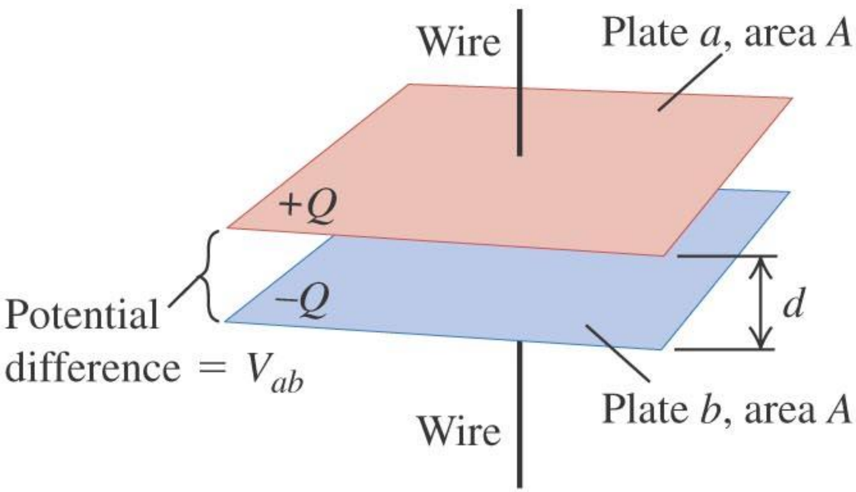
Chapter 24
Capacitors and Dielectrics

What is Capacitance?

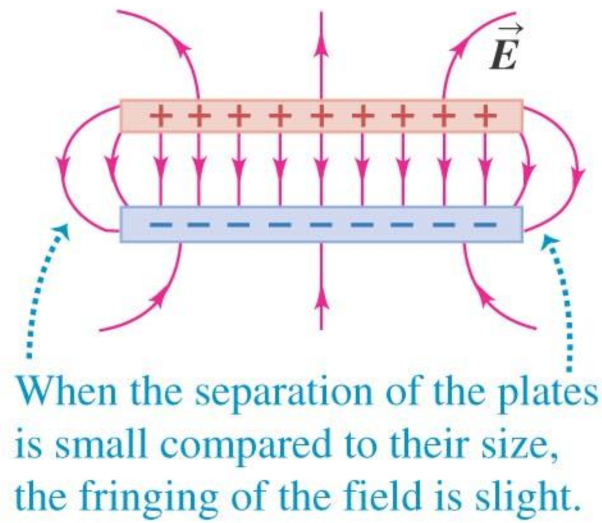
- Capacitance (C) is equal to the Charge (Q) between two charges or charged "regions" divided by the Voltage (V) in those regions.
- Here we assume equal and opposite charges (Q)
- Thus C = Q/V or Q = CV or V=Q/C
- The units of Capacitance are "Farads" after Faraday denoted F or f
- One Farad is one Volt per Coulomb
- One Farad is a large capacitance in the world of electronics
- "Capacitors" are electronic elements capable of storing charge
- Capacitors are very common in electronic devices
- All cell phones, PDA's, computers, radio, TV's … have them
- More common units for practical capacitors are micro-farad (10^{-6} f = μf), nano-farad (10^{-9} f = nf) and pico-farads (10^{-12} f = pf)

A classic parallel plate capacitor

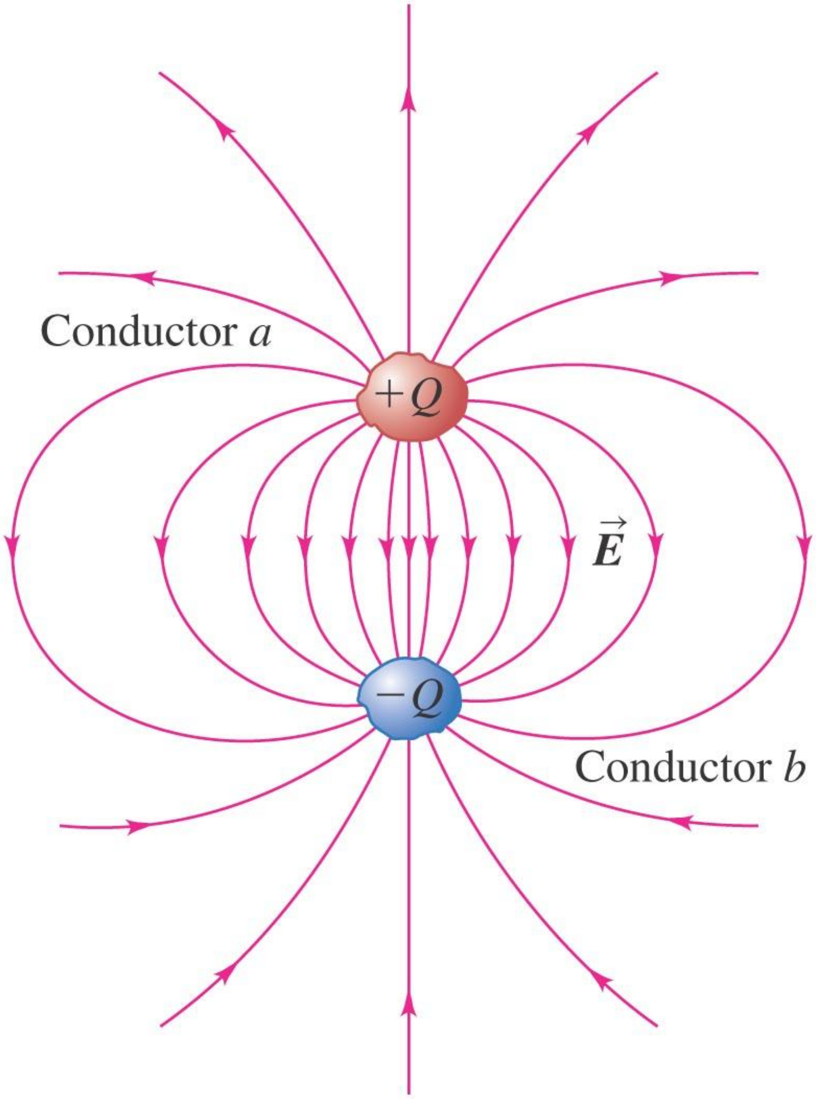
(a) Arrangement of the capacitor plates



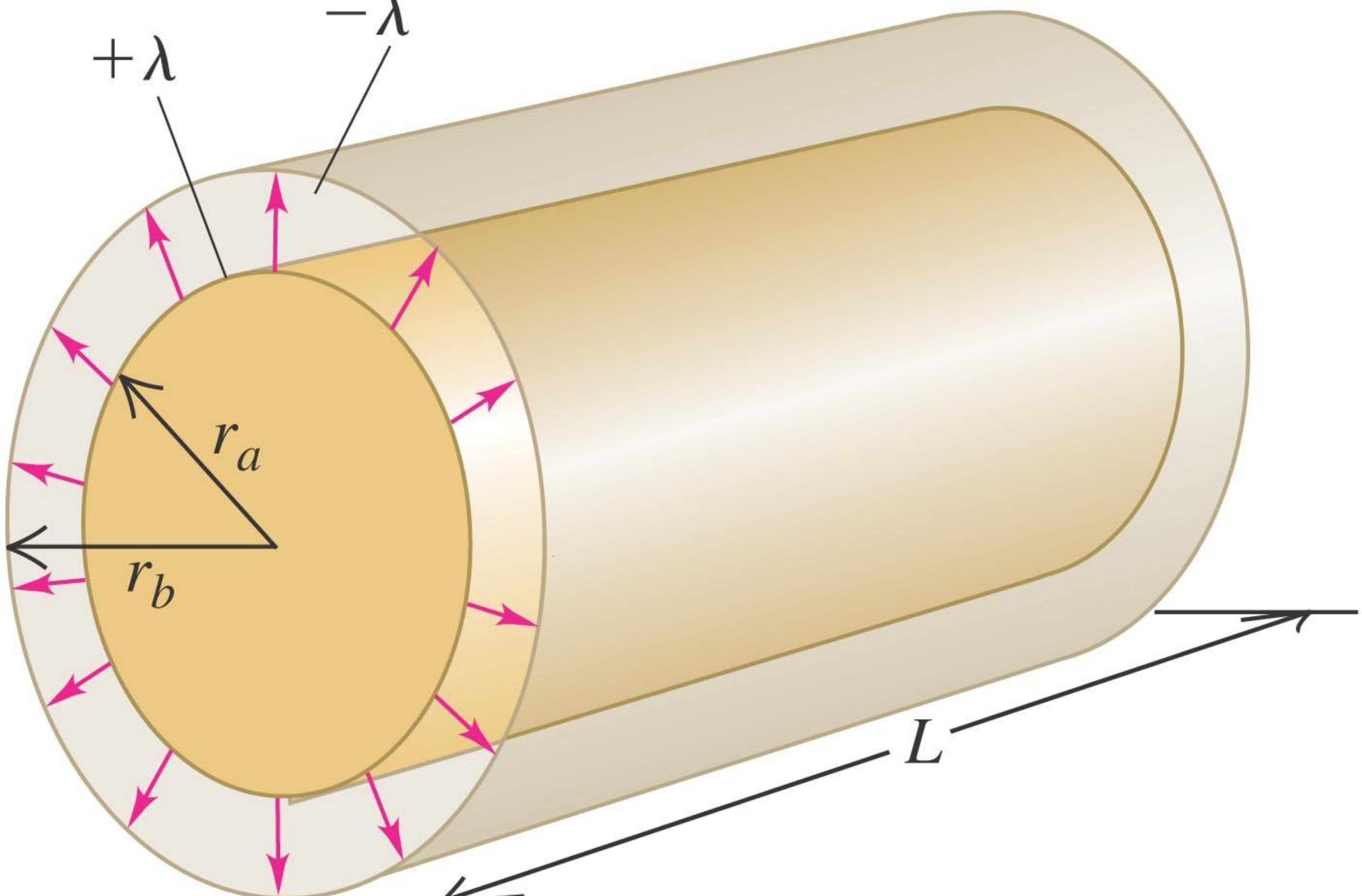
(b) Side view of the electric field $\vec{E}$



