Relativistic solutions to directed energy

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ABSTRACT

This paper analyses the nature and feasibility of using directed energy to propel probes through space at relativistic speeds. Possible mission scenarios are considered by varying the spacecraft mass, thickness of the sail and power of the directed energy array. We calculate that gram-scaled probes are capable of achieving relativistic speeds and reaching Alpha Centauri well within a human lifetime. A major drawback is the diffraction of the beam which reduces the incident power on the sail resulting in a terminal velocity for the probes. Various notions of efficiency are discussed and we conclude that directed energy propulsion provides a viable direction for future space exploration.

Keywords: directed energy, interstellar exploration, relativistic spacecrafts, wafer probes, laser propulsion

1. INTRODUCTION

For more than half a century humans have been on the quest to further space exploration. So far, all attempts to explore the vast expanse of space have relied on chemical propulsion. Rockets powered by combustion are extremely inefficient and need to carry massive fuel payloads. If we continue to follow in the footsteps of chemically powered rockets, intersteller exploration will be a hopeless endeavour. It has recently been proposed by Lubin,[1] and others[23] that space travel using directed energy propulsion could be the next leap in human space exploration. With the rapidly developing technological advancements in directed energy and silicon wafer technology, one can envision a single photon driver sending out an armada of wafer probes.[4] For a sufficiently powerful directed energy source and small wafer probes, relativistic speeds can be reached within a matter of minutes. This technology could allow the human race to study other star systems within a human lifetime and improve our understanding of the cosmos.

In this paper we consider some theoretical aspects of relativistically propelled spacecraft. Using the conservation of momentum we find the rate at which the spacecraft moves. Diffraction effects of the laser are also considered and shown to be the limiting factor in the maximum velocity a spacecraft can reach. We also quantitatively demonstrate that directed energy propulsion is scalable, efficient and is likely to be the next step in our never ending quest for space exploration.

2. METHODOLOGY

Lubin[1] outlines the DE-STAR (**D**irected **E**nergy **S**ystem for **T**argeting of **A**steroids and **E**xplo**R**ation) scale for classifying the size and power of directed energy arrays. Apart from it's applications in planetary defence[5],[6] directed energy arrays can also be used to propel spacecraft at relativistic speeds. In particular, this paper primarily refers to the DE-STAR 4 array which is a square array, 10 km on a side with a power output of 70 GW. Consider such an array emitting a stream of photons of wavelength λ which strike a sail moving at a velocity $v(t)$. In the regime where diffraction effects are small and the spot size of the laser beam is less than the size of the sail, conservation of momentum enables us to find relationship between v and t. In the frame where the DE-STAR is at rest (we call this the ground frame), the momentum of each emitted photon is given by $p_i = \frac{h}{\lambda}$. In the sail's frame this photon is observed to have a Doppler shifted wavelength $\lambda_{obs} = \lambda\gamma\,(1+\beta)$ and is emitted

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with the same wavelength as it was incident with. There is a further factor of Doppler shift associated with transforming the reflected the photon from the sail's frame to the ground frame. $\lambda_{em} = \lambda\gamma^2(1+\beta)^2$.

$$p_i = \frac{h}{\lambda} \qquad p_f = -\frac{h}{\lambda}\left(\frac{1-\beta}{1+\beta}\right) \tag{1}$$

Assuming the DE-STAR has a power P and emits photons at a rate n_0. The rate at which photons are incident on the sail (n_i) is reduced due to the relative motion between the sail and the array; $n_i = n_0(1-\beta)$. Since the rate at which photons strike the sail decreases with velocity, we also expect the impulse imparted on the sail to decrease with velocity (and time). Let n_f be the rate at photons bounce off the sail after reflection. Due to the reflectivity (α_1) of the sail, $n_f = \alpha_1 n_i$. The rate of change of the photons momentum is given by:

$$\frac{\Delta p_{photon}}{\Delta t} = n_f p_f - n_i p_i = -\frac{hn_0\left(1-\beta\right)}{\lambda}\left[\alpha_1\left(\frac{1-\beta}{1+\beta}\right)+1\right] \tag{2}$$

