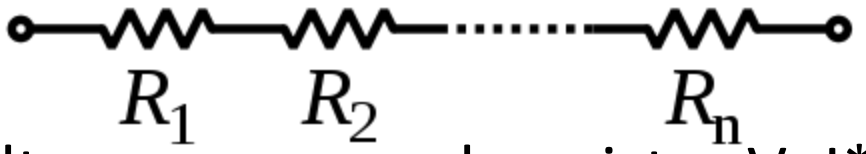
Chapter 26 DC Circuits

Series Resistors

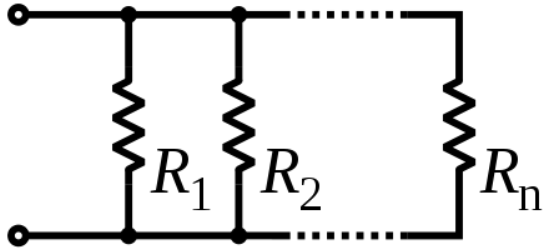
- Resistors in series must have the **same current going thru each resistor** otherwise charge would increase or decrease



- But voltage across each resistor V=I*R can vary if R is different for each resistor
- Since I is the same in each resistor and the **total potential is the sum of the potentials** therefore
- $V_{total} = V_1 + V_2 + \ldots + V_n = I*R_1 + I*R_2 + \ldots + I*R_n$
- $= I* (R_1 + R_2 + \ldots + R_n) = I*R_{total}$
- Thus $\mathbf{R_{Total} = R_1 + R_2 + \ldots + R_n}$

Resistors in Parallel

- Resistors in parallel have the same voltage V (potential)

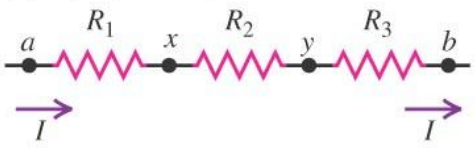


- The current thru each resistor is thus V/R where R is the resistance of that particular resistor
- The total current (flow of charge) must be the sum of all the currents hence:
- $I_{total} = I_1 + I_2 + \ldots + I_n = V/R_1 + V/R_2 + \ldots + V/R_n$
- $= V\,(1/R_1 + 1/R_2 + \ldots + 1/R_n) = V/R_{total}$
- $1/R_{Total} = 1/R_1 + 1/R_2 + \ldots + 1/R_n$

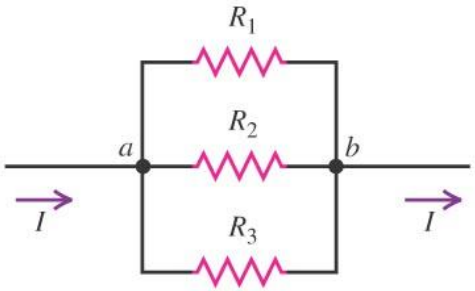
Some example geometries

- Lets looks at some possibilities
- Always try to break up the system into parallel and series blocks then solve for complete system