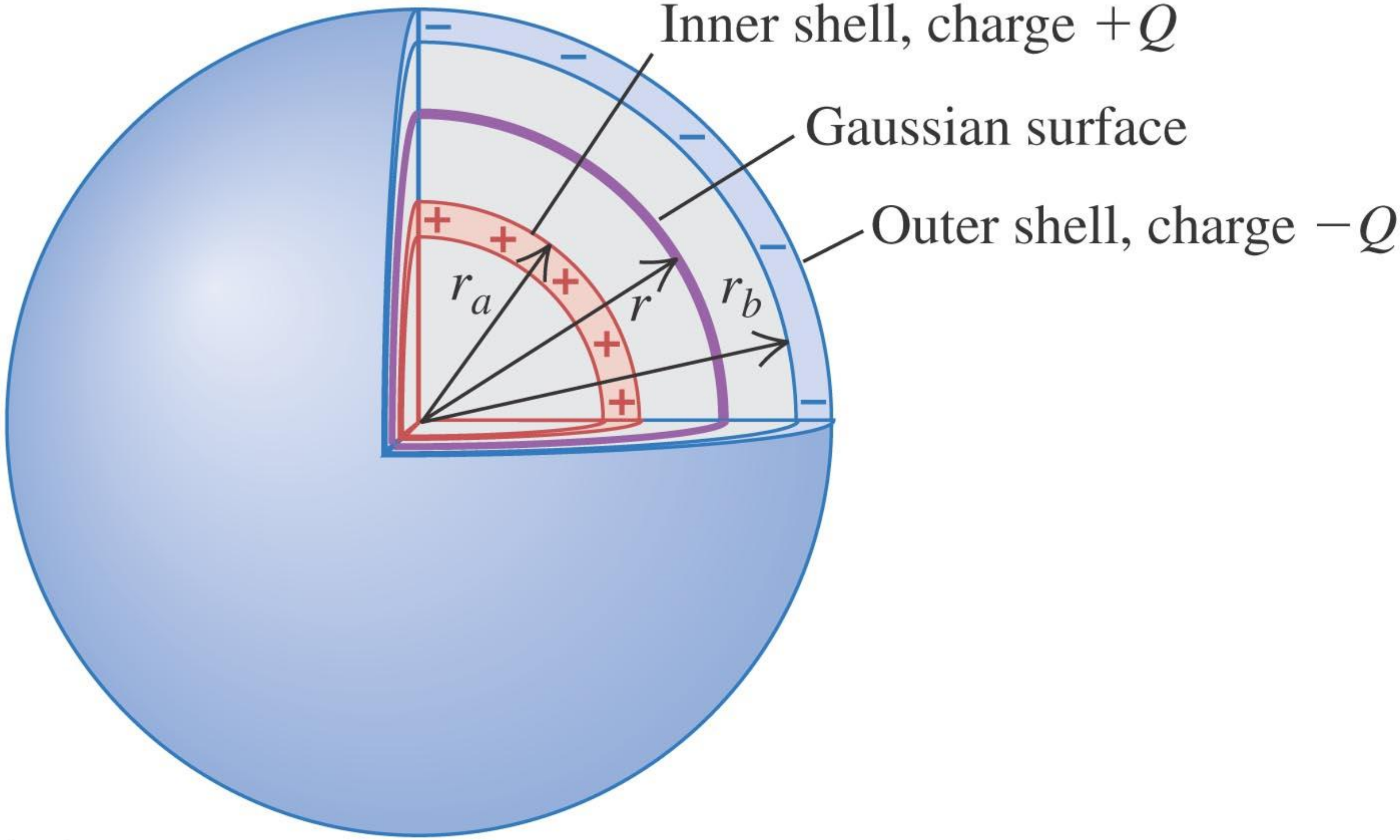
General Surfaces as Capacitors
The Surfaces do not have to be the same