By taking Δt to be very small we can approximate eq.2 by a continuous time derivative. The quantity hcn_0/λ can be interpreted as the rate at which energy is expelled from the DE-STAR. By definition, $hcn_0/\lambda = P$ and our results can interpreted in terms of the power emitted by the laser array. Since momentum is conserved in the DE-STAR spacecraft system, we can say: $\frac{dp_{photon}}{dt} + \frac{dp_{sc}}{dt} = 0$

$$\frac{dp_{sc}}{dt} = \frac{P}{c}\left(1-\beta\right)\left[\alpha_1\left(\frac{1-\beta}{1+\beta}\right)+1\right] \tag{3}$$

The relatvivistic momentum for the spacecraft at a given time is $p_{sc} = mv\left(t\right)\gamma\left(t\right)$. Differentiating this and plugging into eq.3, we find:

$$m\dot{\beta}\gamma^3 = \frac{P}{c^2}\left(1-\beta\right)\left[\alpha_1\left(\frac{1-\beta}{1+\beta}\right)+1\right] \tag{4}$$

Equation.4 is a differential equation governing how the velocity of the spacecraft increases as a function of the power output. In the special case of a perfectly reflecting sail at all incident wavelengths ($\alpha_1 = 1$), eq.4 simplifies considerably.

$$\dot{\beta} = \frac{2P}{mc^2\gamma^3}\left(\frac{1-\beta}{1+\beta}\right) \tag{5}$$

Equation.5 can be integrated to yield a closed form expression relating β and t. The constant of integration is chosen such that the spacecraft starts at rest at $t = 0$.

$$t = \frac{mc^2}{6P}\left[\frac{\left(1+\beta\right)\left(2-\beta\right)\gamma}{\left(1-\beta\right)} - 2\right] \tag{6}$$

Equation.6 gives us the time it takes the spacecraft to get to a certain velocity. As consistency check, it is helpful to examine what happens in the limit where $\beta << 1$. The time taken to reach a certain velocity to first order in $\frac{v}{c}$ is $t = \frac{mcv}{2P}$. In the non-relativistic limit, the force due to radiation pressure is $F = \frac{2P}{c}$. Combining these two, the small velocity limit of equation.6 can be expressed as:

$$mv = Ft \tag{7}$$

In this non-relativistic limit, we recover the Newtonian equation of impulse for radiation pressure acting on a mass m for a time t. This is expected since our analysis of this system was based upon momentum transfers between the photons and the sail.

For a given mass and power of the array, inverting eq.6 results in a plot of velocity of the probe as a function of time.

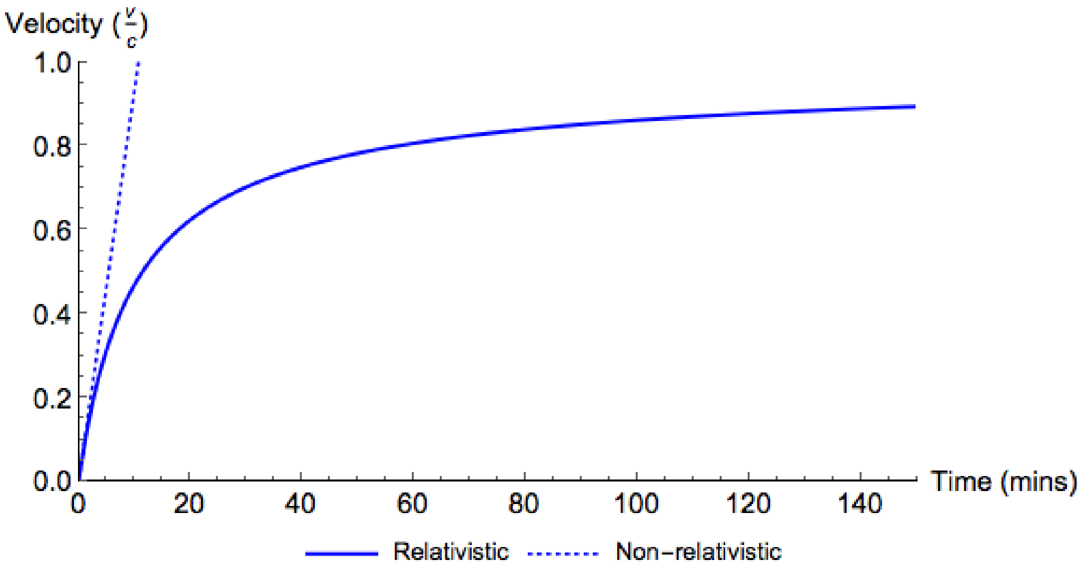

Figure 1: Velocity of a 1 gram probe propelled by a 70 GW array as a function of time as measured by an observer in the ground frame. Relativistic effects become significant very quickly. In a little over 10 minutes, the probe is traveling at half the speed of light. By the fourth day, the probe is moving at more than 99% the speed of light. The naive non-relativistic calculation is bad approximation for the large velocities the probe attains.