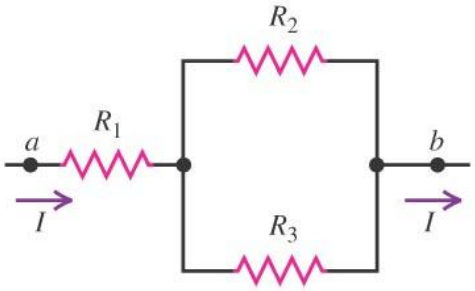
(a) R_1, R_2, and R_3 in series



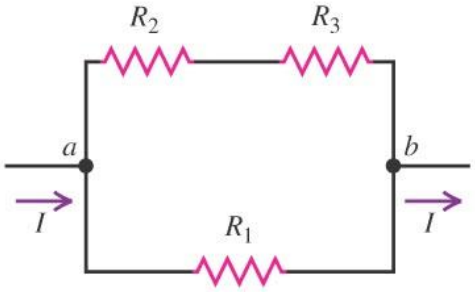
(b) R_1, R_2, and R_3 in parallel



(c) R_1 in series with parallel combination of R_2 and R_3



(d) R_1 in parallel with series combination of R_2 and R_3



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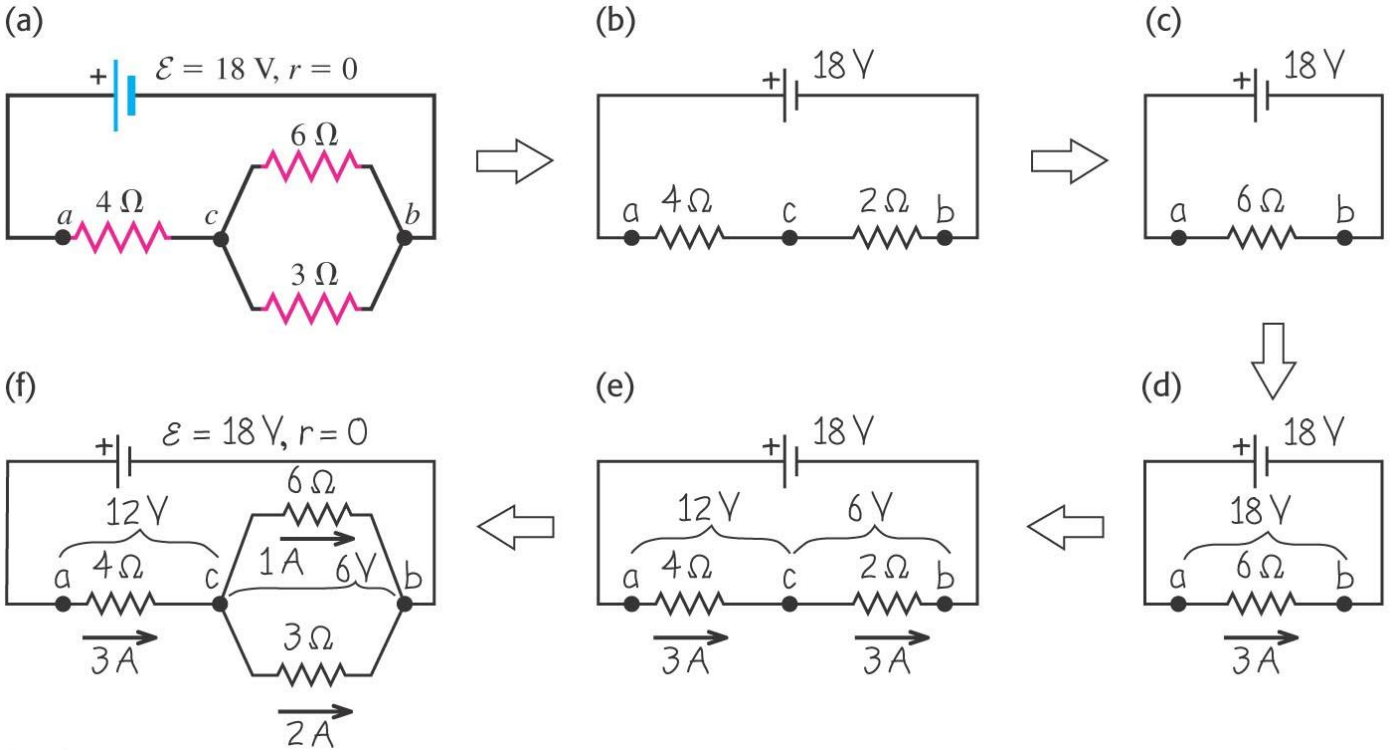
An example

- First find the equivalent resistance of the 6Ω and 3Ω in parallel the use the result of that in series with the 4Ω. The results is 6Ω total. Hence the current flowing is 18/6 = 3 amps



