MIDTERM SOLUTIONS

Problem 1:

From definition of moment of inertia ($I = \Sigma_i m_i r_i^2$), we can see that the system on the right has the movable masses closer to the rotation axis, which gives it a lower moment of intertia and from $\tau = I\alpha$ the angular, and thus the linear, accelertation will be faster. So the mass on the left will hit the ground first

Problem 2:

There are a number of ways to do this question, here is the easiest way I found.

First we recognize that the acceleration will be constant because all of the forces involved are constant. If the 14 kg mass falls 10 m in 2 seconds, and start from rest, we can write

$$x(t) = \frac{1}{2}at^2 \tag{1}$$

Solve for a and plug in our numbers

$$a = 2x(t)/t^2 = 2 \times 10/2^2 = 5 \text{ m/s}^2 \tag{2}$$

Next we sum up the forces on the block:

$$\Sigma F = mg - T = ma \tag{3}$$

Rearranging for T:

$$T = m(g - a) \tag{4}$$

Now to solve for the moment of inertia, we use the torque equation:

$$\tau = I\alpha \tag{5}$$

Rearranging for I, and using the fact that we are rolling without slipping ($\alpha = a/R$):

$$I = \tau/\alpha = \frac{TR}{a/R} = \frac{TR^2}{a} = \frac{m(g-a)R^2}{a} \approx 54 \text{ kg} \cdot \text{m}^2 \tag{6}$$

Problem 3:

Given mass of the boy, m_b, and his distance from pivot point, $-x_b$ (negative because he is to the left of the pivot point), and similarly for the girl, m_g and x_g. If they are balanced, then we have:

$$\Sigma\tau = -m_b g x_b + m_g g x_g = 0 \tag{7}$$

This gives us:

$$m_b x_b = m_g x_g \tag{8}$$

If we change their distances by any factor, as long as it is the same factor for both sides, the previous equation will still hold, thus the seesaw will still balance

Problem 4:

This problem involves using torques. Since there are two sources providing unknown torques we will need to set the pivot point at one site where an unknown torque is being applied to solve for the other unknown torque. Lets first solve for the force applied at point A, F_A, by placing our pivot point at B.

$$F_A(1.6) = mg(5.0 - 1.6) \tag{9}$$

$$F_A = \frac{mg(5.0-1.6)}{1.6} = \frac{82 \times 9.8 \times 3.4}{1.6} = 1.71 \text{ kN} \tag{10}$$

The force of the diver is oriented downward, and since F_A is on the opposite side of the pivot, F_A must also be downward.
Now to solve for F_B we place the pivot at point A:

$$0 = F_B(1.6) + mg(5.0) \tag{11}$$

$$F_B = \frac{-mg(5.0)}{1.6} = \frac{-82 \times 9.8 \times 5.0}{1.6} = -2.51 \text{ kN} \tag{12}$$

The force from A is negative because it must be directed opposite from the force of the diver because they are on the same side of the pivot and the system is balanced.

Problem 5:

For this problem we will employ Bernoulli's equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant \tag{13}$$

If we keep the height, h, fixed, the for this equation to be satisfied, if we increase the speed of the fluid v, the the pressure, p, must decrease.

Problem 6:

Here we use the equation $p = \rho g h$. We can ignore air pressure because it is essentially the same on all sides.

$$p = 5 \times 10^4 \text{ Pa} = \rho g h = (1000 \text{ kg/m}^3)g(3.0 \text{ m}) + (510 \text{ kg/m}^3)g(x \text{ m}) \tag{14}$$

Solving for the depth of the oil, x:

$$x = \frac{5 \times 10^4 \text{ Pa} - (1000 \text{ kg/m}^3)g(3.0 \text{ m})}{(510 \text{ kg/m}^3)g} = 4.1 \text{ m} \tag{15}$$

Problem 7:

In this problem we assume twice the size means twice the radius. The escape velocity is given by:

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}} \tag{16}$$

If we double both the mass and radius we get a new escape velocity given by:

$$v'_{\text{escape}} = \sqrt{\frac{2G(2M)}{2R}} = \sqrt{\frac{2GM}{R}} = v_{\text{escape}} \tag{17}$$

Thus the escape velocity is unchanged.

Problem 8:

We can first find the orbital velocity by setting centripetal acceleration to gravitational acceleration:

$$\frac{v^2}{r} = \frac{GM}{r^2} \tag{18}$$

Solving for v:

$$v = \sqrt{\frac{GM}{r}} \tag{19}$$

Since it is a circular orbit, the distance travelled in one orbit is $2\pi r$. We divide distance by orbital speed to obtain the orbital period:

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}} = \frac{2\pi(3.4 \times 10^6 + 0.1 \times 10^6 \text{ m})^{3/2}}{\sqrt{6.67 \times 10^{-11}\text{N} \cdot \text{m}^2/\text{kg}^2 \times 6.42 \times 10^{23} \text{ kg}}} = 1.75 \text{ hours} \tag{20}$$

Problem 9:

The frequency of a spring is set by the mass and the spring constant ($\omega = \sqrt{k/m}$), and is independent of the amplitude of the oscillations.

Problem 10:

The magnitute of the restoring force of a spring is the same on both sides of the equilibrium position, so in this system, the total spring constant is the sum of the individual spring constants:

$$\omega = \sqrt{k_{\text{tot}}/m} = \sqrt{(k_1 + k_2)/m} = \sqrt{(7.6 + 5.0)/2} = 2.5 \text{ rad/s} \tag{21}$$