These numbers are unfathomable by today's chemically powered rockets. Traveling at this rate, the probe will reach Proxima Centarui in under four and a quarter years in the Earth's frame of reference. In considering the general problem of DE-STAR array propelling a spacecraft forward, the above analysis is incomplete since a key issue has been left out; the diffraction effects of the laser.

3. SAIL OF FINITE SIZE

The calculations until now have only been valid for a regime in which the spot size of the DE-STAR is smaller than the size of the sail. At larger distances, the spot size of the sail increases due to diffraction thus the sail doesn't receive all the power from the DE-STAR. A fourth generation DE-STAR emitting light at a wavelength of 1054 nm with a side length of 10 km has a beam divergence given by:

$$\theta = \frac{2\lambda}{d} \approx 2 \times 10^{-10} rad \tag{8}$$

These spacecraft are expected to reach distances of over a light year. At a distance of a light year, the sail size to catch the entire DE-STAR beam would be on the order of a million meters. Clearly, this is unrealistic and the situation where the spot size is larger than the reflector must be considered. Each probe comprises of a sail of mass m_s attached to a spacecraft of mass m_0. Given the rate at which technology is improving, it is modest to assume a reflector 1 micron in thickness made out of a material with density 1.4 g/cc. Such a reflector having a mass of 1 g, would be 85 cm on a side. Let L_0 be the maximum distance at which the spot size of the DE-STAR fully illuminates a sail of side length D.

$$L_0 = \frac{dD}{2\lambda} \tag{9}$$

At any distance x, where $x > L_0$, the laser spills beyond the area of the sail and the power incident on the sail is reduced by a factor of $\left(\frac{x}{L_0}\right)^2$. Equation 5 is only valid when $x < L_0$. In general, the motion of the spacecraft will be determined by eq.10.

$$\dot{\beta} = \begin{cases} \frac{2P}{mc^2\gamma^3}\left(\frac{1-\beta}{1+\beta}\right) & x < L_0 \\ \frac{2P}{mc^2\gamma^3}\left(\frac{1-\beta}{1+\beta}\right)\left(\frac{L_0}{x}\right)^2 & x > L_0 \end{cases} \tag{10}$$

When $x < L_0$, the acceleration depends only upon the velocity and a closed form solution of $t(\beta)$ can be obtained. When $x > L_0$, the acceleration depends on both the velocity and the position. This results a nonlinear second order differential equation and finding a closed-form solution for $x(t)$ is difficult.

Instead, it is insightful to find how the velocity varies with distance by rewriting $\dot{v} = \frac{dv}{dx}v$ and integrating to find a relationship between v and x.

$$\frac{2\beta - 1}{3\gamma(1-\beta)^2} + \frac{1}{3} = \frac{2P}{mc^3}x \tag{11}$$

Equation.11 is only valid in the $x < L_0$ regime. The constant of integration has been chosen such that at $x = 0$, $v = 0$. Consider a 2 g probe consisting of a 1 g spacecraft and a 1 g sail under constant illumination by a 70 GW DE-STAR. From eq.9, $L_0 \approx 4 \times 10^9$ m. To put these distances in perspective, the distance to Proxima Centauri is 10 million times more than L_0. If the spacecraft wants to get there, it spends almost all it's travel time in the $x > L_0$ regime. From eq.11 we find that under constant illumination from a 70 GW array, the velocity of the spacecraft at $x = L_0$ is $\beta_0 = 0.13$.