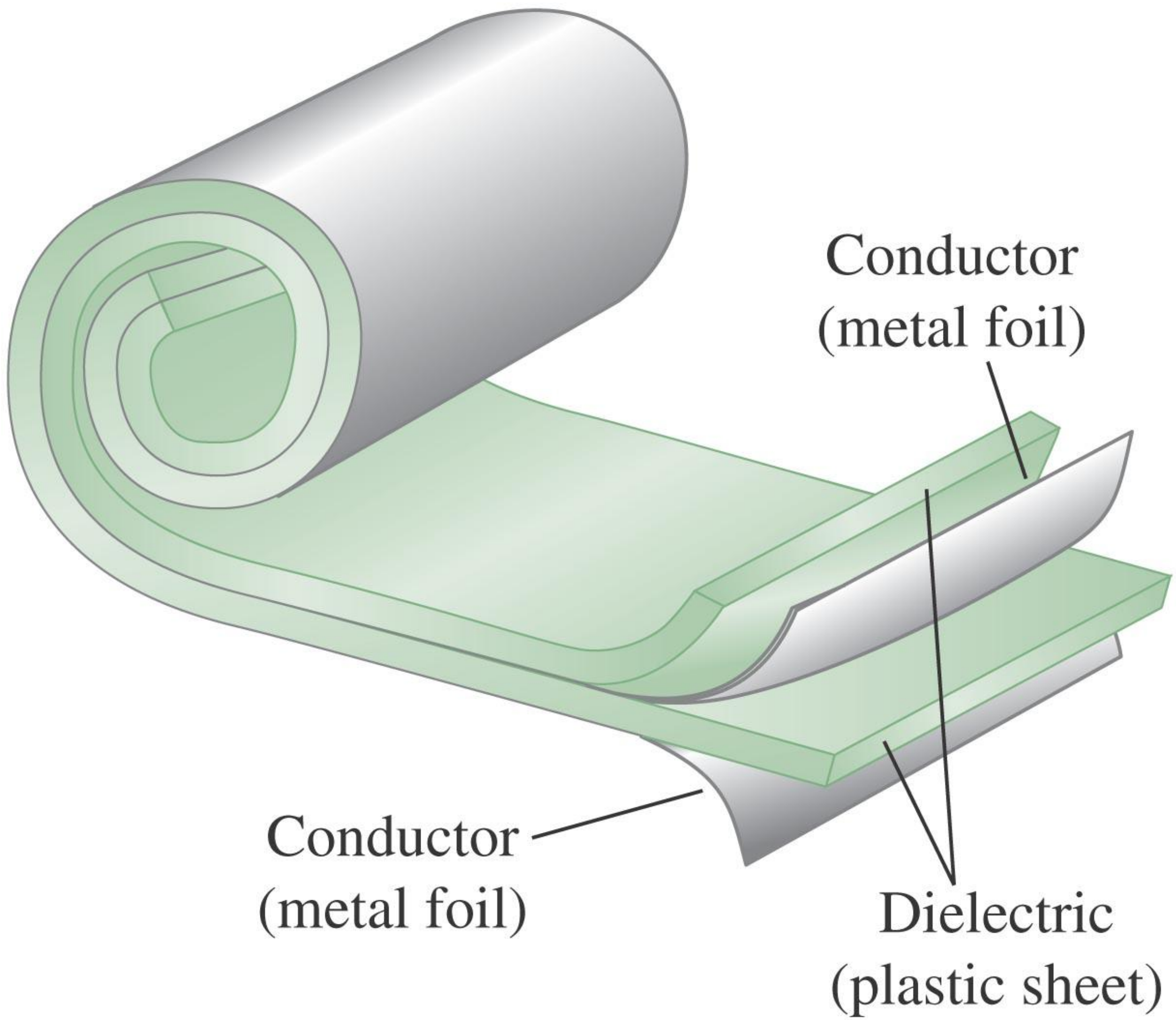
Cylindrical – “Coaxial” Capacitor



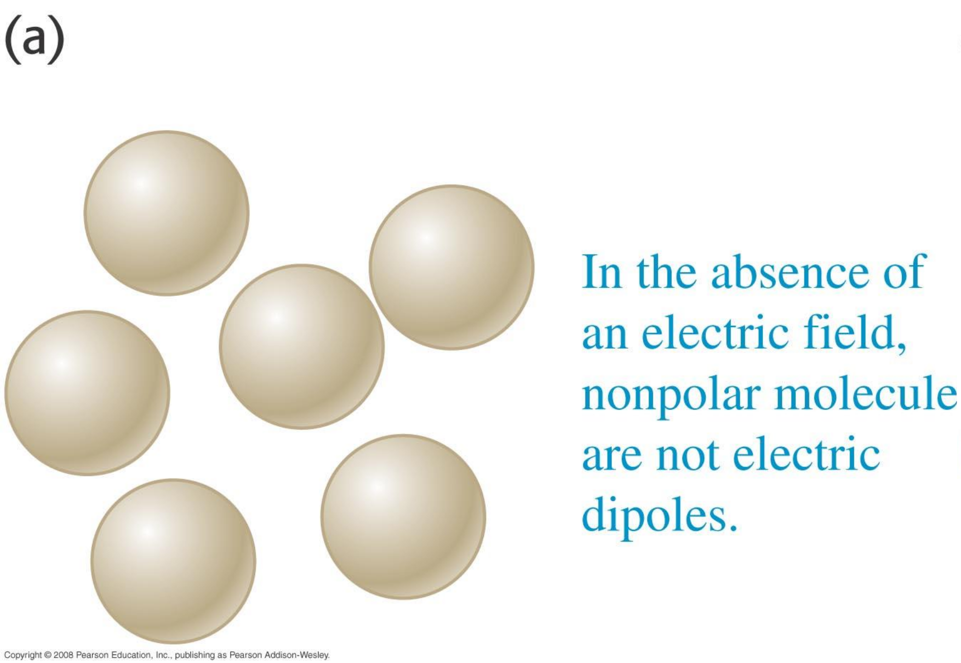
Spherical Shell Capacitor

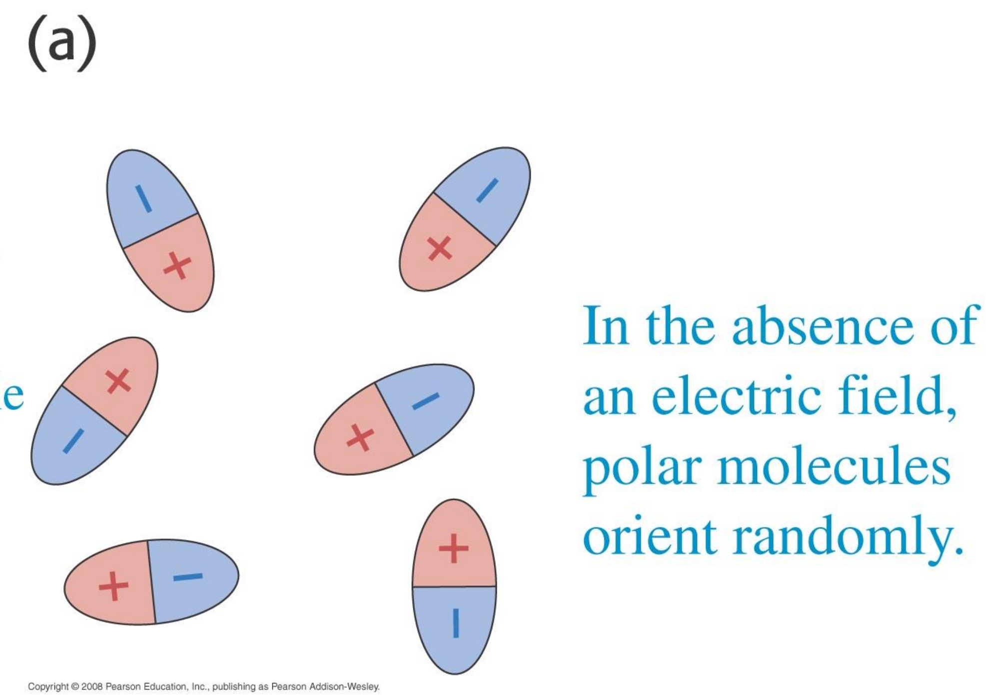


How most practical cylindrical capacitors are constructd

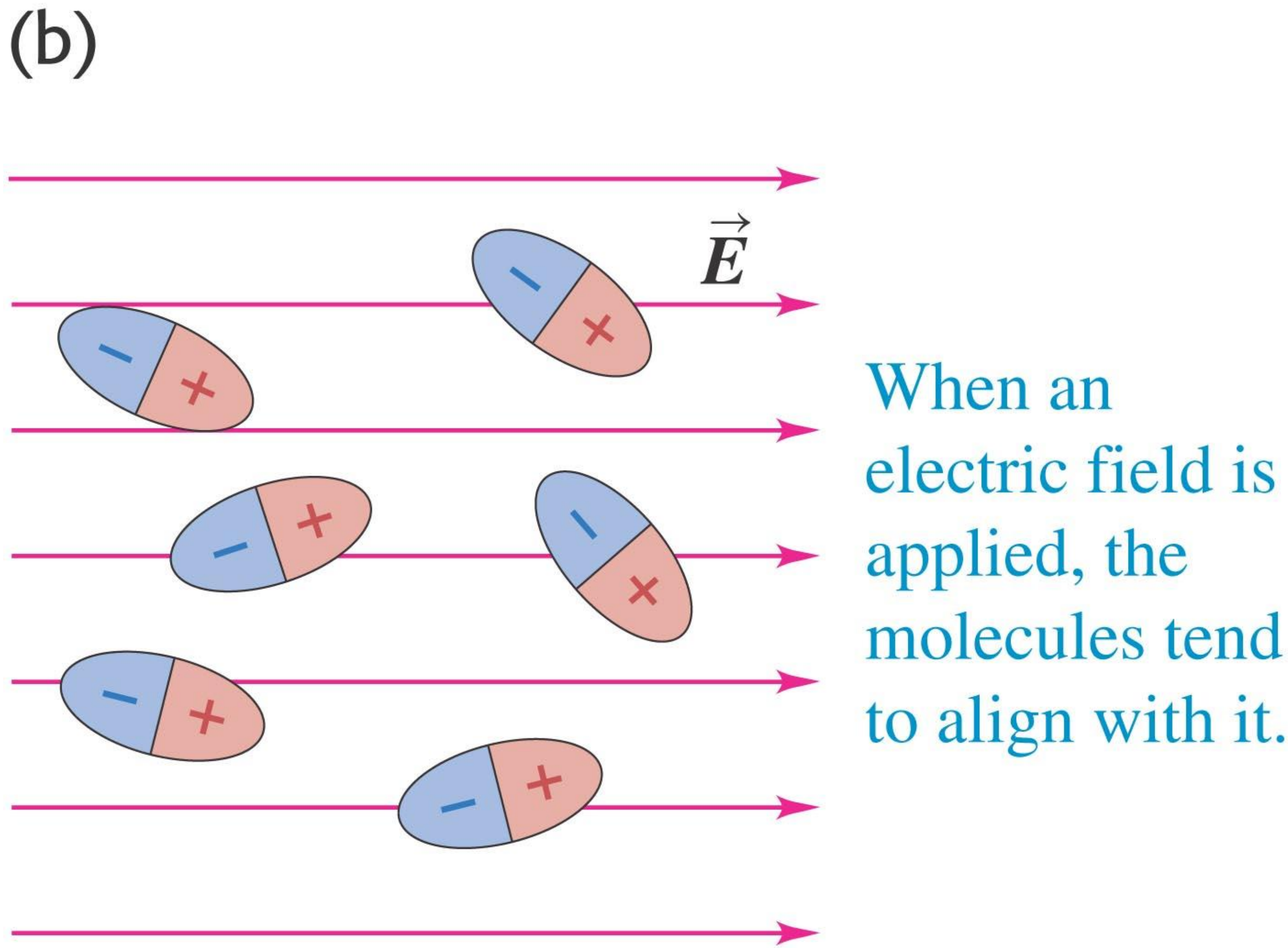


Dielectrics – Insulators – Induced and Aligned Dipole Moments

