Small-Scale Anisotropies: The Final Frontier

Jatila Van der Veen, Philip Lubin, Paolo Natoli, and Michael Seiffert

In the early 1960s Arno Penzias and Robert Wilson, then at Bell Labs, noticed a small discrepancy in their microwave instruments that indicated an excess of radiation coming in from space. Not content to ignore it, they soon made one of the profound discoveries of the twentieth century: they had found the embers of the early universe. This radiation, which is nearly constant in every direction, in all seasons, and at all times of day, is now called the Cosmic Microwave Background (CMB) radiation, and is almost universally accepted to be evidence of a hot dense beginning to our universe. When viewed in the light of Edwin Hubble's discovery some 30 years earlier of the redshifts of galaxies, this microwave background was interpreted as none other than the highly redshifted and cooled relic radiation from a very hot infant universe, now seen at a black-body temperature of around 3 K. (This corresponds to a black-body radiation curve that peaks near 200 GHz.)

The CMB photons are the oldest observable radiation, coming to us from the time when matter and radiation first separated, when the universe was cool enough to become transparent. This epoch is sometimes referred to as the Recombination Era, for this was the first time the universe was cool enough for protons to capture an electron, and form neutral hydrogen some 300,000 years after the Big Bang. The Recombination Era is seen now as if we are looking outward at the inner surface of a sphere that surrounds us at a redshift (z) of approximately 1000, and is often referred to as the Last Scattering Surface (LSS). This is depicted schematically in Fig. 1. We can't say for sure how old the LSS is, or how far away; only that the universe has stretched by a factor of approximately 1000 since that time. (See Box 1: Redshifts Explained.)

For nearly three decades after its discovery, the only variation found in the CMB was the so called "dipole anisotropy," which revealed a slightly warmer temperature in the direction of the Virgo supercluster and a slightly cooler temperature in the opposite direction. (That is approximately 11h Right Ascension, -7^{o} Declination on the Celestial Sphere.) This bipolar temperature variation is thought to be due to our motion relative to the CMB and is well explained as a "Doppler shift." Once this motion was corrected for, the CMB remained isotropic to one part in 10^4, at least as of 1980 (Fig. 2).

The question then arose: If this microwave background is totally uniform, how can we

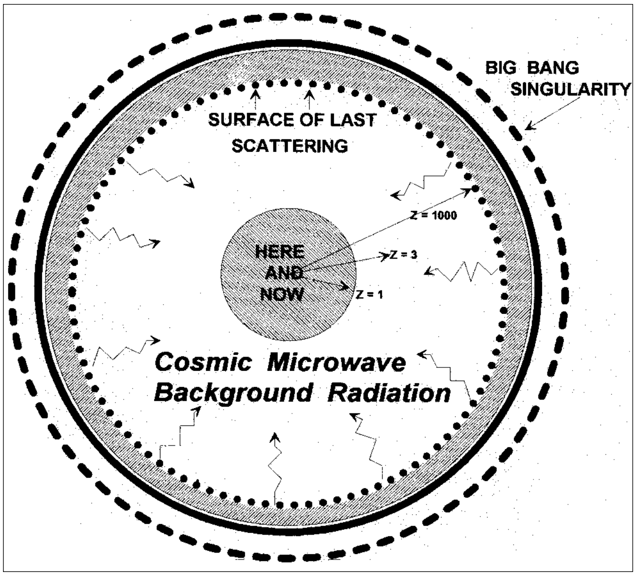
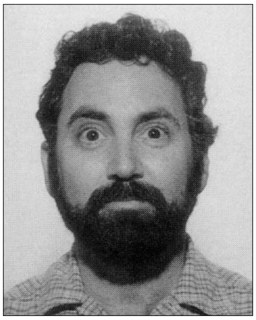


Fig. 1. Looking back in time from anywhere in the universe at the Last Scattering Surface. LSS represents a small span of time during which the universe cooled down below the ionization temperature of hydrogen, and the first neutral hydrogen could form.

***Jatila Van der Veen** teaches physics and astronomy at Adolfo Camarillo High School, a National Blue Ribbon School, in Camarillo, California. She has spent every summer since 1990 working with Prof. Philip Lubin, head of the Experimental Cosmology group at the University of California, Santa Barbara, developing curricula in basic astronomy, observational astrophysics, and cosmology as part of the Remote Access Astronomy Project. Jatila has conducted a number of image-processing workshops at National AAPT Meetings, and is a past chair of the AAPT's Astronomy Education Committee.*

Adolfo Camarillo High
4660 Mission Oaks Blvd.,
Camarillo, CA 93012

Center for Particle Astrophysics
Univ. of California
Santa Barbara, CA 93106
jatila@cfi.ucsb.edu



Philip Lubin is a professor of physics at the University of California, Santa Barbara. He has worked in the area of early universe studies for more than 20 years and done observations of the CMB from the South Pole, balloon flights and with the NASA COBE satellite. He is currently a part of the ESA/ NASA PLANCK satellite program .