By the time the spot size equals the sail size, the spacecraft is already moving at a fairly relativistic speed of $0.13c$. In order to further analyze the spacecraft's motion, the $x > L_0$ regime needs to be considered. Integrating and using the condition that at $x = L_0$, $\beta = \beta_0$, we get a relation between β and x in the $x > L_0$ regime.

$$\frac{1 - 2\beta}{3\gamma(1-\beta)^2} = \frac{2PL_0^2}{mc^3x} + 0.31 \tag{12}$$

From eq.12, we find that as $x \to \infty$, $\beta_\infty = 0.18$. The maximum velocity that this spacecraft can reach is $0.18c$. The instant the spacecraft enters the $x > L_0$ regime, the incident power on the sail decreases quadratically resulting in a terminal velocity for the spacecraft. The effect of diffraction is even more hindering the larger the spacecraft. Consider a 2000 kg probe consisting of a 1000 kg sail 850 m on a side and a 1000 kg spacecraft. When the spot size equals sail size at a distance of $L_0 = 27$ AU, the velocity of the spacecraft is $0.0046c$. The terminal velocity of this spacecraft is $0.0064c$. The larger the spacecraft, the lower it's terminal velocity. A spacecraft's velocity is limited by the velocity it can attain before entering the $x = L_0$ regime. Thus we see that directed energy propulsion is heavily diffraction limited.

3.1 Terminal velocity in non-relativistic limit

In the non-relativistic limit, eq.10 reduces to:

$$m\dot{v} = \begin{cases} \frac{2P}{c} & x < L_0 \\ \frac{2P}{c}\left(\frac{L_0}{x}\right)^2 & x > L_0 \end{cases} \tag{13}$$

The same approach as before to find $v(x)$ in the two different regimes yields:

$$v\left(x\right) = \begin{cases} \sqrt{\frac{4P}{mc}x} & x < L_0 \\ \sqrt{\frac{4PL_0^2}{mc}\left(\frac{2}{L_0} - \frac{1}{x}\right)} & x > L_0 \end{cases} \tag{14}$$

When at infinity, the maximum velocity that can be reached is given by:

$$v(\infty) = v_{\max} = \sqrt{\frac{8PL_0}{mc}} = \sqrt{2}v_0 = 1.414v_0 \tag{15}$$

For massive spacecrafts that don't reach relativistic speeds before entering the $x > L_0$ regime, eq.15 is a very good approximation. Over 70% of the spacecraft's terminal velocity is reached before $x = L_0$. Thus, optimizing spacecraft speed is thus a matter of maximizing the speed attained before diffraction effects start becoming relevant.

3.2 Optimizing spacecraft design

Assume a spacecraft of mass m_o is to be accelerated as fast as possible, what is the optimum sail size to maximize velocity? On one hand the bigger the sail, the more time the spacecraft spends in the $x < L_0$ regime. However, the spacecraft is also more massive so it's acceleration is reduced. Letting $x = L_0$ in eq.11:

$$\frac{2\beta_0 - 1}{3\gamma_0(1-\beta_0)^2} = \frac{2P}{mc^3}L_0 - \frac{1}{3} \tag{16}$$

It can be shown that the left hand side of eq.16 monotonically increases with β_0. Thus, maximizing β_0 is equivalent to maximizing the right hand side. Define g such that:

$$g \equiv \frac{2PL_0}{mc^3} = \frac{PdD}{mc^3\lambda} = \frac{PdD}{(m_o + \rho D^2 h)\,c^3\lambda} \tag{17}$$

The total mass of a probe is $m_0 + m_s$. For a sail of side length D with a fixed thickness h and density ρ, $m_s = \rho D^2 h$. Maximizing g with respect to the side length of the sail yields:

$$\frac{\partial g}{\partial D} = \frac{PD}{c^3\lambda}\left(\frac{\rho D^2 h - m_o}{(m_o + \rho D^2 h)^2}\right) = 0 \tag{18}$$

This condition is met when $\rho D^2 h = m_o = m_s$. Thus, the probe reaches its maximum velocity when the mass of the sail equals the mass of the spacecraft.

3.3 Possible mission scenarios

Under the optimum condition of $m_s = m_0$ several possible mission scenarios for propelling various spacecrafts can be considered. Table 1 lists a variety of missions that span 8 orders of magnitude in spacecraft mass. While directed energy propulsion is optimal for propelling gram scaled wafer crafts at relativistic speeds of $0.2c$, it also has possible applications to for manned interplanetary expeditions in large spacecraft. For instance a 100,000 kg spacecraft capable of containing a manned crew attains a terminal velocity of 0.2% the speed of light. The versatility and scalability of missions make directed energy propulsion an attractive prospect for future space exploration missions. With advances in materials science and nanotechnology, it's possible to envision sails which are a fraction of a micron thick. For a given mass, the thinner the sail is and the larger it can be; thus spending more time in the $x < L_0$ regime. This is demonstrated in table 2