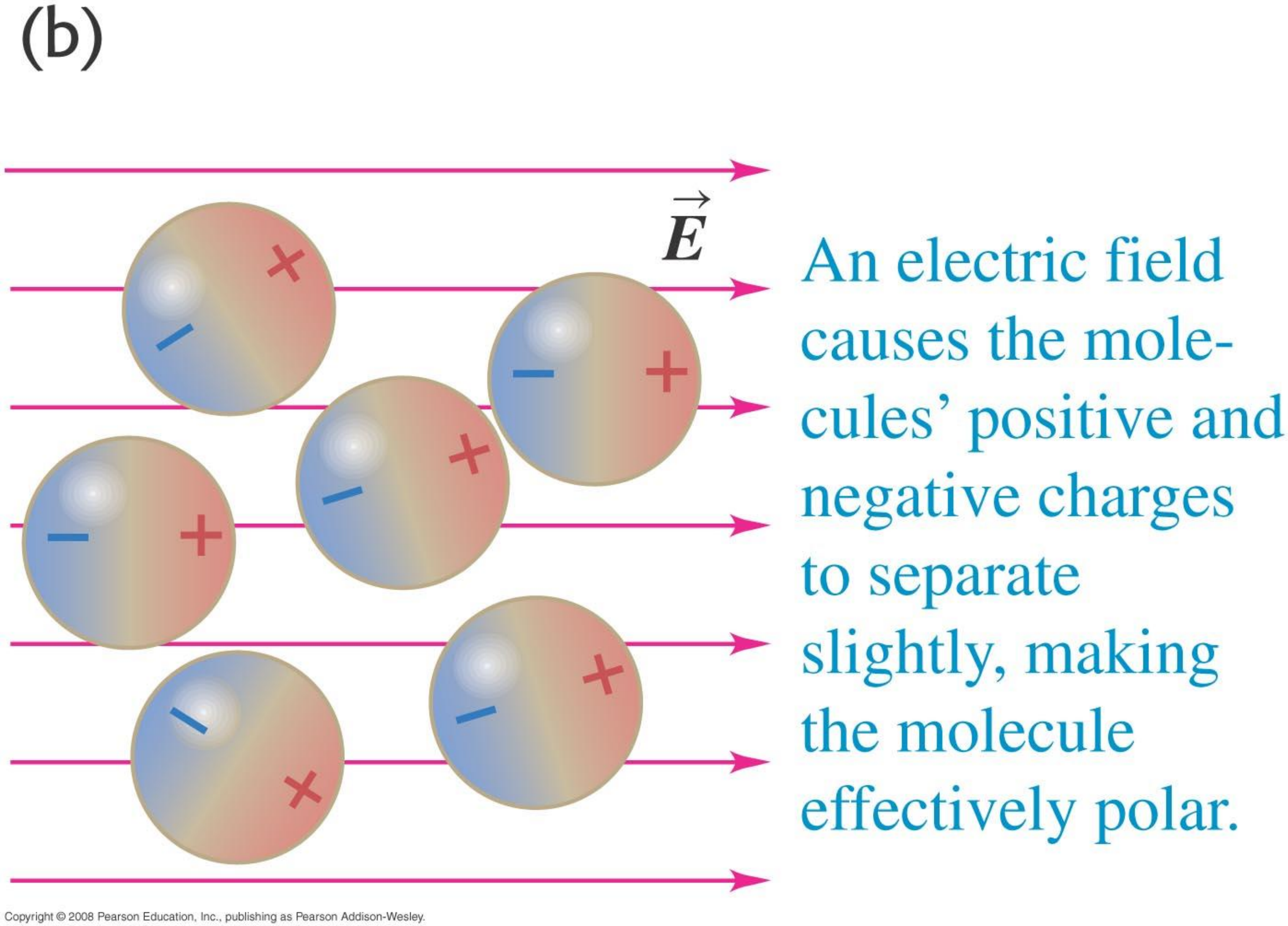




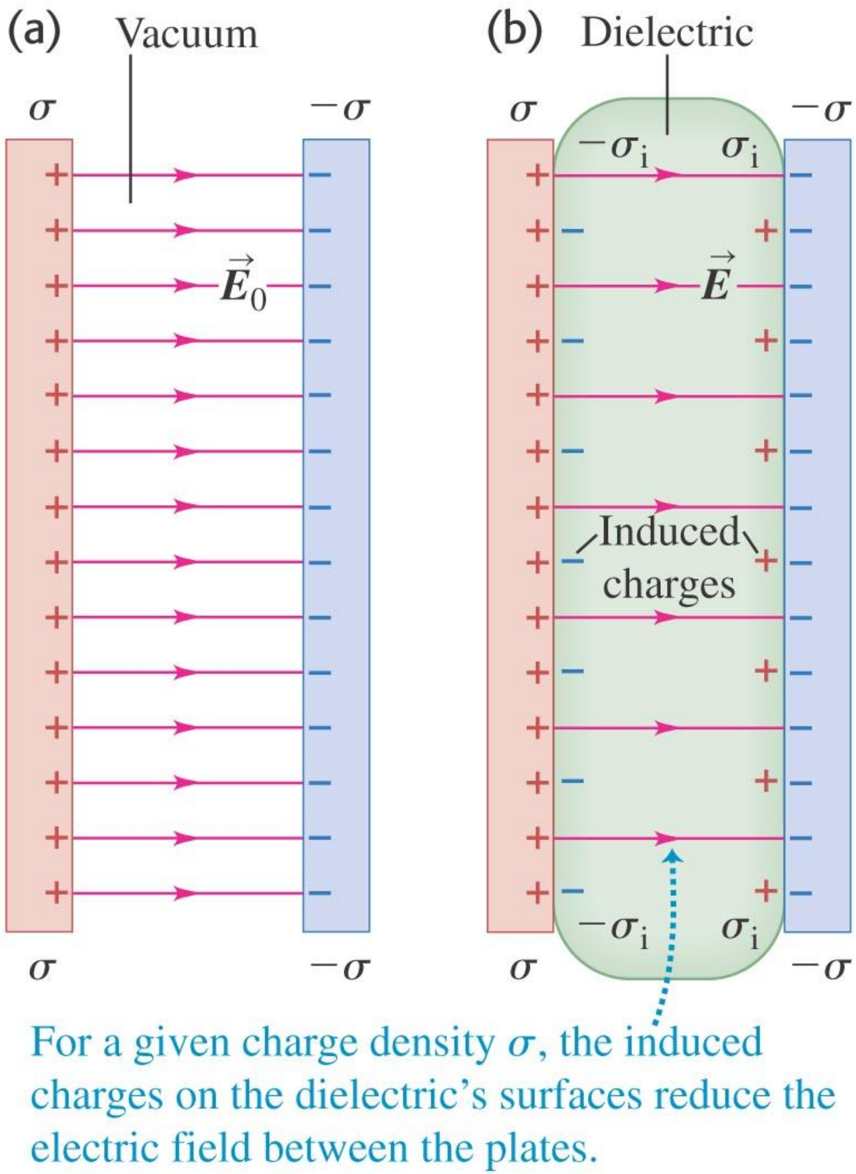
Aligning Random Dipole Moments



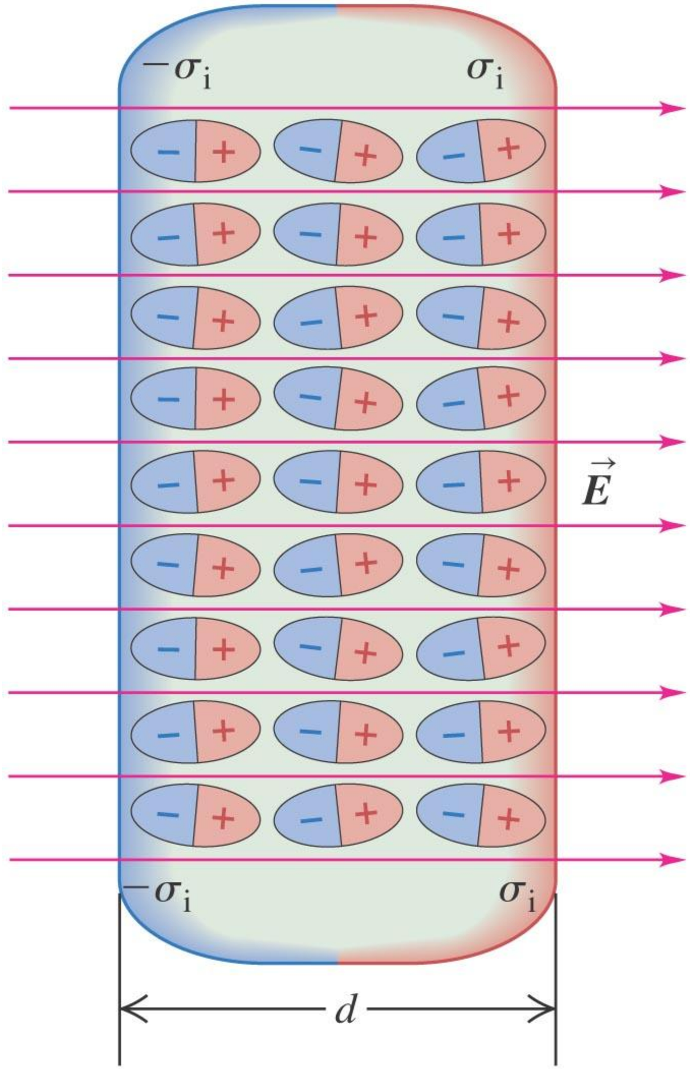
Creating Dipole Moments – Induced Dipoles

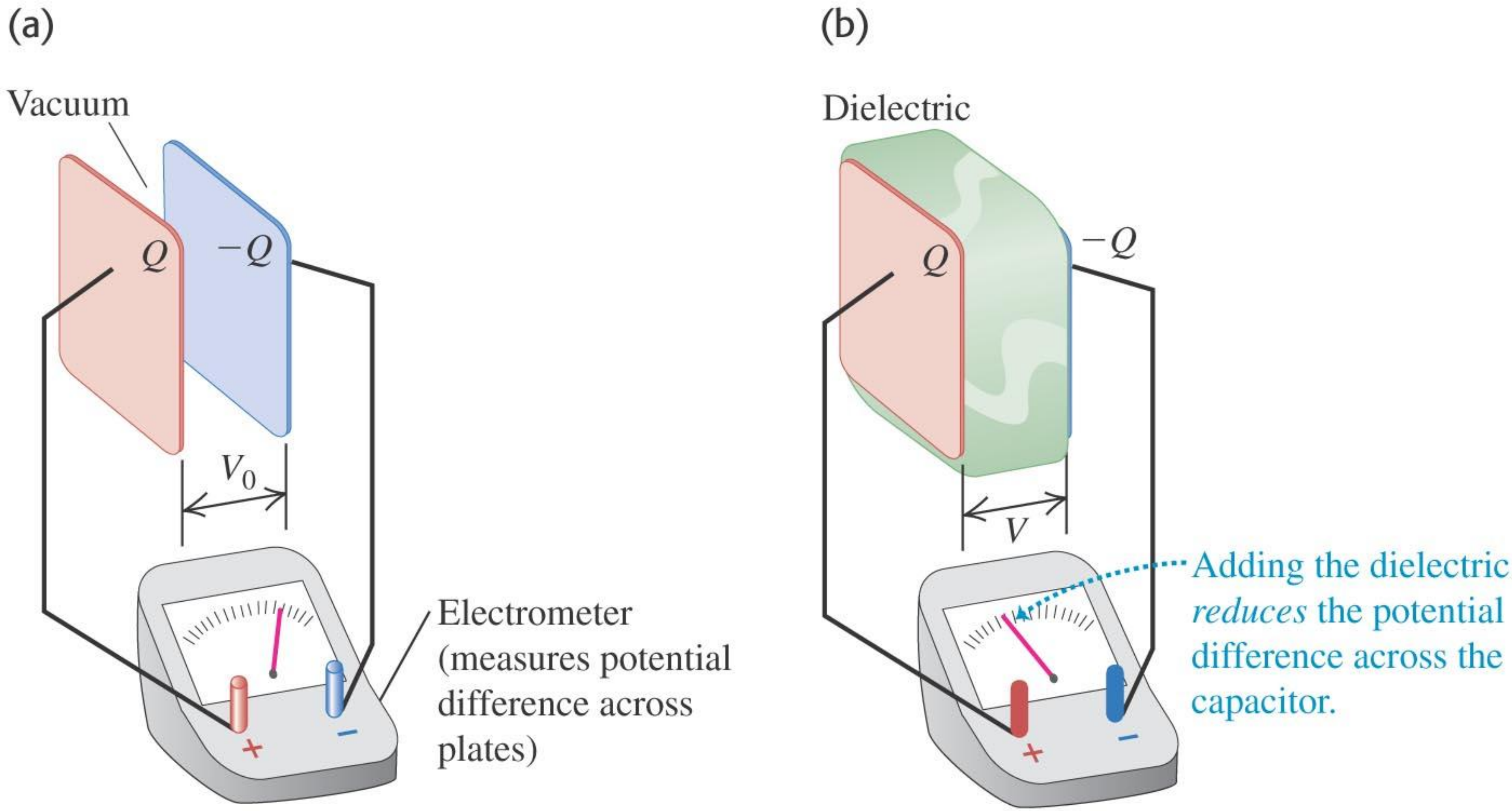


Adding Dielectric to a Capacitor INCREASES its Capacitance since it DECREASES the Voltage for a GIVEN Charge