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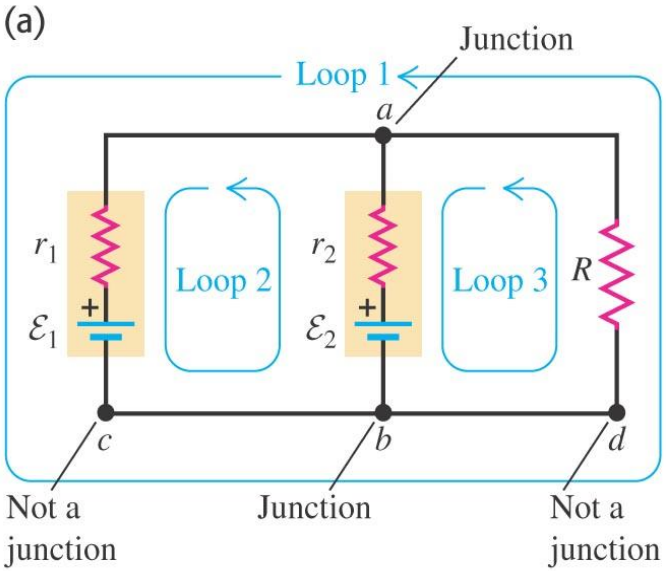
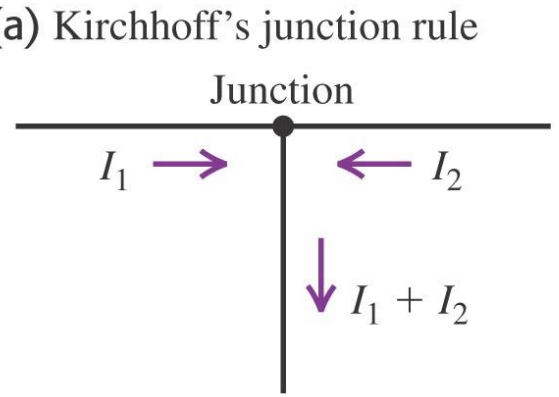
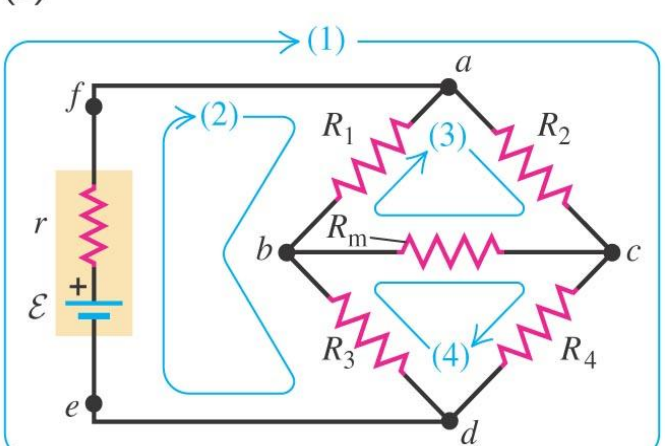
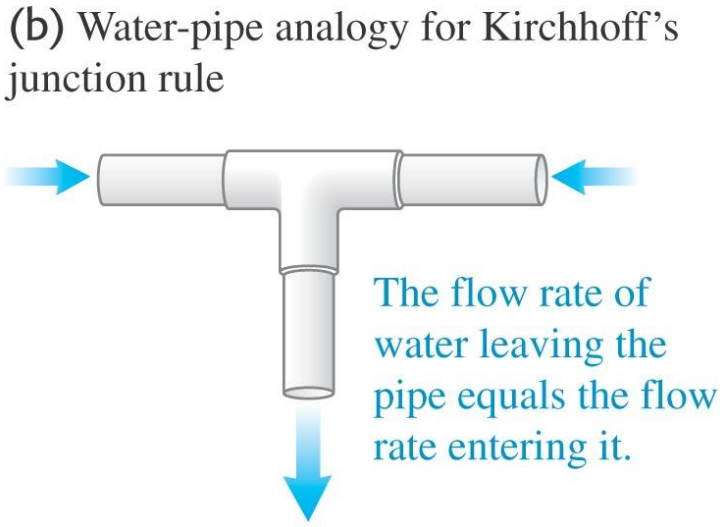
Kirchoff's Rule - I - current into a junction

- The algebraic sum of the currents into any junction is zero. This is Kirchoff's Rule – it is nothing more than a statement of charge conservation. Charge is neither created nor destroyed.









