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LIMITS ON COLD DARK MATTER COSMOLOGIES FROM NEW ANISOTROPY BOUNDS ON THE COSMIC MICROWAVE BACKGROUND

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ABSTRACT

The recent UCSB South Pole experiment on the cosmic microwave background anisotropy imposes stringent constraints on cold dark matter cosmologies with variable baryonic and dark matter content. We have taken into account both the sampling strategy and the instrumental noise of the experiment by performing Monte Carlo simulations of the theoretically predicted microwave sky and have analyzed the experimental and simulated data using both the likelihood ratio test and a χ^2 analysis. The latter provides a slightly more stringent upper limit on the rms differential (single subtraction) temperature anisotropy: $\delta T/T < 4.0 \times 10^{-5}$, on the angular scale probed by the UCSB South Pole experiment. This limit, in practice independent of the parameters of the CDM models considered, has a 95% confidence level and a power of the test $\beta = 55\%$. In addition we test Gaussian correlation function models and place a 95% confidence level, 41% power upper limit of $T/T < 3.5 \times 10$ at an angular scale of 20″–30″. We also set a lower limit (95% confidence level, $\beta = 55\%$) on the density parameter of cold dark matter universes with $\gtrsim 3\%$ baryon abundance and a Hubble constant of 50 km s^{-1} Mpc^{-1}: $\Omega_0 \gtrsim 0.6b^{-1}$, where b is the bias factor ($1 \lesssim b \lesssim 2$), equal to unity only if light traces mass.

Subject headings: cosmic background radiation — cosmology — dark matter

1. INTRODUCTION

Detection of anisotropies in the cosmic microwave background (CMB), induced by the primordial matter fluctuations which gave rise to the observed large-scale structure, remains one of the most challenging and elusive goals in modern cosmology. Observations of galaxy clustering (e.g., Davis & Peebles 1983) and superclustering (Bahcall 1988) of large voids (Geller, Huchra, & de Lapparent 1986) and apparently coherent bulk flows (e.g., Gunn 1989) have constrained possible options for the gravitational instability theory that describes how infinitesimal fluctuations in the early universe account for these observations. The cold dark matter (CDM) model (e.g., Blumenthal et al. 1984), wherein the universe contains a critical density of weakly interacting matter and a baryonic fraction of about ~10% of this density, remains an attractive option, both for its successes in accounting for most of the large-scale structure observations and for its theoretical appeal in the context of the inflationary cosmology (e.g., Blau & Guth 1987). It provides unambiguous predictions for radiation anisotropies (Vittorio & Silk 1984; Bond & Efstathiou 1984), since the primeval fluctuation spectrum is uniquely specified, and sets the standard against which rival theories, such as a baryonic dark matter universe with primeval isocurvature fluctuations (Peebles 1987), the explosive galaxy formation scenario (Ostriker & Cowie 1981), or cosmic strings (e.g., Turok 1985), are judged.

In the context of a CDM scenario, intermediate angular scales (0°.1–10°) provide one of the best opportunities for studying primordial CMB anisotropies, as most of the predicted power in the radiation power spectrum is on these scales. Erasure of fluctuations due to the fuzziness of the last scattering surface is negligible unless the universe has undergone relatively early reionization, a situation that is not expected in the standard CDM model. One directly probes the horizon scale at last scattering, and any primeval fluctuations should remain undiluted.

Hitherto, theorists have generally ignored the subtleties involved in comparing predictions with upper limits on the CMB temperature anisotropy, especially when these upper limits are derived from a small number of independent points on the sky. An unambiguous approach for comparing theory and observation in a statistically significant way is to perform Monte Carlo simulations of the microwave sky, expected in a given theoretical scenario, and analyze them as is done in the actual experiment. In this way, one can take into account such effects as data sampling, sky coverage, receiver noise, and chopping strategy, that contribute to the final uncertainties in any quantitative upper limit on $\delta T/T$. In principle, as the simulations are based on a given model, the derived upper limits on the CMB anisotropy become model dependent. This approach was already applied to the Tenerife experiment result (Davies et al. 1987) by simulating the CMB temperature pattern expected on large scales for scale-free density fluctuation power spectra (Vittorio et al. 1989).