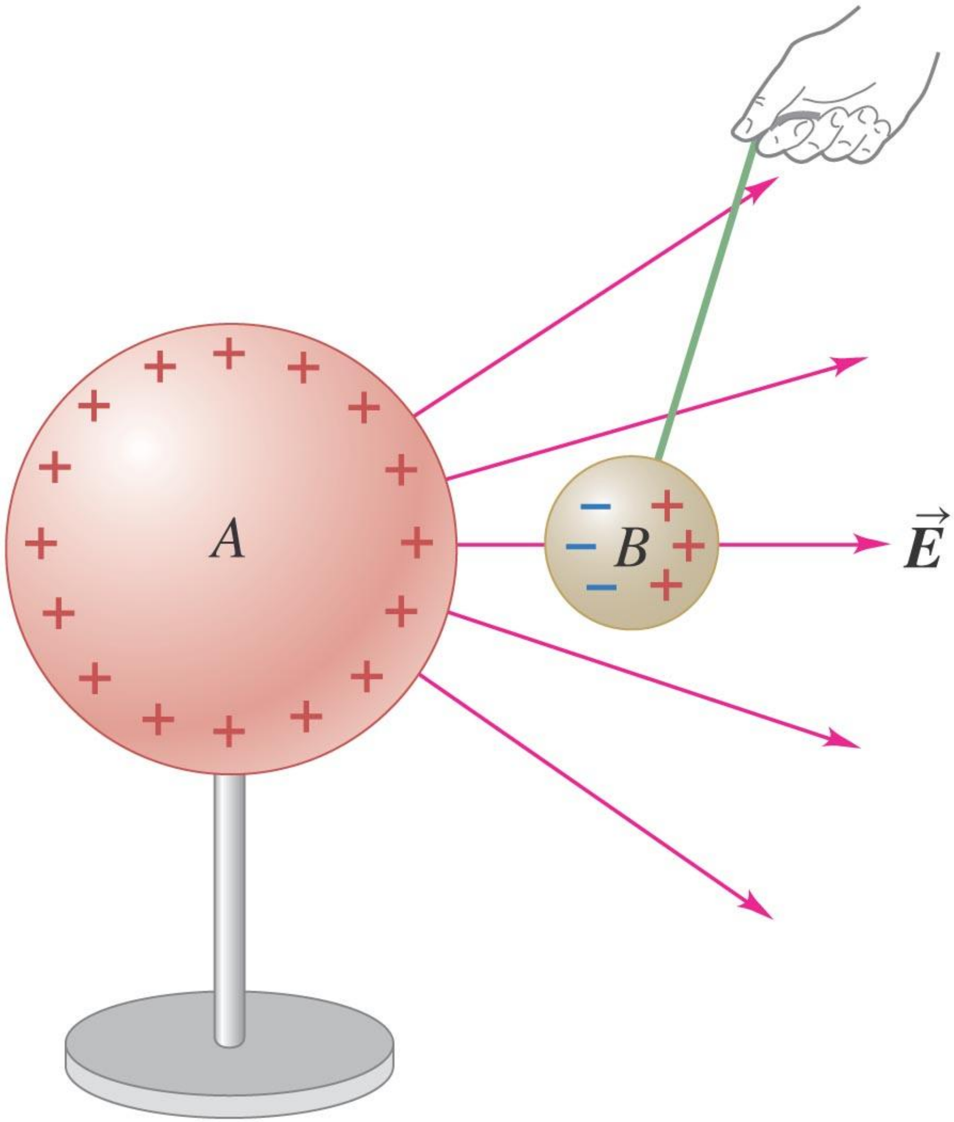


Induced Dipole Moments in a Normally Unpolarized Dielectric




(a)
Vacuum
Q
$-Q$
V_0
+
−
Electrometer
(measures potential
difference across
plates)
(b)
Dielectric
Q
$-Q$
V
+
−
Adding the dielectric
reduces the potential
difference across the
capacitor.

Electrostatic Attraction – "Cling" Forced on Induced Dipole Moments



Energy Stored in a Capacitor

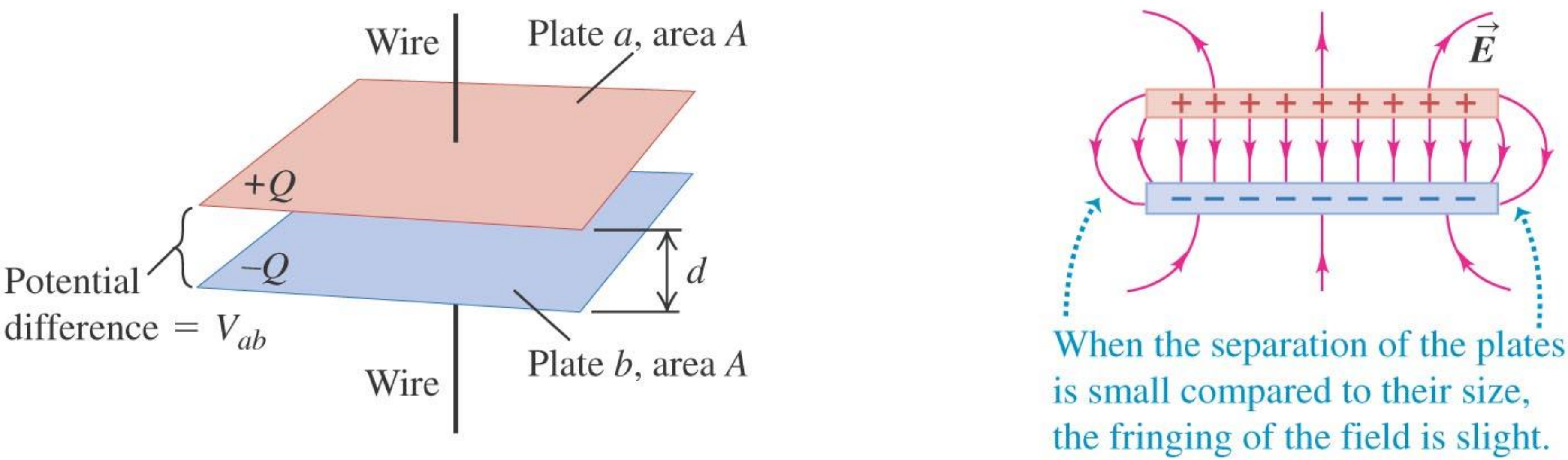
- Capacitors store energy in their electric fields
- The force on a charge in E field E is $F=qE$
- The work done moving a charge q across a potential V is qV
- Lets treat a capacitor as a storage device we are charging
- We start from the initial state with no charge and start adding charge until we reach the final state with charge Q and Voltage V.
- Total work done $W = \int V(q)\, dq$ (we charge from zero to Q)
- BUT $V = q/C$
- We assume here the Capacitance in NOT a function of Q and V BUT only of Geometry
- Thus the energy stored is $W = 1/C \int q\, dq = \frac{1}{2} Q^2/C = \frac{1}{2} CV^2$

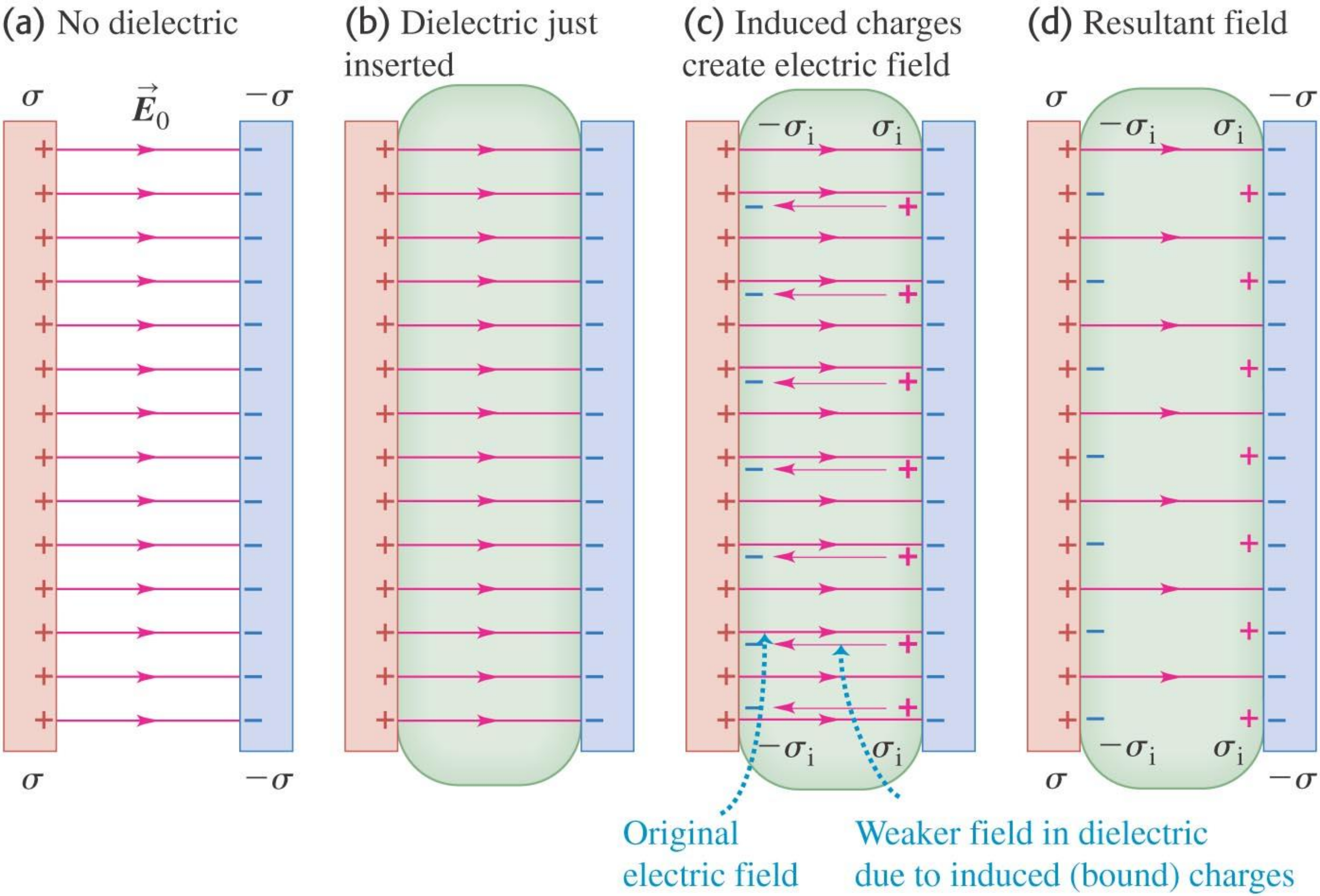
Calculating Parallel Capacitor Capacitance

- Assume two metal plates, area A each, distance d apart, Voltage V between them, Charge +-Q on Plates
- $\sigma = Q/A \quad V = \int E\,dx = Ed \quad E = \sigma/\epsilon_0$ (from Gauss)
- Therefore $C = Q/V = \sigma A/(Ed) = \boldsymbol{\epsilon_0 A/d}$
- **Note – As d decreases C increases**