Spacecraft mass (Kg)	Sail size (m)	t_0 (s)	L_0 (m)	β_0	β_{max}
10^{-3}	0.85	195	4.0×10^9	0.13	0.18
10^{-2}	2.7	1070	1.3×10^{10}	0.077	0.11
10^{-1}	8.5	5950	4.0×10^{10}	0.044	0.062
10^0	27	3.33×10^4	1.3×10^{11}	0.025	0.035
10^1	85	1.86×10^5	4.0×10^{11}	0.014	0.020
10^2	270	1.05×10^6	1.3×10^{12}	0.0081	0.011
10^3	850	5.88×10^6	4.0×10^{12}	0.0046	0.0064
10^4	2700	3.32×10^7	1.3×10^{13}	0.0026	0.0036
10^5	8500	1.86×10^8	4.0×10^{13}	0.0014	0.0020

Table 1: A table of possible mission scenarios for the 70 GW DE-STAR 4 for the optimal case where spacecraft mass equals sail mass. A micron thick square reflector made out a material with a density of 1.4 g/cc is assumed. L_0 is the distance at which the sail size equals the DE-STAR spot size. t_0 is the time taken to accelerate from rest to a distance L_0. β_0 is the speed of the spacecraft at L_0 and β_{max} is the spacecraft's terminal speed.

Sail thickness (µm)	Sail size (m)	t_0 (s)	L_0 (m)	β_0	β_{max}
0.01	8.5	682	4.0×10^{10}	0.34	0.43
0.05	3.8	434	1.8×10^{10}	0.25	0.33
0.1	2.7	359	1.3×10^{10}	0.22	0.29
0.5	1.2	233	5.7×10^9	0.15	0.21

Table 2: A 2g probe under constant illumination from a 70 GW DE-STAR under the optimal case of $m_s = m_0$. The sail is constructed out of a material whose density is 1.4g/cc.

Another case of interest is the Breakthrough Starshot case. This involves propelling a variety of low mass spacecraft by a 100 GW DE-STAR array. By numerically solving eq.10, a plot of the velocity of the spacecraft with time can be generated in fig. 2.

The results in figure 2 indicate the possibility of sending spacecrafts at extremely fast speeds. By further optimizing parameters such as using a larger, more powerful array with lighter spacecraft and thinner sails, it is possible to reach tremendously relativistic velocities. Under these conditions in figure 3, probes with sail thicknesses of 1 µm, 0.1 µm and 0.01 µm can reach Alpha Centauri in 25 yr, 15 yr and 10 yr respectively after which it would take 4 years for earth to receive any transmitted data. Given these short time scales, one can imagine interstellar exploration using millions of tiny wafer spacecraft propelled by directed energy systems.