Physics Department
Center for Particle Astrophysics
Univ. of California
Santa Barbara, CA 93106
lubin@cfi.ucsb.edu

account for the structures we observe in the universe today? Small variations in the CMB would indicate temperature and density variations in the infant universe, which could have formed the initial "seeds" around which large scale structure could eventually form. Throughout the 1980s, several groups around the world used balloon-borne and ground-based microwave antennas, radiometers, and bolometers to search for this elusive structure. Their measurements showed that any structure in the CMB was at or below the few parts in 10^5 level. The effect of these measurements was to pressure the theoretical understanding toward more exotic explanations such as inflation and cold dark matter, since the structure, not yet seen, was uncomfortably small.

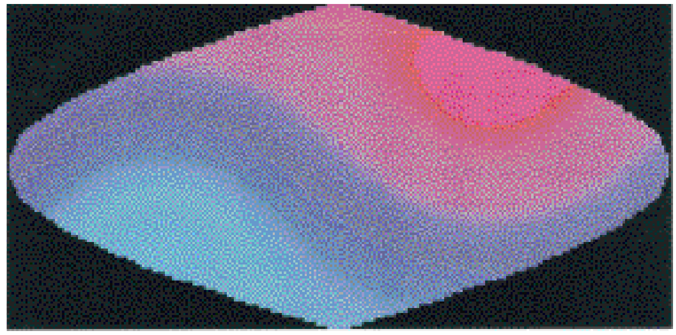


Fig. 2. Dipole anisotropy of the CMB at 53 GHz. Magenta color represents a warming of about 3 millikelvins, and cyan an equal amount of cooling, relative to the black-body background of 2.725 ± 0.002 K.[1] This apparent dipole anisotropy is not a relic of the early universe, but a Lorentz contraction of the CMB radiation field due to our net motion relative to the general expansion of the universe (co-moving frame).

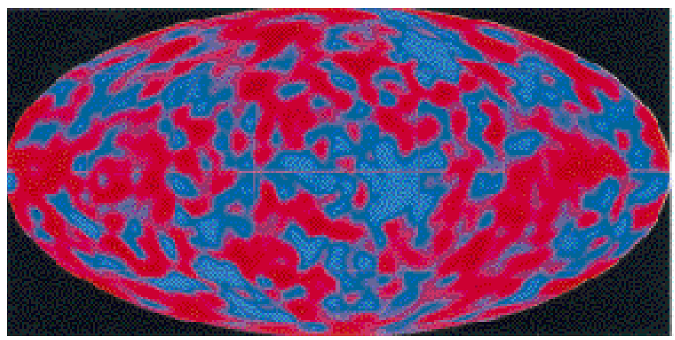


Fig. 3. COBE two-year map of temperature anisotropies in the CMB. The variation is about 30 microkelvins above or below the black-body background of 2.725 K. In this projection, the galaxy would be across the equator.

In 1992, data from the Cosmic Background Explorer (COBE) satellite, launched by NASA in 1989, showed evidence for minute temperature variations (anisotropy) in the CMB at a level of just one part in 10^5 at angular scales near 10° or higher (Fig. 3). (For reference, 10° is about the angle subtended by a typical fist at arm's length.) The smallest anisotropy that COBE could measure was about 7°, the limiting size of the beam. At about the same time (even prior in some cases), data from the South Pole and balloon-borne experiments showed anisotropies at 1° scales at a similar amplitude level.

As we will see, it is particularly important to measure at angular scales smaller than 1°, since this is the scale below which matter has been in "causal" contact. By definition, two points are in causal contact if there has been enough time since the beginning of the universe for light to travel from one point to the other. Above this size scale, there had not been enough time for one region to influence another. This is a critical scale and, not unexpectedly, the anisotropies show a marked transition at this angular scale.

Although COBE represented a major breakthrough in our understanding of cosmology, the anisotropies it mapped are at an angular scale that is larger than we can study with optical astronomy for even the largest structures we can observe today, such as the Great Wall[2] of galaxies discovered by Margaret Geller and John Huchra of Harvard in the early 1990s.

Still, questions such as: "What is the actual density of the universe?", "What fraction of all the matter in the universe is 'baryonic?'"(i.e., composed of normal matter), and "What is the correct value of the Hubble parameter?" (i.e., expansion rate of the universe) remain unanswered. More accurate measurements of the small-scale fluctuations in the CMB should help us answer these and other questions about the physical processes in the very earliest stages of the universe, and which have profound implications for our understanding of fundamental particles, forces, and symmetries in nature.

Two satellites currently under construction, one by NASA and the other by the European Space Agency (ESA), which are scheduled for launch early in the next century, should help us answer these questions. Called *MAP* and *Planck,* respectively, these two satellites will map the entire microwave sky at angular scales of a few arc minutes to 180°, thereby allowing us to specify the temperature fluctuations that were present in the universe at the recombination time. When the data are analyzed, we may be able to resolve many of today's unanswered questions in cosmology.

Most students do not attempt a sophisticated study of cosmology until graduate school, yet cosmology offers one of the most intriguing applications of physics imaginable. In this article we will describe a simplified method of modeling CMB spectra[3] and maps whereby undergraduates and advanced high-school students can participate in this most exciting branch of modern physics research. (For an extensive list of mapping experiments and their respective websites, see Ref. 4, page 35.)