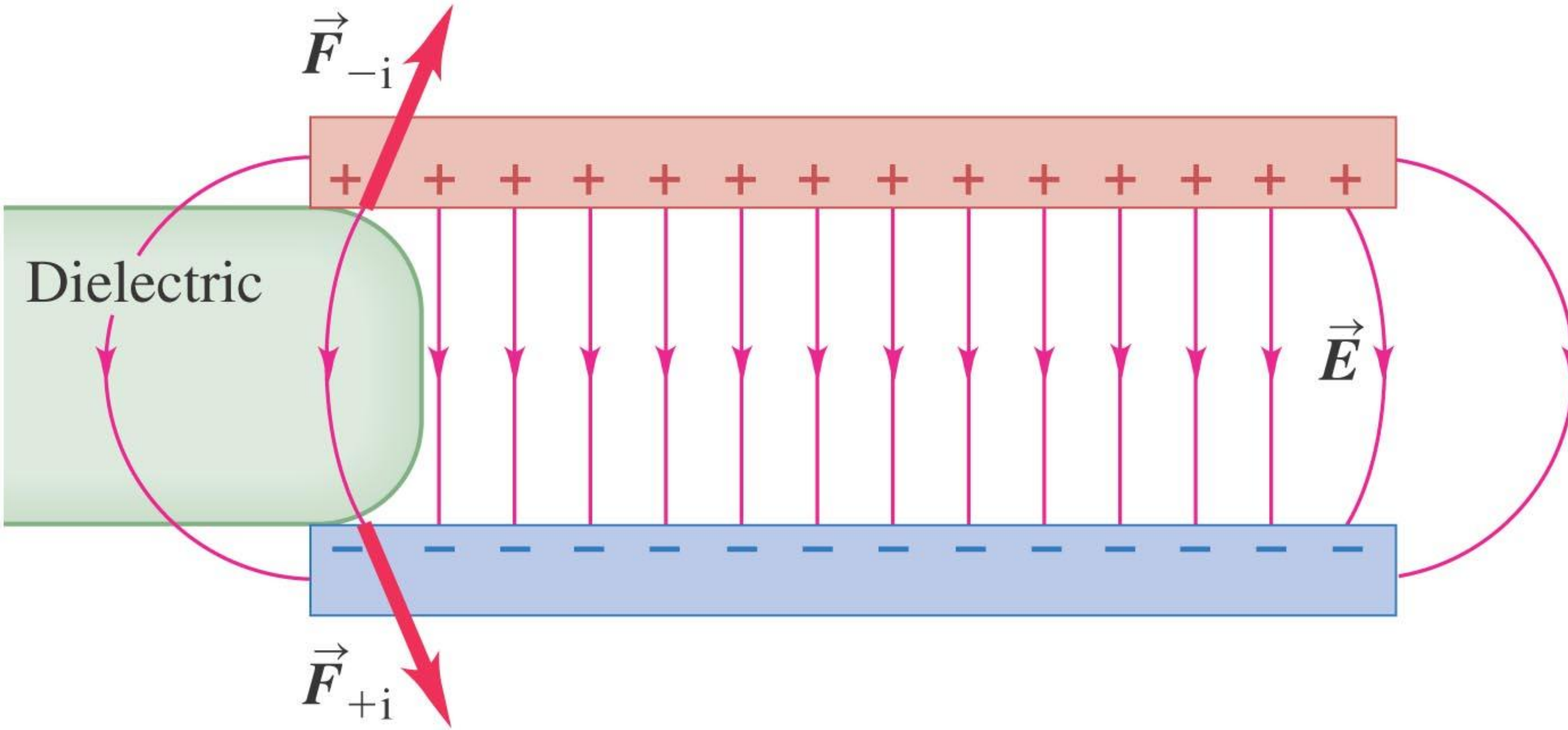
(a) Arrangement of the capacitor plates

(b) Side view of the electric field $\vec{E}$




(a) No dielectric
(b) Dielectric just inserted
(c) Induced charges create electric field
(d) Resultant field
σ
$\vec{E}_0$
$-\sigma$
$-\sigma_i$
σ_i
Original electric field
Weaker field in dielectric due to induced (bound) charges

Force on a Dielectric inserted into a Capacitor



Force on Capacitor Plates

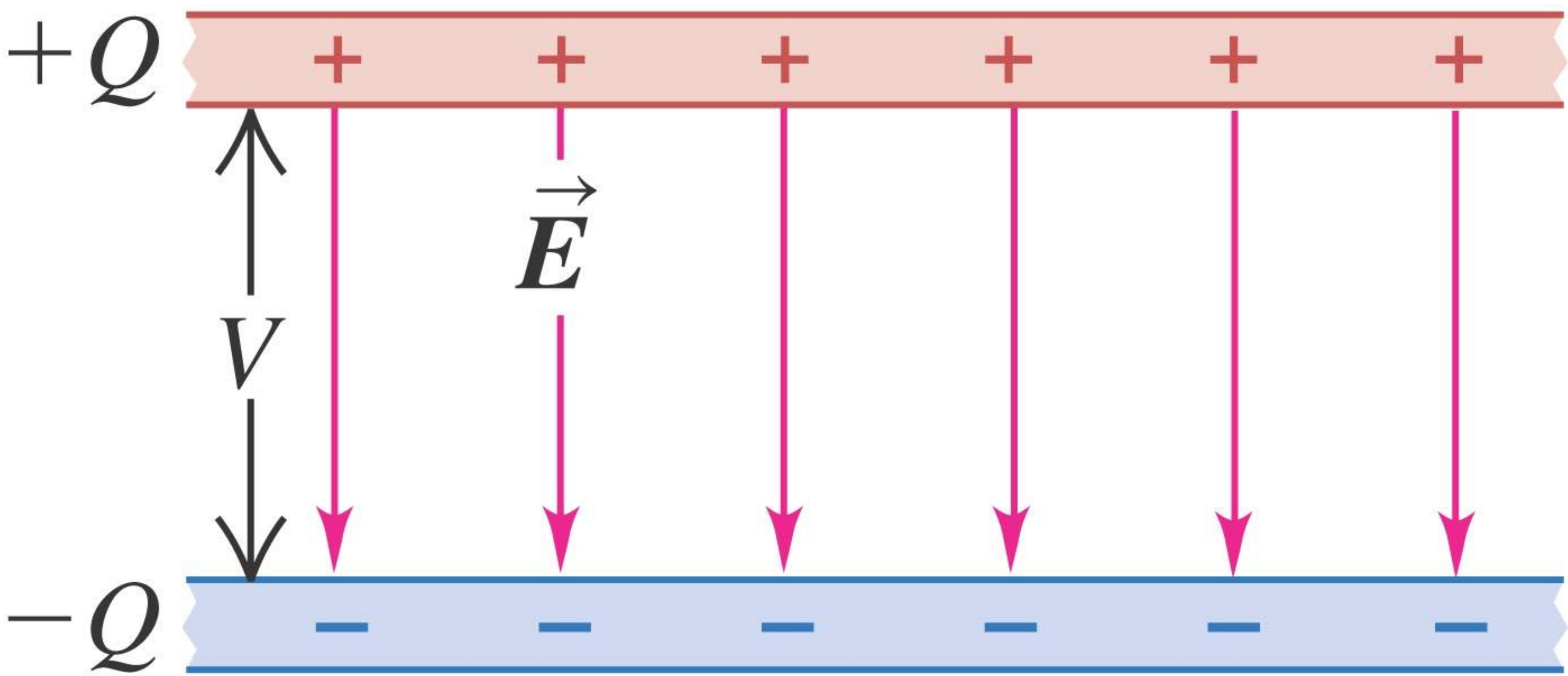
F=QE   V=Ed (d separation distance)