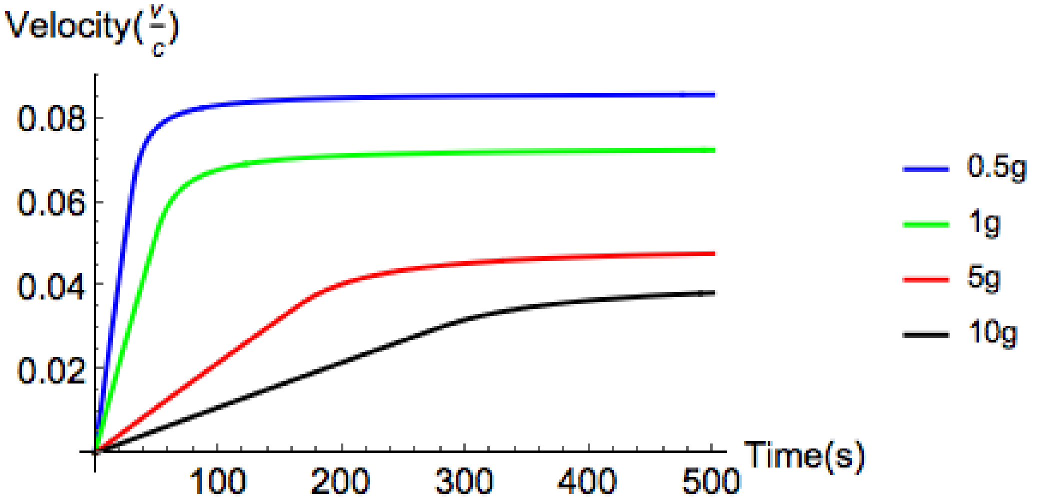

Figure 2: Velocity of various spacecraft propelled by a 1 km, 100 GW DE-STAR array for the optimal case where $m_s = m_0$. We assume a micron thick reflector constructed from a material whose density is 1.4 g/cc. Although the array is more powerful than the array in table 1, the spacecrafts in this plot archive much lower velocities due to the larger diffraction effects. The Breakthrough Starshot case involves a more compact array thus for a given sail, L_0 is reduced by a factor of 10 compared to a DE-STAR 4. Regardless, the spacecraft manage to reach speeds between 0.03c and 0.09c. The spacecrafts accelerate extremely quickly effectively approaching their terminal velocity within a matter of minutes.

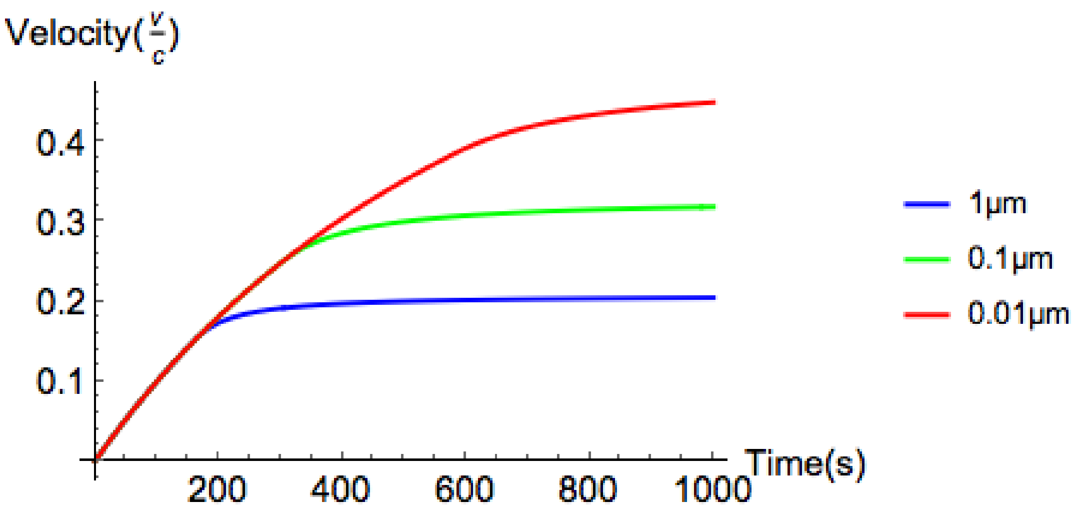

Figure 3: The plot shows the velocity of a 2 gram probe ($m_0 = m_s$) propelled by a 10 km, 100 GW DE-STAR array. $v(t)$ is plotted for different sail thicknesses. The three trajectories are identical early on when they are all in the $x < L_0$ regime and are completely determined by eq.6. As each spacecraft subsequently moves out the $x > L_0$ regime, diffraction effects become important and the spacecraft quickly approaches a terminal speed.

4. ENERGY TRANSFER BETWEEN SPACECRAFT AND ARRAY

So far the dynamics of the spacecraft has been viewed as a consequence of the momentum transfer between the array and the probe. Another equally useful way to view this system is an energy transfer between the spacecraft and the array. As each photon bounces off the sail, it get's increasing redshifted. The energy loss through the redshift of the photons goes into the kinetic energy of the probe. This can be demonstrated quantitatively. During any individual collision momentum is conserved.

$$\Delta p^{sc} = \tilde{p}_i - \tilde{p}_f = \tilde{p}_i - \left(-\frac{1-\beta}{1+\beta}\tilde{p}_i\right) = \frac{2\tilde{p}_i}{1+\beta} \tag{19}$$

Where Δp^{sc} is the change in the spacecraft's momentum during a collision and $\tilde{p}_i$ and $\tilde{p}_f$ are the photons momentum before and after the collision. All quantities are measured with respect to the ground frame. Any given photon is going to transfer a small amount of momentum which causes a small change in the spacecrafts energy. These can be approximated as differentials $d\left(E^2\right) = d\left(c^2p^2 + m^2c^4\right)$.

$$dE^{sc} = \beta c dp^{sc} = \frac{2\beta\tilde{p}_i c}{1+\beta} \tag{20}$$

In a similar manner, the photons energy before and after the collision can be calculated.

$$\Delta\tilde{E} = c\tilde{p}_f - c\tilde{p}_i = \left(\frac{1-\beta}{1+\beta}c\tilde{p}_i\right) - c\tilde{p}_i = -\frac{2\beta\tilde{p}_i c}{1+\beta} \tag{21}$$

The loss in the photons energy due to Doppler shift equals the gain in the probe's kinetic energy during a collision. When stated in this manner, this result seems almost trivial. However demonstrating this result naturally lends itself to a discussion on the efficiency with which the photons transfer energy to the spacecraft.

