p \text{ - momentum } \quad U \text{ - energy incident} \]

Pressure \[ p = \frac{151}{c} \] absorbed

\[ p = 2 \frac{151}{c} \] reflection

\[ p = N/m^2 \text{ - pressure} \]

\[ U \text{ - energy incident} \]

momen \[ p = \frac{U}{c} = \frac{52\pi}{c} \]

\[ F = \frac{dp}{dt} = \frac{5a}{c} \]

Pressure \[ p = \frac{F}{\pi} = \frac{5}{2} \text{ check} \]

Also \[ S = u < u \text{ - energy down} \]

\[ p = u > \frac{151}{c} \]

Note units of pressure \[ \text{ J/m}^2 \]
prove: Pressure = energy density outside surface (partial reflection)

let $f$ be reflected fraction

1) Absorbed part $(1-f)$

\[ P_a = \frac{(1-f)S}{c} \]

2) Reflected part $(f)$

\[ P_r = \frac{2fS}{c} \]

\[ P_T = P_a + P_r = \frac{(1+f)S}{c} \]

energy density in incident wave

\[ \psi_i = \frac{S}{c} \]

reflected energy density \( \psi_r = f \frac{S}{c} \)

\[ \psi_T = \psi_i + \psi_r = \frac{(1+f)S}{c} \]

\[ P_T = \psi_T \]
Radiation Pressure for sunlight

\[ P = \frac{151}{c} \]

\[ 151 = 1400 \text{ W/m}^2 \]

\[ P = 4.7 \times 10^{-6} \text{ N/m}^2 = 6.9 \times 10^{-10} \text{ atm} \]

Recall: 1 atm = 14.7 psi = 10^5 N/m^2

1 N/m^2 = 1 PASCAL

Force of sunlight over entire Earth

\[ F = PA = 4.7 \times 10^{-6} \times \left( \pi \times (6 \times 10^6)^2 \right) \]

\[ r_e \approx 6000 \text{ km} = 6 \times 10^9 \text{ m} \]

\[ F = 5.3 \times 10^{14} \text{ N} = 5.3 \times 10^{18} \text{ pascals} \]

(recall: \( F = mg \) \( g \approx 9.8 \text{ m/s}^2 \), 2.2 "lb" = 1 kg)

2.2 "lb" = 9.8 N \( \Rightarrow 1 \text{ lb} = 4.5 \)
1) Coax

![Image of a coaxial cable]

"Transverse Electromagnetic"

2) Waveguide - follow no inner conductor:
- TE or TM mode
- No TEM mode

Limiting case of TEM mode
When wavelength << size of guide

Why no TEM?

\[ E = 0 \text{ in conductor} \rightarrow V = \text{constant} \]
\[ \text{in conductor} \rightarrow V = \text{constant} \]
inside (Since \( V \) is harmonic \( \nabla^2 V = 0 \))

See Buch, Adv. Calculus, P. 471

\[ E = 0 \text{ inside} \rightarrow \nabla \times \vec{E} = 0 \quad (\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}) \]

\[ \Rightarrow \text{no wave} \]

\[ \Rightarrow \text{No TEM wave in coaxial waveguide} \]
3) Rectangular Waveguide

TE mode

\[ E \text{ along } y \]
\[ B \text{ in plane } x \times b \text{ but not } x \times a \]

ie. some component of \( B \) along \( z \)

\[ \rightarrow \text{ not purely } \frac{1}{2} \text{ but some } \frac{1}{2} \]

Reflection at wall

\[ \tan \alpha = \frac{\lambda/2}{\lambda/2} = \frac{\lambda_0}{\lambda_g} \]
\[ \sin \alpha = \frac{\lambda_0}{\lambda_g} \quad \cos \alpha = \frac{\lambda_0}{\lambda_g} \quad \tan \alpha = \frac{\lambda_0}{\lambda_g} = \frac{1}{\lambda_0} \]

\[ \sin \alpha = \frac{\lambda_0}{\lambda_g} = \sqrt{1 - \cos^2 \alpha} = \sqrt{1 - \left(\frac{\lambda_0}{\lambda_g}\right)^2} \]

\[ \lambda_g = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_g}\right)^2}} > \lambda_0 \]

1) \( \lambda \), must be \( < 2a \) - high power fibre

2) \( V_{\text{phase}} = c / \sin \alpha = \frac{c}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_g}\right)^2}} > c \)

3) \( V_{\text{group}} = c / \sin \alpha = c \sqrt{1 - \left(\frac{\lambda_0}{\lambda_g}\right)^2} < c \)

as \( \lambda_0 \to 2a \quad \lambda_g \to \infty \quad \lambda_g \to \infty \quad \lambda_g \to 0 \)
Cylindrical cavity

Consider a capacitor at high freq.

\[ \oint E \cdot ds = -\frac{d}{dt} \Phi \]

\[ \oint B \cdot ds = \mu_0 I + \mu_0 \epsilon_0 \frac{d}{dt} \]

Changing \( E \rightarrow \) Changing \( B \rightarrow \) Changing \( E^* \)

\( \rightarrow \) Changing \( B^* \rightarrow \)

\( E^*, B^* \) opposite to \( E, B \)

Layer \( W \rightarrow \) Layer \( E^*, B^* \)

\[ E_{\text{total}} = E + E^* + \ldots \]

\[ B_{\text{total}} = B + B^* + \ldots \]

Maybe \( E = 0 \) at some \( r \)

In fact \( E = E_0 \sin \omega t \) \( J_0 \left( \frac{\omega r}{c} \right) \)

\( J_0 = \) Bessel function

\( J_0 = 0 \) for \( \frac{\omega r}{c} \approx 2.405 \)

\[ B = B_0 \cos \omega t \) \( J_1 \left( \frac{\omega r}{c} \right) \)

To enclose capacitor with side walls at \( \frac{\omega r}{c} = 2.405 \)

Resonant cavity

\( B \rightarrow 0 \) at walls because currents in walls
\[ v = 500 \text{ MHz} \text{ resonant wire} \]

\[ \frac{\text{wavelen}}{c} = 2.405 \text{ for resonance} \]

\[ w = 2\pi v \]

\[ \Rightarrow r \approx 23 \text{ cm} \quad (9 \text{ in}) \]
1) \[ S = \int_{x_1}^{x_2} (T-U) \, dt \quad \text{Feynman} \quad \text{pg 19-20} \]

\[ = \int_{x_1}^{x_2} L \, dt \quad \text{Theoretical Physics} \quad \text{pg 24} \]

\[ \delta S = 0 \]

\[ \delta \left( \int_{x_1}^{x_2} p \, dr \right) = 0 \quad \text{Theoretical Physics} \quad \text{pg 27} \]

\[ \delta \left( \int_{x_1}^{x_2} \sqrt{E - U(x)} \, dx \right) = 0 \quad \text{pg 27} \]

\[ E \quad \text{total energy} \]

\[ \text{Also: Powell & Enzena} \]

\[ \text{DM 93} \]
Clarification of last lecture

\[ p = \frac{U}{c} \]
- momentum

\[ S = \text{energy absorbed} \]

\[ p = \frac{IS}{c} \]
- pressure

\[ \vec{S} = \text{Poynting vector} \]

\[ g_0 = \text{over radiation pressure} \]

Momentum

\[ p = \frac{U}{c} = \frac{SAx}{c} \]

\[ F = \frac{dL}{dt} \]

\[ p = \frac{F/A}{c} = \frac{S}{c} \]
- pressure

\[ S = Uc \]
- energy density

\[ \rho = U \]