F=QV/d

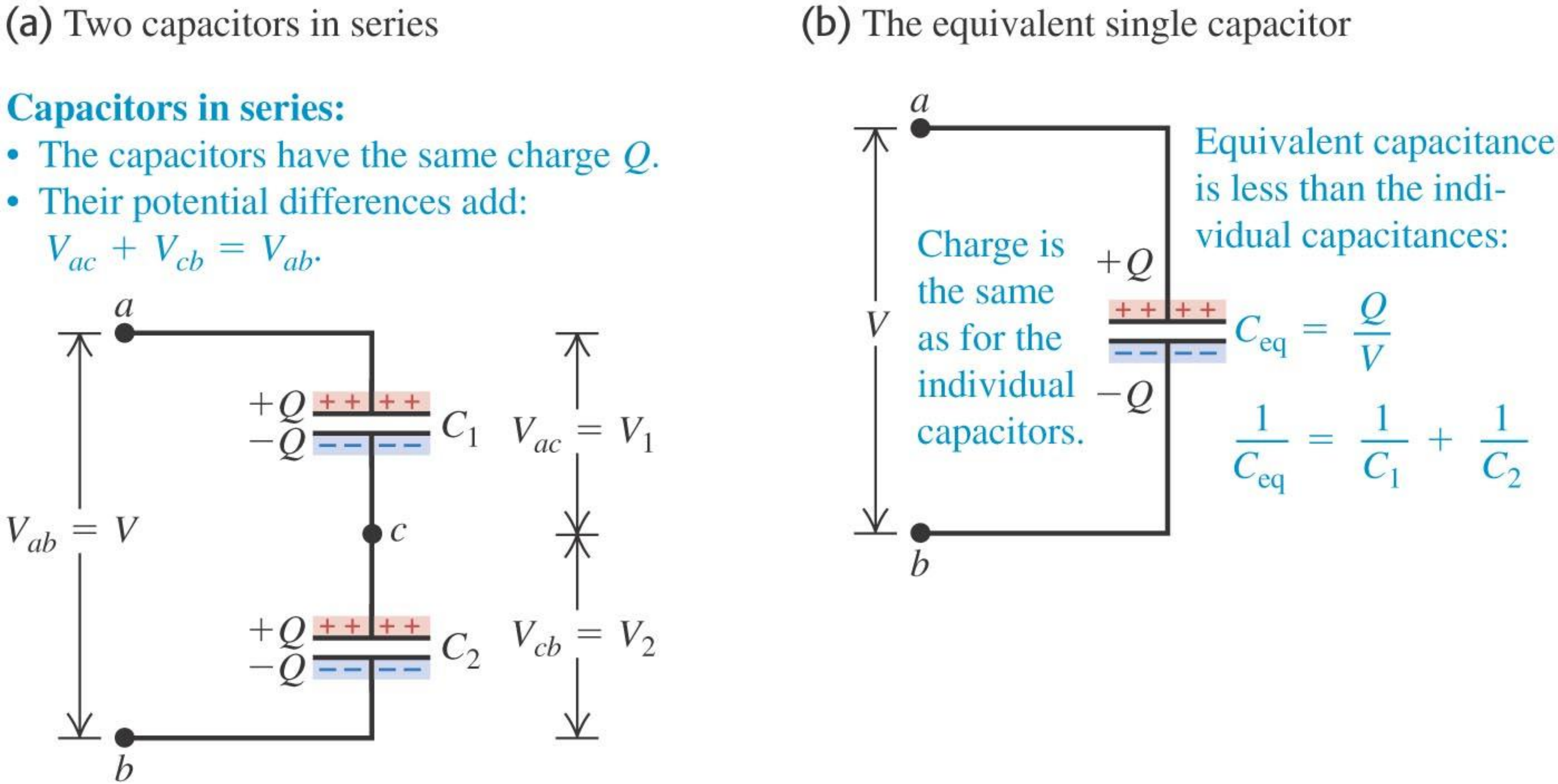
Q=CV -> F =CV^2/d

Recall W (Stored Energy) = ½ CV^2

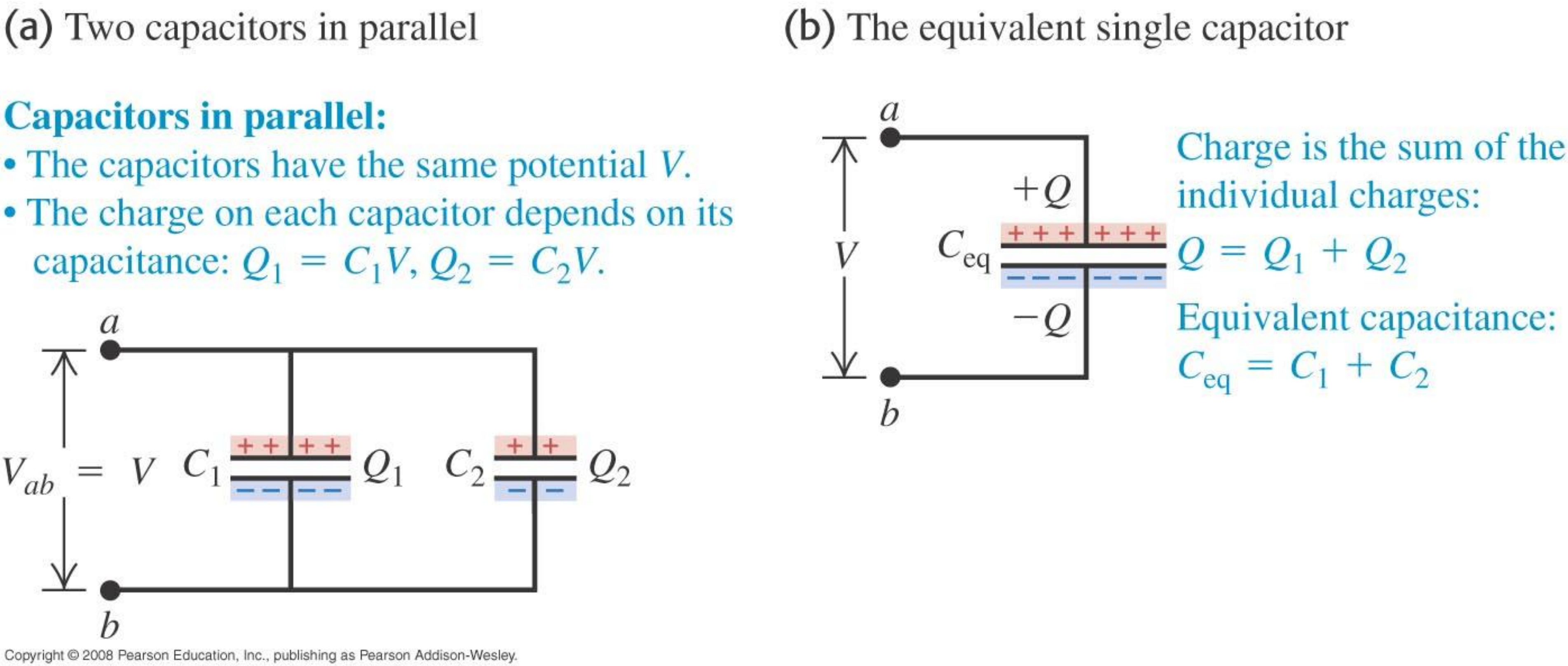
Hence F = 2W/d or W = ½ Fd



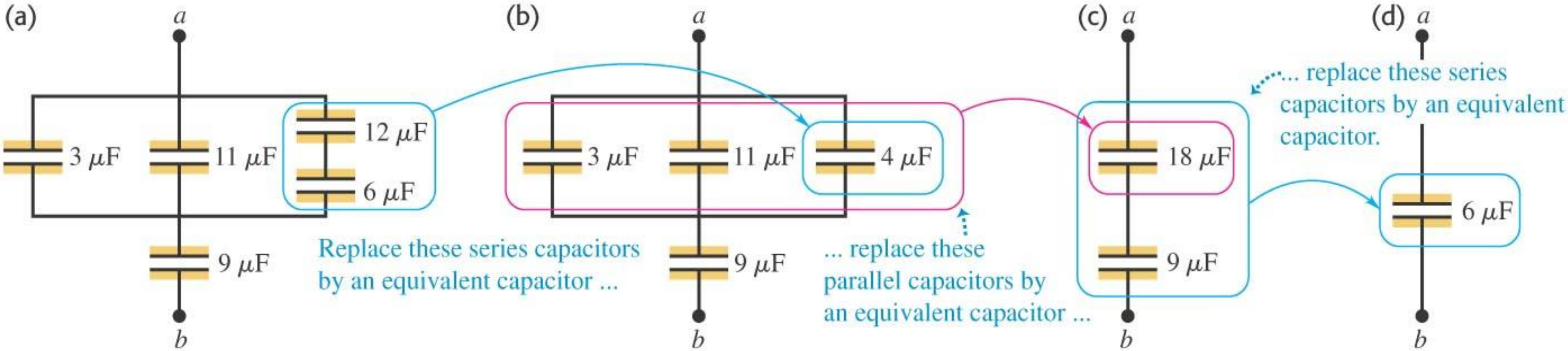
Capacitors in Series



Capacitors in Parallel



Series and Parallel Capacitors

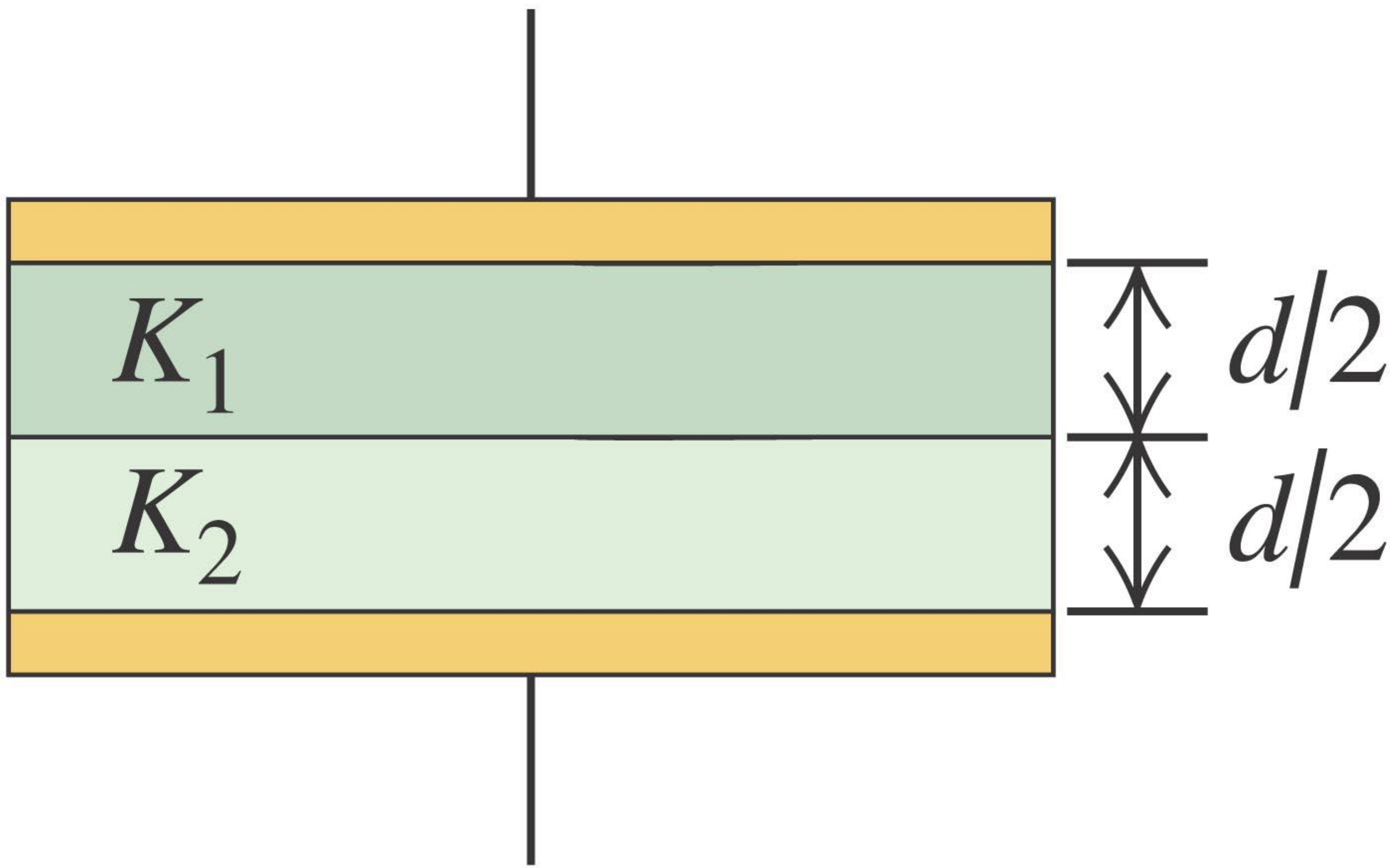


Dielectric Constants of Some Common Materials

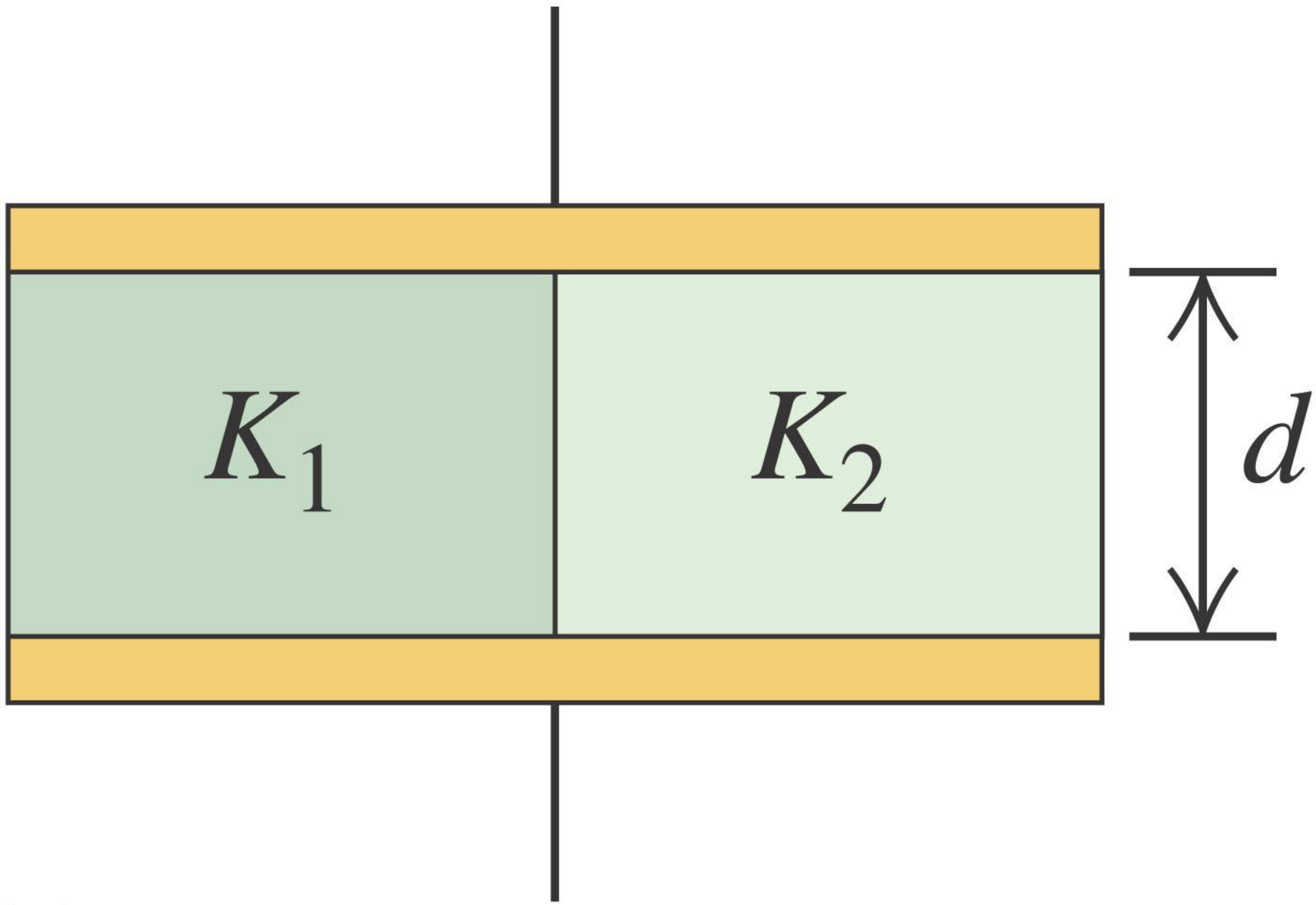
Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

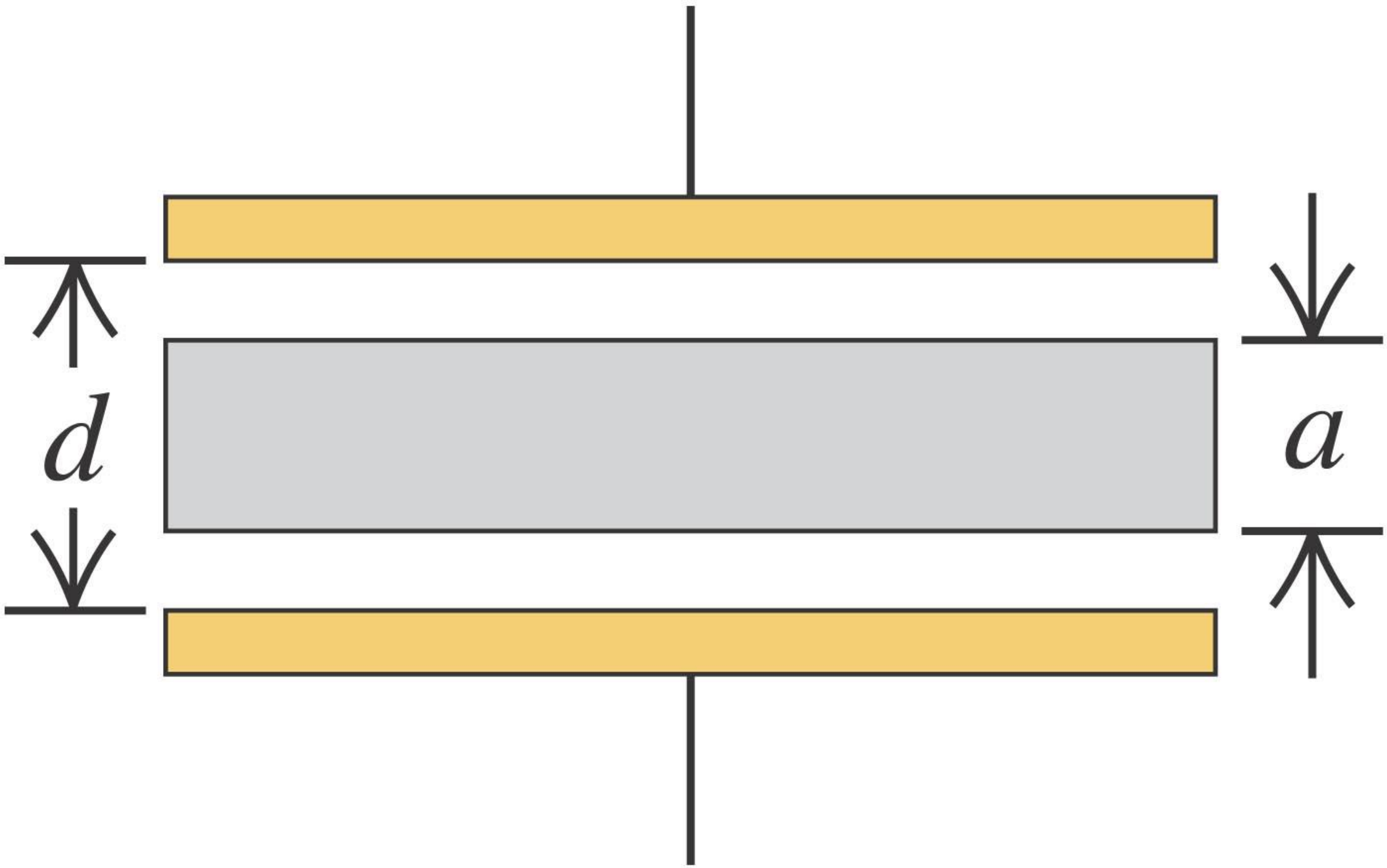
Two Dielectric Constants – As if capacitors in Series



Two Dielectric Constants – As if two capacitors in Parallel



Dielectric Plus Vacuum (Air)
Treat is if three capacitors in Series



Capacitance of simple systems

Type	Capacitance	Comment
Parallel-plate capacitor	$\varepsilon A/d$	A: Area d: Distance
Coaxial cable	$\dfrac{2\pi\varepsilon l}{\ln\left(a_2/a_1\right)}$	a_1: Inner radius a_2: Outer radius *l*: Length
Pair of parallel wires[17]	$\dfrac{2\pi\varepsilon l}{\operatorname{arcosh}\left(\frac{d^2}{2a^2}-1\right)}=\dfrac{\pi\varepsilon l}{\operatorname{arcosh}\left(\frac{d}{2a}\right)}=\dfrac{\pi\varepsilon l}{\ln\left(\frac{d}{2a}+\sqrt{\frac{d^2}{4a^2}-1}\right)}$	a: Wire radius d: Distance, d > 2a *l*: Length of pair