Modeling Small-Scale Fluctuations in the CMB

Even before the data arrive from *MAP* and *Planck*, cosmologists are making computer models of the spectrum of possible anisotropies. Using the data we already have, we can already place constraints on the various cosmological parameters, such as the expansion rate of the universe, density of matter, and whether the universe is open or closed.

The Standard Cosmological Model, often called the Big Bang theory, states that the universe began from an initially hot, dense state, from which it is still expanding today. The experimental pillars of this model are the observed redshifts of galaxies, the cosmic microwave background radiation, and the measured abundances of hydrogen, deuterium, and helium.[5] In addition, we observe that the universe is homogeneous and isotropic at sufficiently large scales.

One consequence of this model is the so-called Hubble law, which predicts that the velocity of a galaxy as seen from Earth is a linear function of its distance: $v(r) = H_0 r$. Strictly speaking, this is an approximation, and holds only for velocities much less than the speed of light.

One important parameter in the context of this model is Ω_0 ("Omega naught"), the ratio of the total density of all matter in the universe to the so-called critical density:

$$\rho_{\text{total}}/\rho_{\text{critical}} = \Omega_0 \tag{1}$$

(See Box 2 for a definition of ρ_{critical} and how it is related to H_0.)

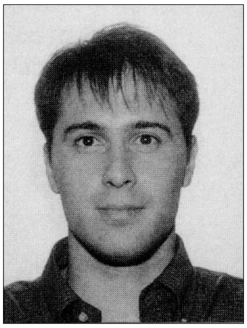
Universes with $\Omega_0 > 1$ are said to be *closed*, implying that there is sufficient matter to cause the universe to eventually collapse back on itself ("Big Crunch"). Universes with $\Omega_0 < 1$ are said to be *open*, implying that the universe will expand forever. Universes with $\Omega_0 = 1$ are called *flat*, implying that the curvature of space is zero when the total density of all matter equals the critical density of the universe. This is equivalent to saying that the total kinetic energy of expansion exactly balances the gravitational potential energy of all the matter in the universe, thus producing an expansion rate that asymptotically approaches zero. (This is strictly true only if we don't consider vacuum energy density.)

Within the Standard Cosmological Model, there are basically two theories proposed to describe the growth of perturbations in the CMB, which lead to different expectations in the spatial distribution of CMB anisotropies and their spectra: random density fluctuations and topological defects in space-time itself.

The Inflationary Model is based on a random distribution of density perturbations in the earliest moments of the universe. A very rapid expansion known as *inflation* suddenly stretched out the universe by a factor of 10^{20} or so. Inflation may have started somewhere around 10^{-35} of a second after the Big Bang, and lasted perhaps 10^{-33} of a second (no one is really sure),[6] but it had the effect of smoothing out the amplitudes of the original density perturbations. This model provides one explanation of why the original density perturbations would have left such small-scale anisotropies in the CMB.

During the first few minutes of the universe, all the primordial hydrogen and helium nuclei were formed, but for the next 300,000 years the density and temperature were so great that the universe was opaque to all forms of electromagnetic radiation. This era is often called the "tight coupling era" because baryons and photons were tightly coupled by electromagnetic interactions in a "photon-baryon fluid" (PB fluid).

The remnants of the original density fluctuations from the Big Bang provided the mechanism for gravity-driven oscillations in the PB fluid. Competition between local grav-



Paolo Natoli is a graduate student at the University of Rome, "Tor Vergata" in physics. He is specializing in work on the CMB and is preparing a thesis on implications of small-scale anisotropy measurements. He has developed a very fast code to generate large CMB maps from the power spectra. Figures 7, 8 and 9 were generated using his methods.

UCSB Physics Dept., on leave from Università di Roma "Tor Vergata"
Viale della Ricerca Scientifica,
1 I-00133 Roma, Italy
Natoli@cfi.ucsb.edu

itational collapse due to excess mass density, and adiabatic expansion due to radiation pressure caused oscillations in the PB fluid, which in turn sent out acoustic waves that propagated at the local speed of sound.[7]

At recombination, some 300,000 years after the Big Bang, matter and radiation essentially decoupled from each other, but the density contrasts remained embedded in the young universe. It is conjectured that a large portion of the mass of the universe is locked up in "dark matter," so named because it is only observed by its gravitational interactions and not by any electromagnetic interactions. Photons don't scatter off dark matter as they do off protons and electrons; hence any dark matter formed early on would have been free to collapse gravitationally at a much earlier era.

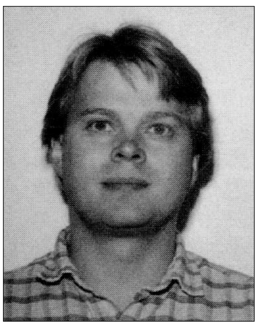
After decoupling, there was much less interaction between the photons and the now neutral matter. The baryons collected in the gravitational potential wells left by the dark matter, later growing into the structures we find today. The photons were free to propagate on their own through the universe, imprinted with the signature of the density perturbations just prior to decoupling.

Recombination was not instantaneous, but is thought to have taken place over a time interval corresponding to a Δz of ~100, or roughly 10% of the redshift of the LSS itself. This had the effect of damping out the smaller wavelength (higher l– see below) peaks in the spectrum.