We have accordingly undertaken a detailed comparison of the microwave anisotropies predicted by the CDM models with recent results that two of us obtained at the South Pole in 1988 December (Meinhold & Lubin 1991; see also Lubin, Meinhold, & Chingcuanco 1990). This experiment (hereafter UCSB South Pole experiment) operated at a frequency of 90 GHz and observed a strip which subtends $\simeq 10°$ on the sky, at constant declination $\delta = -73°$. Nine data points have been obtained, each representing the CMB temperature difference around a given direction. The system consists of a microwave telescope and a very low noise SIS detector, with the main lobe of the telescope response pattern being well approximated by a Gaussian of full width at half-maximum (FWHM) = 30′. The telescope is chopped sinusoidally with an amplitude of 0°.7 on the sky, resulting in an effective beam separation of 1°. Approximately 43 hr of high-quality data were obtained with an overall sensitivity of about 7.3 mK/$(\mathrm{Hz})^{1/2}$. The error per point is approximately 60 μK, measured directly from the data as the dispersion of the individual data points for a given line of sight. The calibration of the instrument was measured to be constant to $\pm 1.5\%$ during the 10 day measurement period. For a complete discussion of the instrument as well as consideration of systematic and statistical errors, see Meinhold (1990).

Previous data analyses (Davies et al. 1987; Readhead et al. 1989) have generally assumed that the CMB temperature field can be described by a Gaussian autocorrelation function. With a primeval scale-free power spectrum of density fluctuations, as in the CDM case, this assumption is not justified. In what follows, we present a brief discussion of the constraints that the data put on the amplitude of a Gaussian correlation function (to facilitate comparison with earlier results), a grid of CDM model predictions for the CMB anisotropies, and a statistically precise comparison of the CDM models with the UCSB South Pole observations.

2. THEORETICAL CONSIDERATIONS

Let us consider a patch of the sky, 12° × 12°. If this patch is much larger than the typical coherence angle of the CMB temperature distribution, then we can consider different patches of the sky as independent realizations of a given statistical ensemble. In each of these patches we can define the CMB temperature correlation function

$$\mathscr{C}(\alpha, \sigma_B) = \left\langle \frac{\delta T}{T}(\hat{\gamma}_1)\, \frac{\delta T}{T}(\hat{\gamma}_2) \right\rangle_{\text{patch}} . \quad (1)$$

Here, $\hat{\gamma}_1$ and $\hat{\gamma}_2$ are two generic lines of sight, an angle α apart; σ_B is the dispersion of the Gaussian approximating the angular response of the antenna $\{\sigma_B = \mathrm{FWHM}/[2(2\ln 2)^{1/2}] = 13'\}$, and the symbol $\langle\,\rangle_{\text{patch}}$ implies an angular average over the patch. Different patches of the sky will then exhibit in principle a different correlation, both in shape and in amplitude. The quantity usually quoted by the theorists is the average of this distribution, $\langle \mathscr{C}(\alpha, \sigma_B) \rangle$.

In fact, in the case of adiabatic fluctuations with an initial Zel'dovich power spectrum, different patches of the sky are not statistically independent but are correlated, due to the long range of potential fluctuations responsible for the anisotropy on large angular scales (see, e.g., Scaramella & Vittorio 1990). This is not crucial here, because we wish to study the differential CMB anisotropy on angular scales $\approx 1°$. In fact, in a single subtraction experiment, such as the one we are considering here, the variance of the CMB temperature differences on a given angular scale is given by