Wire parallel to wall[17]	$\dfrac{4\pi\varepsilon l}{\operatorname{arcosh}\left(\frac{2d^2}{a^2}-1\right)}=\dfrac{2\pi\varepsilon l}{\operatorname{arcosh}\left(\frac{d}{a}\right)}=\dfrac{2\pi\varepsilon l}{\ln\left(\frac{d}{a}+\sqrt{\frac{d^2}{a^2}-1}\right)}$	a: Wire radius d: Distance, d > a *l*: Wire length
Concentric spheres		a_1: Inner radius a_2: Outer radius
Two spheres, equal radius[18][19]	$=2\pi\varepsilon a\left\{\ln 2+\gamma-\frac{1}{2}\ln\left(\frac{d}{a}-2\right)+O\left(\frac{d}{a}-2\right)\right\}$	a: Radius d: Distance, d > 2a D = d/2a γ: Euler's constant
Sphere in front of wall[18]	$4\pi\varepsilon a\displaystyle\sum_{n=1}^{\infty}\frac{\sinh\left(\ln\left(D+\sqrt{D^2-1}\right)\right)}{\sinh\left(n\ln\left(D+\sqrt{D^2-1}\right)\right)}$	a: Radius d: Distance, d > a D = d/a
Sphere	$4\pi\varepsilon a$	a: Radius
Circular disc	$8\varepsilon a$	a: Radius
Thin straight wire, finite length[20][21][22]	$\dfrac{2\pi\varepsilon l}{\Lambda}\left\{1+\frac{1}{\Lambda}\left(1-\ln 2\right)+\frac{1}{\Lambda^2}\left[1+\left(1-\ln 2\right)^2-\frac{\pi^2}{12}\right]+O\left(\frac{1}{\Lambda^3}\right)\right\}$	a: Wire radius *l*: Length Λ: ln(*l*/a)