Both the Inflationary and Topological Defect models predict that in the earliest moments of the universe matter and radiation existed in one state, and the four fundamental forces we observe today were united into one. Somewhere in the tiniest fraction of a second after the Big Bang, the universe passed through a succession of transitions in which first the strong nuclear force differentiated, then the weak nuclear force, and finally electromagnetism.

According to the Topological Defect Models, these transitions broke the original symmetry of the universe and left defects in the fabric of spacetime itself, which could have taken various forms, such as strings, knots, domain walls, and other discontinuities.[8] These regions of discontinuity in spacetime became regions of varying gravitational potential where matter later collected to form the large-scale structures we find in the universe. The mathematics behind the Topological Defect Model is very complicated and not well understood at this time.

The predicted spectrum of anisotropies for each model should look radically different from each other. The Inflationary Model predicts a CMB spectrum that resembles that of a complex sound wave that damps out at high frequencies. The Topological Defect Model predicts a CMB spectrum that rises slowly to one peak and slowly damps out. This contrast is shown in Fig. 4; the spectrum labeled "Strings" is based on the Topological Defect Model, whereas the other three spectra are based on Standard Inflationary Models in which the cosmological parameters are varied, as discussed below.



Michael Seiffert has participated in two major efforts to observe CMB anisotropies at degree angular scales from the South Pole, operated a large-scale anisotropy experiment from White Mountain research station, and contributed to UCSB's most recent CMB balloon flight. He is currently a Research Scientist at the Jet Propulsion Laboratory in Pasadena, California.

Jet Propulsion Laboratory
4800 Oak Grove Drive
Pasadena, CA 91109
michael.d.seiffert@jpl.nasa.gov

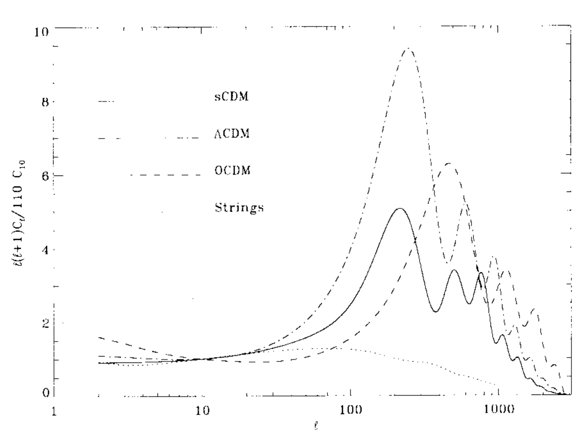


Fig. 4. Predicted CMB spectra for Standard (s), Open (O), and Λ cold dark matter models, and for a String model.

Understanding the Spectrum of CMB Anisotropies in the Standard Inflationary Model

The problem of predicting the CMB spectrum from a chosen set of initial conditions for the perturbations involves solving the first-order general relativity field equations (which may or may not include a cosmological constant, Λ, see description below) for the various components of the cosmic fluid. These include dark matter and neutrinos as well as photons and baryonic matter. These are very complicated equations, but can be viewed as generalizations of the classical fluid dynamic equations, which express the continuity of flow and the conservation of energy and momentum under the influence of gravity.

The PB fluid is visualized more or less as a plasma of relativistic particles, in which the speed of sound is given as

$$v_s = \frac{c}{\quad} \left(\frac{1}{\qquad} \right) \tag{2}$$

where R is the radius of the universe at time t. As $R \to 0$,

i.e., the farther back you go in time, $v_s = \frac{c}{\quad}$.[9]

Each little pocket of matter can be thought of as behaving like a tiny harmonic oscillator. The baryons contribute mass that causes the collapse, and the photons contribute the resistance to collapse in each pocket of material. What results is something like a damped

Box 1. Redshifts Explained

Redshift refers to the apparent stretching of the wavelength of light, $\Delta\lambda$, due to the recession of the source from us, relative to the wavelength we would observe for the same source in a laboratory on Earth, λ. The redshift, z, is defined as follows:

Let λ = wavelength observed for given source in a lab on Earth
Let λ' = wavelength observed for same source, moving away from Earth

Then $z = \dfrac{\lambda' - \lambda}{\lambda} = \dfrac{\Delta\lambda}{\lambda}$

There are different sources of redshift: one is due to the actual velocity of the source relative to Earth, one is due to the stretching of space itself, and another is a gravitational redshift.

For galaxies close to us, the redshift gives a good approximation to the actual velocity of recession of the object, in cases where the stretching of the wavelength of light is much less than one wavelength:

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c} = z$$

For example: the first emission line in the hydrogen spectrum (H-α line) has a wavelength of approximately 656.3 nm. Suppose this line is observed to be shifted toward the red by 0.1%. This gives a velocity of recession of the source of 0.001 c.

The most distant quasars have redshifts close to 5. Using the simple definition above would give a recession velocity of 5 c, which is prohibited by special relativity. For such objects, the relativistic redshift formula must be used, from which the recession velocity can be calculated:

$$\frac{v}{c} = \frac{(z+1)^2 - 1}{(z+1)^2 + 1}$$

Thus, a quasar with a redshift of 4.9 has a recession velocity of approximately 0.944 c.