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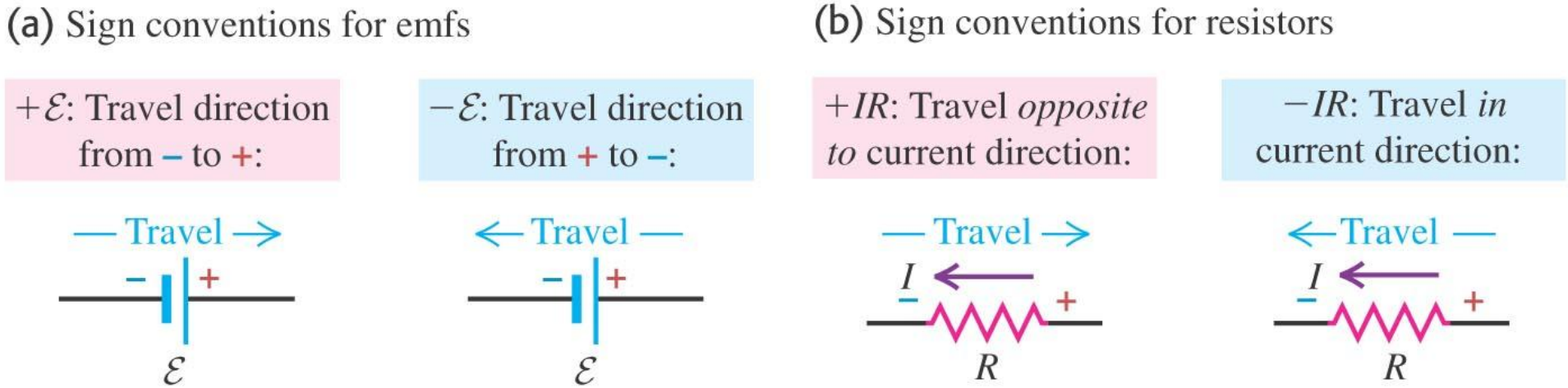
Kirchoff's Rules II – DC voltage loops must be zero

- The algebraic sum of the DC potential differences in any loop, including those associated with emfs (generally batteries here) and those of resistive elements, must equal zero. By convention we treat the charges as though they were positive carriers but in most systems they are electrons and hence negative. This is NOT true in AC systems where a changing current yields a changing magnetic field which yields an AC potential



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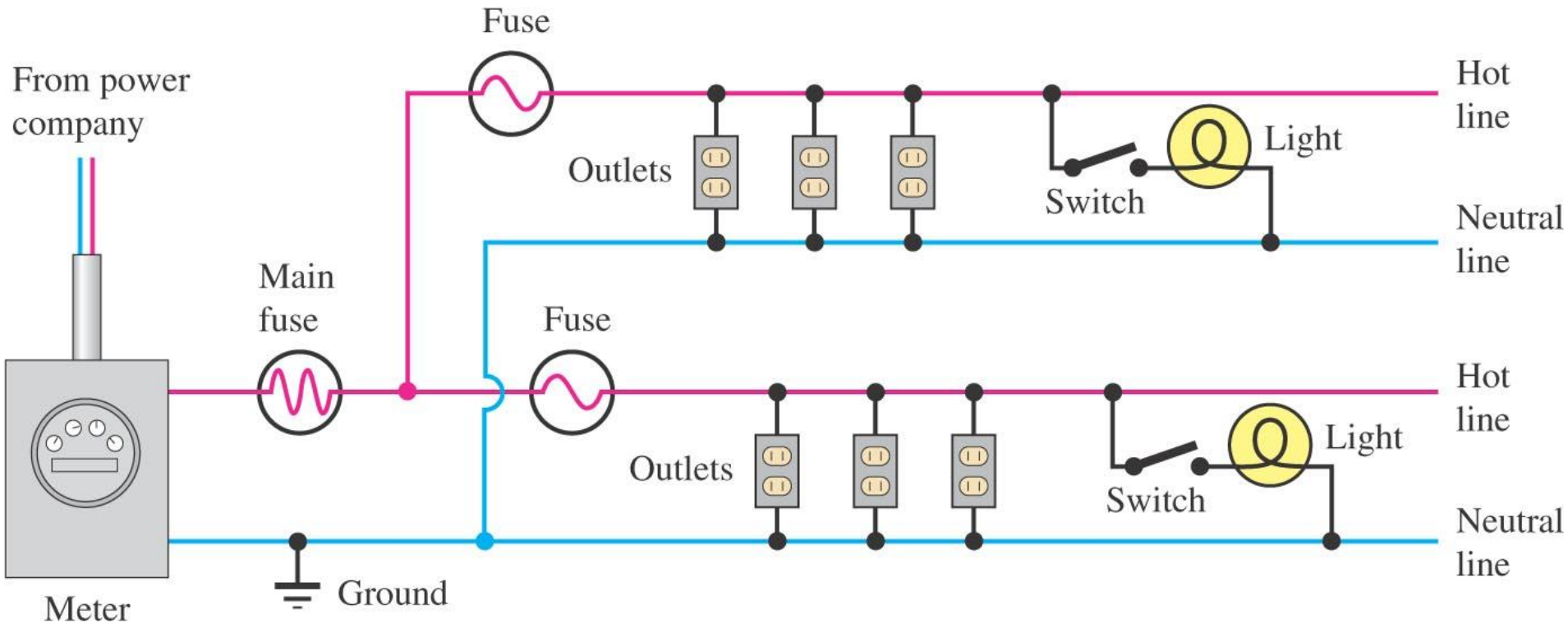
Power in a home – Typical modern US home has 240 VAC split into two 120 VAC circuits. The circuit below shows an older home - 120 VAC only – Hot and Neutral only – no separate ground. Outlets were two pronged not three like today.