$$\Delta^2_{\text{rms}}(\alpha, \sigma_B) = 2[\langle \mathscr{C}(0, \sigma_B) \rangle - \langle \mathscr{C}(\alpha, \sigma_B) \rangle] ; \quad (2)$$

and the long-range correlations cancel out. Provided we consider temperature differences on scales of $\approx 1°$, large patches can indeed be considered independent, thereby justifying our Monte Carlo simulations of the UCSB South Pole experiment on 12° × 12° sky maps. These maps are generated with a standard FFT technique, after calculating for each model the power spectrum of the CMB temperature fluctuations. Although the UCSB South Pole experiment observed a 10° strip on the sky, we used 12° × 12° maps to avoid problems due to the periodic boundary conditions. Considering a patch of the sky 12° × 12° to be flat is a very good approximation for this analysis, introducing an error of $< 2\%$ in our final results.

Numerical simulations of this kind provide a direct means of including effects such as differencing strategy, instrumental noise, sky coverage, etc. and enable us to properly take into account sampling effects. The simulations self-consistently take this into account and constrain the ensemble average amplitude of CMB temperature fluctuations, $\langle \mathscr{C}(0, \sigma_B) \rangle$.

In Table 1, we present theoretical predictions for a grid of CDM models. These predictions are normalized in a standard way to the galaxy clustering on small scales, by requiring that the rms mass density fluctuations, averaged over an $8h^{-1}$ Mpc sphere, are unity [here, as usual, $h \equiv$ Hubble constant/(100 km s^{-1}/Mpc)]. A biasing parameter, b, is usually introduced (see Dekel & Rees 1987 for a review) to reconcile the dynamical determination of density parameter with the $\Omega_0 = 1$ CDM models, and the predictions for the models have to be reduced by this factor, typically $b \simeq 1.5$.

3. DATA ANALYSIS

Given the theoretical predictions of Table 1, we want to assess the constraints that the UCSB South Pole experiment places on these models. The goal of the data analysis is to test two hypotheses. The first, H, assumes that CMB temperature fluctuations do exist in the sky. The microwave pattern is determined, for a given model, by the shape of the CMB temperature autocorrelation function, the only free parameter being its amplitude, $C_0 \equiv \langle \mathscr{C}(0, \sigma) \rangle$. The alternative hypothesis, K, assumes instead a completely uniform CMB sky, with no intrinsic fluctuations, i.e., $C_0 = 0$.

In order to distinguish between these two cases, we apply the likelihood ratio test, the likelihood ratio being defined as $\lambda \equiv L(C_0)/L(0)$, where

$$L \equiv (2\pi)^{-N/2} |\mathbf{M}|^{-1/2} \exp\left[-\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \Delta_i M_{ij}^{-1} \Delta_j \right] \quad (3)$$

is the likelihood function. Here $\{\Delta_i\}$ is the set of N CMB

TABLE 1

THEORETICAL PREDICTIONS FOR CDM COSMOLOGIES

Ω_0	Ω_b	h	$\langle \mathscr{C}(0, 0) \rangle^{1/2}$	$\langle \mathscr{C}(0, 13') \rangle^{1/2}$	$\Delta_{\text{rms}}(1°, 13')$
1.0.........	0.03	0.5	2.97×10^{-5}	2.05×10^{-5}	2.12×10^{-5}
	0.10	0.5	3.57×10^{-5}	2.47×10^{-5}	2.71×10^{-5}
	0.20	0.5	4.57×10^{-5}	3.17×10^{-5}	3.69×10^{-5}
	0.20	0.4	5.55×10^{-5}	3.70×10^{-5}	4.38×10^{-5}
0.8.........	0.03	0.5	3.77×10^{-5}	2.36×10^{-5}	2.74×10^{-5}
0.6.........	0.03	0.5	5.43×10^{-5}	3.02×10^{-5}	3.85×10^{-5}
0.4.........	0.03	0.5	8.96×10^{-5}	4.40×10^{-5}	6.01×10^{-5}