If the recession velocities are due to the expansion of space, then we can use Hubble's law (see definition in Box 2) to calculate the distance to any object, once we have calculated its velocity (for $v/c \ll 1$) from its measured redshift. The problem with this method is that independent measurements place H_0 between about 50 and 80 km/s/Mpc. Assuming no gravitational redshift and no deviation from the "Hubble flow," the only true measurement we can get from an object's redshift is the factor by which space has stretched since the time the light left the object. This is the cosmological redshift.

Let R_0 = the distance to a source now
Let R = our equivalent distance to the source when the light left it

Then, $z = \dfrac{R_0}{R} - 1$

Thus, a redshift for the LSS of 1000 tells us that the universe has expanded by a factor of 1000 since the time the CMB photons we see now decoupled from matter.

harmonic oscillator in which density (instead of displacement) is the variable, the expansion rate of the universe (H_0) provides the damping term, and gravity provides the driving force for each oscillating pocket of PB fluid.[9]

When the equations are solved over a range of spatial scales (or wave numbers), the solution takes the form of a series of sines and cosines (Fourier series), which lead to a particular spectrum of density (and hence temperature) fluctuations.

The elegance of this model is that the details of the CMB spectrum depend heavily on the gravity-driven acoustic oscillations. The amplitudes of the peaks depend

Box 2. Estimating ρ_c and H_0

The critical density of the universe (taken to be homogeneous at sufficiently large scales) is defined as the density necessary to make the gravitational potential energy of the universe just balance the kinetic energy of expansion. We define the gravitational potential of any test mass at infinity, relative to a large mass, as 0. In our Flat Universe model, the kinetic energy of expansion of this test mass also approaches 0 as it approaches infinity. We define the initial kinetic energy of expansion and initial gravitational potential energy at some initial radius r to be K_0 and U_0, respectively.

$$K_0 = \frac{1}{2}mv_0^2 \qquad \text{and} \qquad U_0 = -\frac{GmM}{r}$$

where G is the universal gravitational constant, which has the value 6.67×10^{-11} Nm^2 kg^{-2} in SI units.

By conservation of energy,

$$K_0 + U_0 = K_\infty + U_\infty = 0 \qquad \text{which leads to} \quad \frac{1}{2}mv_0^2 = \frac{GmM}{r}.$$

Isolating v_0, we can calculate the escape velocity necessary for mass m to escape the gravitational pull of M and just barely make it to "infinity," asymptotically approaching zero kinetic energy:

$$v_0 = \sqrt{\frac{2GM}{r}}$$

Now, consider a sufficiently large, spherical region of the universe in which the density is ρ. The mass M of this region is

$$M = \frac{4}{3}\pi r^3 \rho$$

Substituting this expression into our equation for escape velocity, we get

$$v_0 = \sqrt{\frac{8\pi G\rho r^2}{3}}$$

Now recall Hubble's law, which relates the recession velocity of any galaxy to its distance from us, r:

$$v(r) = H_0 r$$

where H_0 is the Hubble parameter, in units of km/s/Mpc. Whether $v(r)$ is equal to the escape velocity necessary for the universe to escape its own gravitational influence (so to speak), depends on whether the average density, ρ, is equal to the critical density, ρ_c, necessary to balance the expansion at some time in the future. To estimate the critical density, we equate $v(r)$ and v_0, giving us

$$v(r) = H_0 r = \sqrt{\frac{8\pi G\rho_c r^2}{3}}$$

which, after a bit of algebraic massaging, leads to an expression for the critical density of the universe in terms of H_0 and the universal gravitational constant, G:

$$\rho_c = \frac{3H_0^2}{8\pi G}$$

If we take the current conservative value of H_0 = 65 km/s/Mpc, after converting units we come up with $\rho_c \approx 8 \times 10^{-30}$ g/cm^3, which turns out to be at least an order of magnitude *larger* than the density estimated from all observable matter!

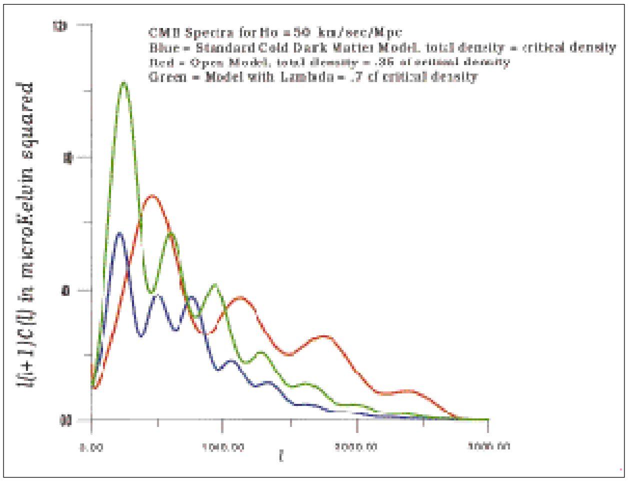


Fig. 5. Spectra for three cosmological models, generated by CMBFAST.[3] H_o = 50 km/s/Mpc; order number is plotted along horizontal axis.