The efficiency η can be defined to be the fraction of the photon's energy transferred at each bounce.

$$\eta \equiv \frac{\Delta\tilde{E}}{\tilde{E}_i} = \frac{2\beta}{1+\beta} \tag{22}$$

As defined in eq.22, initially energy transfer is inefficient since the photons are reflected back without a significant redshift. As the sail picks up speed, the change in wavelength increases resulting in more efficient energy transfer to the sail. As the spacecraft approaches c, the energy transfer becomes perfect since all the photons that strike the sail get infinitely redshifted. Marx arrived at the same result in his original paper on interstellar travel.[7] While this seems to promisingly indicate highly efficient, seemingly limitless space travel, a critical issue has been missed; the travel time of the photons. While energy transferred per bounce becomes extremely efficient as $\beta \to 1$, the photons take increasingly longer to catch up to the spacecraft. An alternate definition of efficiency can by defined as:

$$\varepsilon \equiv \frac{1}{P}\frac{dE_{probe}}{dt} \tag{23}$$

In eq.23, the efficiency has been defined as the rate of change of the probe's energy divided by power of the incident laser. Equation 23 is defined in terms of the ground frame and measures how much of the power emitted by the light source goes into increasing the kinetic energy of the probe per unit time. This can be calculated explicitly using eq.5

$$\varepsilon = \frac{mc^2\dot{\beta}\beta\gamma^3}{P} = 2\beta\left(\frac{1-\beta}{1+\beta}\right) \tag{24}$$

ε is 0 at $v = 0$ and $v = c$ and is maximized at $\beta_{\varepsilon\,\max} = \sqrt{2} - 1 \approx 0.414$. Thus ε can not take on a value larger than 0.35.

This apparent inconsistency in our results stems from our differing definitions of efficiency. We showed that the loss in energy of the photons due to redshift equals the gain in kinetic energy of the probe. Thus, a photon striking the sail of a spacecraft moving arbitrarily close to c transfers all its energy to the kinetic energy to the spacecaft. The situation seems to indicate the efficiency approaches 1 as $v \to c$. However, efficiency has also been defined in a different way according to eq.23 and these two definitions need to be reconciled.

As stated in eq.23, a possible definition of efficiency is the rate at which energy of the spacecraft is changing at time t divided by the rate at which the DE-STAR emits energy at time t. But energy emitted by the array at time t doesn't instantly affect the spacecraft. At a time t and at a distance x, the spacecraft can only be affected by photons emitted at a time $t - \frac{x}{c}$. Accounting for causality, we only examine the change in kinetic energy of the spacecraft at time t within the light cone of the last emitted photons at a time $t - \frac{x}{c}$. This is exactly what the definition in eq.22 corresponds to and is similar to definition of efficiency used by Simmons et al.[8] Written explicitly:

$$\eta' \equiv \frac{\text{The increase in kinetic energy in time } dt}{\text{The total energy output by the laser in time } d(t - \frac{x}{c})} \tag{25}$$

$$\eta' = \frac{dE_{probe}}{dt} dt \times \frac{1}{P\left(dt - \frac{dx}{c}\right)} = \frac{\varepsilon}{1 - \beta} = \eta \tag{26}$$

Thus, efficiency has been defined in 2 ways that seemingly contradict each other yet describe the dynamics of the same system. Defining η to be the efficiency corresponds to the efficiency with which individual photons transfer energy to the sail. This is defined locally and can be physically measured. ε can not be measured locally by a single observer is not very physical due it's problems with causality. Although it is nonphysical, ε can be insightful in examining the manner in which the velocity of the energy of the spacecraft increases as seen by an observer in the ground frame.

The illustration in fig.4 depicts the energy transfers between the array and the sail.