temperature differences, and $\mathbf{M} \equiv \langle \Delta_i \Delta_j \rangle$ is the covariance matrix of these differences. We evaluate the distribution of λ, either under hypothesis H and K, by Monte Carlo simulations of the sky. Under hypothesis H, we generated, for each of the theoretical models, 10,000 maps (12° × 12°), by assigning a value $\sigma_{\rm sky}$ to the parameter C_0. We sample each map as the UCSB South Pole experiment sampled the real sky, explicitly taking into account the sinusoidal modulation of the experiment. We obtain nine CMB temperature differences, to which we add appropriately distributed Gaussian noise, to simulate the experimental errors. In addition, we subtract a linear component from each simulated data set in order to be more consistent with the analysis of the real data (Meinhold & Lubin 1990). As a result, for each of the models considered, we produce 10,000 simulated data sets, $\{\Delta_i\}$. Under hypothesis K, we proceed likewise but impose $C_0 = 0$.

The 9 × 9 covariance matrix has been evaluated numerically, by averaging the product $\Delta_i \Delta_j$ over 10,000 simulations. Without the linear fit subtraction, and with a square wave chop of amplitude $\theta = 1°$ (coincident with the sampling angle) we would have obtained:

$$M_{ij} = 2\langle \mathscr{C}[(i-j)\theta, \sigma_B]\rangle - \langle \mathscr{C}[(i-j-1)\theta, \sigma_B]\rangle - \langle \mathscr{C}[(i-j+1)\theta, \sigma_B]\rangle + \sigma_i^2 \delta_{ij}\,, \quad (4)$$

where $\langle \mathscr{C}(\alpha, \sigma_B)\rangle$ is the expected correlation function of the model we want to test, proportional to C_0, and $\{\sigma_i\}$ are the experimental errors associated with the ith temperature difference. We checked that in this case our numerically derived covariance matrix agrees well (to few percent) with this expression.

We want to have a low probability, γ, of rejecting hypothesis H when H is true (type I error). We tune $\sigma_{\rm sky}$ in the simulations so that only a fraction, $\gamma = 5\%$, of cosmic observers would measure a likelihood ratio $\lambda \le \lambda_{\rm obs}$, where $\lambda_{\rm obs}$ is the likelihood ratio obtained using the actual data. We also evaluate the power of the test, namely the fraction β of cosmic observers that, under hypothesis K, would also measure $\lambda \le \lambda_{\rm obs}$. In this way, by varying $\sigma_{\rm sky}$, we tune γ and we find β *a posteriori.* Ideally, one would like to have a low probability, $1 - \beta$, of accepting H when hypothesis K is true (type II error). For the UCSB South Pole experiment $\beta = 60\%$.

We also evaluate the observed chi-squared, $\chi^2_{\rm obs} = \sum_{i=1}^{9} \Delta_i^2/\sigma_i^2 = 6.7$, and tune $\sigma_{\rm sky}$ in the simulations to have $\chi^2 < \chi^2_{\rm obs}$ only a fraction $\alpha = 5\%$ of the time. In this case the power of the test (i.e., the probability of having under hypothesis K $\chi^2 < \chi^2_{\rm obs}$) is $\beta = 55\%$.

We normalize the upper limits on the CMB anisotropy of a given model to the corresponding theoretical predictions of Table 1. Table 2 contains these normalized upper limits, $\mathscr{K}$ say, at the 95% confidence level, and with the appropriate power β. A given theoretical model is then accepted or rejected if $\mathscr{K}$ is larger or less than unity for unbiased models, and larger or less than b^{-1} for the biased flat models. For example, the 95% upper limit derived with the condition $\chi^2 < \chi^2_{\rm obs}$ for the $\Omega_0 = 0.4$ CDM model provides $\mathscr{K} = 0.70$, i.e., the upper limit is a factor of 1.4 smaller than the theoretical predictions in Table 1. The 95% confidence level upper bounds are $\mathscr{K}$ times the theoretical predictions of Table 1. In this case, $\langle \mathscr{C}(0, 0)\rangle^{1/2} < 0.70 \times 8.96 \times 10^{-5}$, $\langle \mathscr{C}(0, 13')\rangle^{1/2} < 0.70 \times 4.4 \times 10^{-5}$, and $\Delta_{\rm rms}(1°, 13') < 0.70 \times 6.01 \times 10^{-5}$, with a power of the test $\beta = 55\%$.