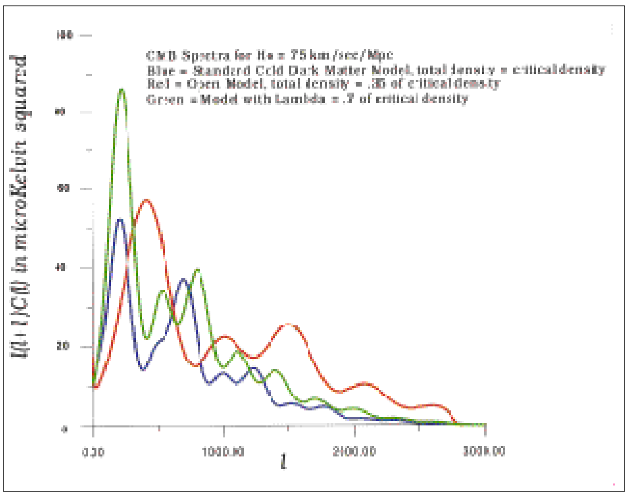


Fig. 6. Same spectra as in Fig. 5, except H_o is set at 75 km/s/Mpc.

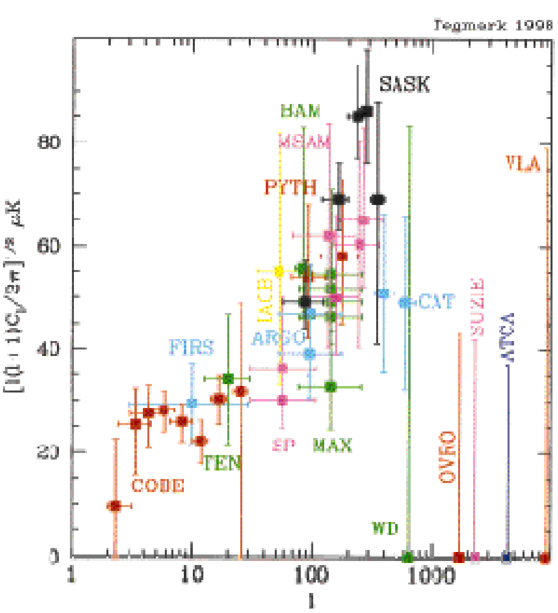


Fig. 10. Data points thus far from COBE, ground-based, and balloon-borne experiments. Taken from Max Tegmark[10] with permission. For a complete explanation and listing of experimental results, see Ref. 7.

on the ratio of baryons (inertia) to photons (restoring force), as well as the percent of dark matter. The baryon density (Ω_b) itself depends on the rate at which space is stretched as the universe expands—the Hubble parameter, H_0. The interdependence of Ω_b and H_0 is often expressed in the parameter $\Omega_b h^2$, where $h = H_0/100$. The initial conditions of the universe determine the phase shift of the peaks in the spectrum. Thus it should be possible to extract a wealth of cosmic information from the spectrum of CMB anisotropies.

All of the above description is a simplified version of how the model predicts the spectrum of CMB anisotropies we *should* measure, given a certain set of initial conditions. The problem of experimentally *deriving* the CMB spectrum from observations of microkelvin fluctuations in the background temperature of the universe is a separate issue! This problem is similar to trying to decipher the sound signals from an orchestra playing random tunes from a great distance away, when there is noise contamination from nearby traffic and shouting children that you

Figs. 7, 8, 9. Maps of expected CMB anisotropies for spectra shown in Fig. 5. Values are in microkelvins above or below zero. The 2.725-K black-body background temperature has been subtracted out. Red represents warmer-than-average and blue cooler-than-average regions on LSS. Maps have no particular coordinate system; fluctuations are purely statistical and do not correlate with any known structures.

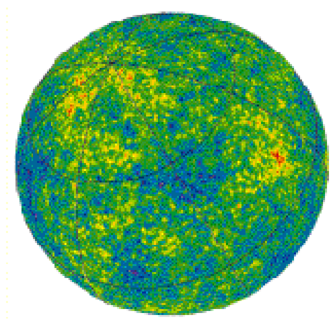


Fig. 7. Standard Model

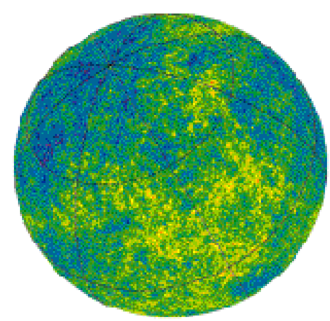


Fig. 8. Open Model

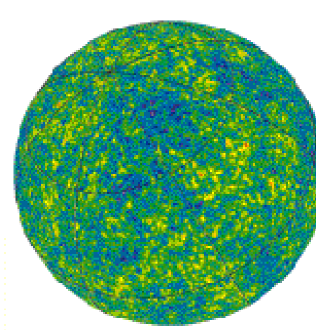


Fig. 9. Flat Model with $\Omega\Lambda$ = 0.7 ot total density.

must first interpret and remove. In measuring the CMB, we have to remove the signal contamination that comes from looking through our galaxy, intergalactic dust, stray radiation, and instrument noise.