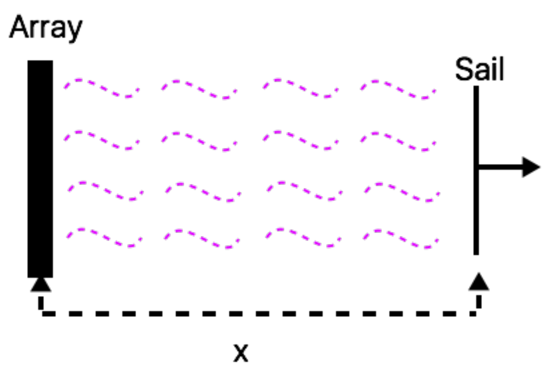

Figure 4: A rough schematic of the array, spacecraft and the photon column between the two (represented by the magenta squiggles). As the spacecraft velocity increases, so does the length of the photon column behind it. As the spacecraft approaches the speed of light, the photons take a long time to catch the sail and most of the energy output from the array goes into elongating the column of photons rather than transferring energy to the sail.

5. CONCLUSION

In this paper, various aspects of directed energy propulsion were analyzed. Using momentum conservation, we were able to find the motion of the spacecraft. The diffraction effects of the laser array placed additional constraints since after a certain distance, only a fraction of the light from the array intercepts the sail. These effects result in the inverse square power loss on the sail and restrict it to a maximum terminal velocity. We also show that when the spacecraft mass equals the sail mass, this terminal velocity is maximized. Due to diffraction effects, large mass spacecraft might never make it to relativistic speeds while gram scaled wafer probes have the potential to reach relativistic terminal velocities. High powered arrays, light spacecraft and sufficiently thin sails make it possible to reach extremely fast relativistic speeds. Directed energy propulsion is also an efficient way of delivering power between the laser and the sail. Due to the effects of Doppler shift, the faster the spacecraft travels, the more efficient the energy transfer is. However, this is a double edged sword since although a fast moving space craft is efficient, the flux of photons on the sail is reduced.

Future work is needed on finding ways to improve the efficiency of energy transfer. One way called 'photon recycling' involves repeatedly bouncing the emitted photons between the sail and array to maximize the fraction of the photon's energy transferred to the sail. Photon recycling is useful in the initial stages of flight when the photon travel time between the sail and array is small. As the spacecraft moves further and faster, it becomes less useful. More work also needs to be done to take into account sources of drag such as interstellar dust that a spacecraft might encounter while traveling relativistically through space.

From the analysis in this paper, we conclude that that directed energy systems for wafer-craft propulsion is a viable solution for unmanned interstellar exploration. While the construction of a 10 km space based laser might seem far fetched today, it's possible that technology advances to a point where this might become a reality.

6. ACKNOWLEDGMENTS

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REFERENCES

1. P. Lubin, "A roadmap to interstellar flight," *arXiv preprint arXiv:1604.01356* , 2016.
2. R. L. Forward, "Roundtrip interstellar travel using laser-pushed lightsails," *Journal of Spacecraft and Rockets* **21**(2), pp. 187–195, 1984.
3. Y. K. Bae, "Prospective of photon propulsion for interstellar flight," *Physics Procedia* **38**, pp. 253–279, 2012.
4. K. A. Beals, M. Beaulieu, F. J. Dembia, J. Kerstiens, D. L. Kramer, J. R. West, and J. A. Zito, "Project longshot: An unmanned probe to alpha centauri," 1988.
5. G. B. Hughes, P. Lubin, J. Bible, J. Bublitz, J. Arriola, C. Motta, J. Suen, I. Johansson, J. Riley, N. Sarvian, *et al.*, "De-star: phased-array laser technology for planetary defense and other scientific purposes," in *SPIE Optical Engineering+ Applications*, pp. 88760J–88760J, International Society for Optics and Photonics, 2013.
6. K. Kosmo, P. Lubin, G. B. Hughes, J. Griswold, Q. Zhang, and T. Brashears, "Directed energy planetary defense," in *2015 IEEE Aerospace Conference*, pp. 1–9, IEEE, 2015.
7. G. Marx, "Interstellar vehicle propelled by terrestrial laser beam," *Nature* **211**, 1966.
8. J. Simmons and C. McInnes, "Was marx right? or how efficient are laser driven interstellar spacecraft?," *American journal of physics* **61**(3), pp. 205–207, 1993.