We have also studied the constraints placed on Gaussian correlation functions by the UCSB South Pole results. This is useful for comparing our results with those from other experiments and should indicate how the Gaussian correlation function upper limits compare with those of the full model calculations. We use likelihood analysis to compare simulations to the data. For each coherence angle θ_c, we produce 1000 strips of sky 1° by 10°, with correlation function

$$\langle C(\theta, \theta_c)\rangle = C_g \exp\left[-\frac{1}{2}\frac{\theta^2}{\theta_c^2}\right], \quad (5)$$

measuring each with the experimental beam throw and beam response. Next, we add Gaussian noise with the proper dispersion to each data point and remove a linear component from each simulated data set. By varying the amplitude of fluctuations in the simulations, we determine how large $\sigma_{\rm sky}$ must be for 95% of the simulated data sets to give C_{10} (the value of C_g at which the likelihood function is 1/10 of its maximum value) larger than the real data. Note that now C_g does not necessarily coincide with $\sigma_{\rm sky}$, being a free parameter in the likelihood analysis of the simulated data set. We then obtain a 95% confidence level upper limit on the amplitude of intrinsic structure in the CMB for Gaussian correlation functions. The value obtained by this procedure is 95 μK, or $C_g^{1/2} < 3.5 \times 10^{-5}$ for our most sensitive scale $\theta_c = 20'$–$30'$. The power of this test, given by the probability that, under hypothesis K, we would have obtained a C_{10} this low, is $\beta = 41\%$. The results of these calculations are summarized in Figure 1.

4. DISCUSSION AND CONCLUSIONS

The purpose of this *Letter* was to present a self-consistent method for comparing theoretical predictions of and observational upper limits on CMB anisotropy and to show new

TABLE 2

UCSB SOUTH POLE EXPERIMENT CONSTRAINTS ON CDM COSMOLOGIES

Ω_0	Ω_b	h	$\lambda \le \lambda_{\rm obs}$ ($\beta = 60\%$)	$\chi^2 \le \chi^2_{\rm obs}$ ($\beta = 55\%$)
1.0.........	0.03	0.5	2.32	1.88
	0.10	0.5	1.78	1.48
	0.20	0.5	1.29	1.09
	0.20	0.4	1.07	0.89
0.8.........	0.03	0.5	1.75	1.47
0.6.........	0.03	0.5	1.25	1.05
0.4.........	0.03	0.5	0.80	0.70

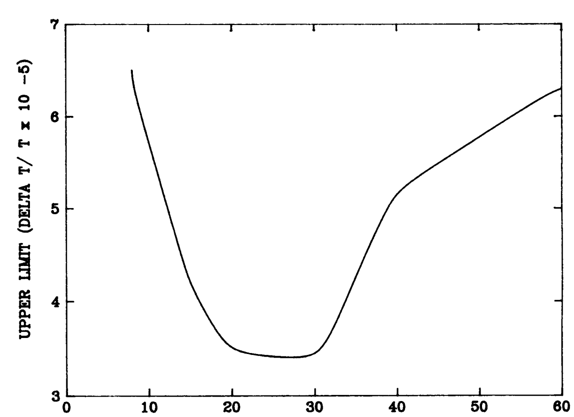

FIG. 1.—95% confidence level upper limits for CMB fluctuations with Gaussian autocorrelation functions. The limits are at 41% power and include a 4% correction for atmospheric attenuation.

bounds on CDM cosmologies set by the UCSB South Pole experiment on the 1° angular scale. Our theoretical predictions are obtained by considering only the antenna beam smearing, among all the possible experimental effects, and by operating an ideal average over all the sky (equivalent, in this context, to an ensemble average). Most experiments of this type deal with only a limited number of points, and comparing the theoretical predictions directly to the observational upper limits can be misleading. We need to specify the confidence level with which a model is accepted or rejected, and we have done this by Monte Carlo simulations of the data, given both experimental and theoretical input.