Let us digress for a moment to the case of a vibrating guitar string, with a characteristic length, density, diameter, and tension. If the string is plucked, a pleasant sound will be produced, which is a combination of the normal modes of vibration of the string. The spectrum of the sound wave will tell you the relative power in each mode, and the actual waveform can be approximated by adding a series of sine waves with frequencies that are integral multiples of the fundamental (longest wavelength). The more terms you include in the series, the more closely you will approach the actual sound.

Now consider a circular drumhead with a characteristic diameter, thickness, density, and tension. If the drumhead is struck, it will vibrate in a characteristic manner that is a combination of its normal modes of vibration. Because it is a two-dimensional surface, however, its normal modes include vibrations with nodes that form concentric circles as well as nodes that are radii. The superposition of these two-dimensional normal modes produces the sound you hear, which can be modeled with a series of time-varying Bessel functions in radius and azimuthal angle.

Finally, imagine a giant circular balloon filled with water, supported by strings that allow it to vibrate freely if tapped, but not roll away. You tap the balloon in several places, which causes pressure waves to travel through the balloon and also around its surface. After a few seconds of oscillation, the entire balloon is suddenly frozen, so that the lumps in its surface at that instant are frozen into it forever. The variations in the surface height of the spherical balloon can be described with a spherical harmonic series as a function of two angles, one measured between "equator and poles" so to speak ("small circles"), and the other around the equator with respect to some arbitrary reference ("great circles").

Since we are really looking out toward the LSS at the inside surface of an imaginary sphere, the spectrum of anisotropies is best described with a spherical harmonic expansion in temperature:

$$T(\theta,\varphi) = \Sigma a_{l,m} Y_l^m (\theta,\varphi) \tag{3}$$

where T is the measured sky temperature, in microkelvins, as a function of two orthogonal angles, θ and φ, and the Y_l^m are the spherical harmonics. (The Y_l^m are products of a function of $\cos(\theta)$ and $[\cos(m\varphi) + i \sin(m\varphi)]$. For a full treatment, see any good book on partial differential equations!

The number of nodes between equator and poles is represented by l, while m represents the number of longitudinal nodes; m varies from $-l$ to $+l$, with $l \geq 0$, and for each l the sum is taken over all possible m's. The $a_{l,m}$ are the expansion coefficients, which are like the individual amplitudes in a Fourier series.

We define a new quantity, C_l, which is the squared average of the expansion coefficients for any l, averaged over all possible m's:

$$C_l = \langle \, | a_{l,m}|^2 \rangle \tag{4}$$

The angular power spectrum, defined as $l\,(l + 1)C_l$, gives the relative strength of the temperature variation we can measure for any order l, averaged over all possible m's for that value of l.

The $l = 0$ term is often called the monopole term, and corresponds to the surface of a completely smooth and featureless sphere. In terms of the CMB, the monopole term is the 2.725 $\pm$ 0.002-K microwave background, which is uniform out to a few millikelvin.

The $l = 1$ term is called the dipole, and corresponds to a sphere with one part more positive than average and the other more negative. In terms of the CMB, "positive" means warmer than background, and "negative" means cooler. The dipole variation of the CMB is approximately $\pm$ 0.003 K, relative to the monopole term. (Technically speaking, the origin of the dipole anisotropy in the CMB is not cosmological, but is an effect of our motion relative to the CMB, which causes the sky to appear warmer in the direction of our motion, and cooler in the opposite direction.)

The $l = 2$ term is the quadrupole and represents a sphere with two warmer regions around the equator and a cooler region at the north and south poles. The quadrupole term of the CMB has an amplitude that is two orders of magnitude smaller than the dipole fluctuation. A qualitative representation of the first three terms of the spherical harmonics for the CMB are shown on the cover.

The angular size $\Delta\theta$ of a temperature fluctuation on the sky depends on the order number l, approximately as follows:

$$\Delta\theta \approx \frac{\pi}{l} \text{ radians} \tag{5}$$

Thus, COBE measured to $l \sim 20$, or somewhat less than 10° of arc on the sky.

The important lesson in all this is that when we plot the CMB power spectrum as a function of l, the distribution and amplitude of the peaks in the spectrum depend heavily upon the values of the cosmological parameters such as baryon density versus radiation density, H_0, whether or not there was cold (nonrelativistic) or hot (relativistic) dark matter, and whether the universe was seeded by these random density/temperature perturbations or by some other, mechanism as yet poorly understood (see Fig. 4.)

Hence the reasoning: *if we can map the anisotropies and measure the power spectrum of the CMB, we can understand the cosmological parameters that constrain the fundamental physics of the universe.*

Predicting the Spectra for Inflationary Scenarios

The cosmological parameters that we are seeking to measure include:

H_0, the Hubble parameter. This is assumed to be a constant expansion rate per Megaparsec (1 Mpc $\approx$ 3.26 million light years). Independent measurements place H_0 somewhere between 50 and 80 km/s/Mpc.

Ω_0, the ratio of the total density of the universe to the critical density.

Ω_b, the ratio of the total density of baryonic (normal matter) to the critical density of the universe.

Ω_{cdm}, the ratio of the density of cold dark matter to the critical density of the universe. Cold dark matter refers to nonluminous yet gravitationally interacting matter, whether baryonic or exotic, that is nonrelativistic.

Ω_{hdm}, the ratio of the density of hot (i.e., relativistic) dark matter to the critical density of the universe. Hot dark matter could take the form of massive neutrinos, or some other unknown particle.

Ω_Λ, the ratio of the vacuum energy density to the critical density of the universe. Λ (lambda) is Einstein's cosmological constant, some unknown vacuum energy density that contributes to the expansion of the universe. Recent independent measurements of high-z Type Ia supernovae and massive galaxy clusters appear to support a non-zero value of Λ.[11] If true, this would have profound consequences on cosmology and in particular on the CMB anisotropy.

Another important parameter that must be considered is the fraction of neutral helium that existed at the Recombination Era. Helium atoms form at a higher temperature than hydrogen atoms, thus a certain portion of all baryons became locked up in neutral helium before the formation of neutral hydrogen. The fraction of neutral helium in the primordial universe is thought to have had a value between 0.23 and 0.26 of the total baryon density.

One of the codes for modeling CMB anisotropies is called CMBFAST.[3] This user-friendly Fortran code accepts input for any combination of cosmological parameters, and returns the power spectrum as a function of order number, in plain text format. This output can then be used with any standard graphing program (that will accept a large number of data points), and the resulting spectrum can be examined for each input model.

One of our group (Natoli) has developed a code that will take the output of CMBFAST and generate temperature anisotropy maps at varying scales. Using these two routines, students can see the expected effects on the CMB itself for the various cosmological parameters (such as H_0 and Ω_b) at a particular length scale. Since each peak in the spectrum falls at a particular l value, and each l corresponds to a particular angular scale on the sky, by choosing the appropriate mapping scale students can see the changes in the predicted CMB temperature fluctuations for different input parameters. The first peak at $l \sim 200$ corresponds to a length scale on the sky of approximately one degree. This should be discernable on a full sky map of 1024 pixels in width—the width of most PC monitors. The flat universe models run in a few minutes on a reasonably fast (120 MHz or faster) Pentium PC, with at least 16 Meg of RAM (although the open models require a main frame at this time). The same models take an hour or more to run on a 486-33. Thus it is possible for advanced high-school students to model CMB anisotropies in the same manner as is done by professional cosmologists.

The unmodifiable executable file of CMBFAST will be available early in 1999 from our website, as well as the code to take the output spectrum and map the anisotropies, along with instructions for inputting the parameters. Interested readers can download these files from www.deepspace.ucsb.edu, under Interactive Astrophysics, CMB models. Students will be able to input their combinations of cosmological parameters, take the output from CMBFAST, run it through a graphing program, and plot the resulting spectrum of anisotropies.

Figure 5 shows the expected spectra for three models: a standard model (blue curve), with $\Omega_b = 0.05$, $\Omega_{cdm} = 0.95$, and $\Omega_\Lambda = 0$, giving $\Omega_0 = 1$ for a Flat Universe, an Open model (red curve), with $\Omega_b = 0.05$, $\Omega_{cdm} = 0.30$, and $\Omega_\Lambda = 0$, giving a total density of 0.35 of the critical density, and a Flat Universe with a positive vacuum energy density (green curve), in which $\Omega_b = 0.05$, $\Omega_{cdm} = 0.25$, and $\Omega_\Lambda = 0.7$, as suggested by recent measurements of high-redshift supernovae. H_0 is 50 km/s/Mpc. The order number, l, is plotted on the horizontal axis, and the $l(l + 1)C_l$ are plotted on the vertical axis in units of μK^2 (microkelvin squared).

For comparison, in Fig. 6 we show the effect of using $H_0 = 75$ km/s/Mpc, while keeping all the other model parameters the same. You can see that changing the value of H_0 has more consequence for temperature anisotropies at length scales much less than 1° (higher order than $l = 200$).

The maps of expected temperature fluctuations in microkelvins, relative to the accepted value of the average temperature of the universe ("monopole term") of 2.725 K for the spectra are shown in Figs. 7, 8, and 9 for the Standard, Open, and "Lambda" models, respectively. The color scheme is such that red represents regions that are warmer than average, and blue represents slightly cooler regions. The location of the first Doppler peak in the spectrum has a noticeable effect on the scale of the temperature fluctuations that can be discerned by comparing the maps for the Standard and Open models. In the Standard model, the first peak occurs at $l \approx 200$, and the anisotropy scale is at 1.0°. In the Open model, the first peak occurs at $l \approx 500$, and you can see the clustering of anisotropies at a finer scale.

Figure 10 shows the (normalized) angular power spectrum in microkelvins, derived from experimental CMB measurements made to date, plotted according to the l values. The clustering of power around $l = 200$ or so is quite suggestive, although the uncertainties in the higher modes do not yet allow us to accurately extract the cosmological parameters we are looking for.

We are finding these modeling exercises useful with undergraduate astrophysics students at UCSB. For high-school seniors who have taken the calculus-based advanced placement physics course, the study of cosmology offers a unique and fascinating motivation to come to school in those last few weeks after the AP exam, and before graduation! This work is part of the Remote Access Astrophysics Project (RAAP) at UCSB. Now in its tenth year, RAAP is dedicated to encouraging the study of astronomy and astrophysics at the secondary school and college levels. The curricula and program for this paper will be available on our website in January 1999, from www.deepspace.ucsb.edu.

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