The most stringent upper bounds are provided by the χ^2 statistics. The upper limits on the rms differential (single subtraction) temperature anisotropy are approximately *independent* of the specific CDM parameters considered [note that the limits on $\langle C(0, 0)\rangle$ and $\langle C(0, \sigma)\rangle$ do depend on the models considered]. We obtain $\Delta_{\rm rms}(1°, 13') < 4.8 \times 10^{-5}$ (for $\lambda < \lambda_{\rm obs}$, $\gamma = 5\%$ and $\beta = 60\%$), and $\Delta_{\rm rms}(1°, 13') < 3.8 \times 10^{-5}$ (for $\chi^2 < \chi^2_{\rm obs}$, $\gamma = 5\%$ and $\beta = 55\%$), only by chance comparable to the upper limit on $C_g^{1/2}$. These values are the average of the upper limits obtained on single models. The dispersion around these means are at most 6% of the mean itself. The upper limit on $C_g^{1/2}$ is a factor 1.4–1.8 smaller than that obtained with the full model calculations for $\langle \mathscr{C}(0, 0)\rangle^{1/2}$, using the likelihood ratio test. For these reasons, we believe it is important to simulate the specific model one wants to test. We use the results of the χ^2 analysis for testing models, although similar conclusions can be reached on the basis of the λ analysis: the limits obtained with the λ statistics are $\sim$20% higher than obtained with the χ^2 analysis.

Flat unbiased CDM models with a baryonic fraction of 20%, the maximum allowed by standard nucleosynthesis (e.g., Boesgard & Steigman 1985), are marginally consistent with the UCSB South Pole result if $h = 0.5$; for lower values of h, a biasing parameter greater than unity must be considered. For $b = 1$ and $h = 0.5$, $\Omega_b \lesssim 0.2$ at the 95% confidence level ($\beta = 55\%$). The standard $\Omega_0 = 1$, $\Omega_b = 0.03$, $h = 0.5$ unbiased model predicts an anisotropy which is only a factor of $\sim$1.88 below the observational UCSB South Pole experiment upper limit. A biasing factor $b \simeq 1.5$, suggested by the large-scale observations, implies that at the present level of sensitivity the UCSB South Pole experiment would have missed a detection only by a factor of 3. The UCSB telescope was taken back to the South Pole in late 1990, data analysis is underway, and we hope to improve the noise significantly. In addition, the telescope is being used for a multifrequency balloon-borne anisotropy measurement (for improved Galatic contamination subtraction) at the same angular scales.

The analysis of CDM open universes performed here leads to a revision of the lower limit on the density parameter ($\Omega_0 \gtrsim 0.4$) previously obtained (Vittorio & Silk 1984; Bond & Efstathiou 1984) on the basis of the Uson & Wilkinson (1984) upper limit. Assuming $h = 0.5$ and $\Omega_b = 0.03$, we conclude that open CDM universes must have, at the 95% confidence level ($\beta = 55\%$), $\Omega_0 \gtrsim 0.6b^{-1}$. The improvement relative to the previous lower bound consists mainly in quantifying its statistical significance. In fact, the previous calculations directly compared theoretical rms predictions to observational bounds and ignore the power of the test of the Uson & Wilkinson data set (which is low; Vittorio & Muciaccia 1991). The lower limit presented here has a reasonably high power of the test and, thus is more reliable than the previous one.

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