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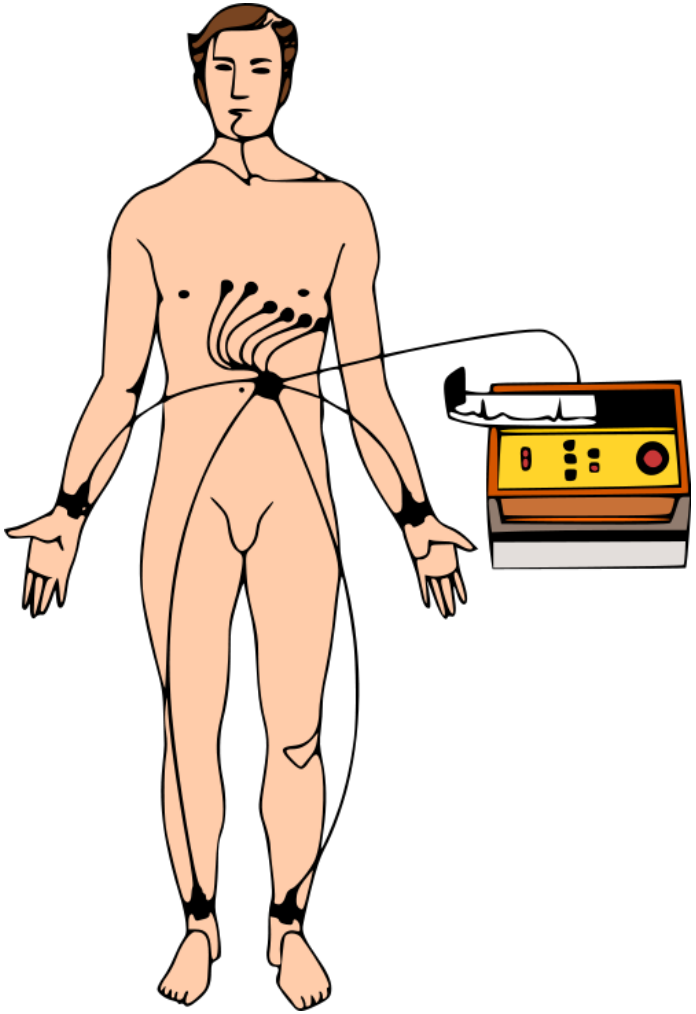
GFI – Ground Fault Interrupter

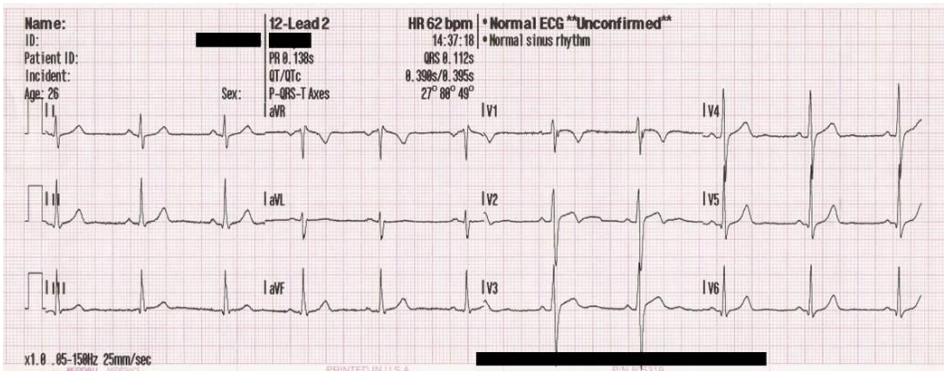
- These are very important safety devices – many lives have been saved because of these
- Also known as GFCI (Ground Fault Circuit Interrupter), ACLI (Appliance Current Leak Current Interrupter), or Trips, Trip Switches or RCD (Residual Current Device) in Australia and UK
- Human heart can be thrown in ventricular fibrillation with a current **through the body** of 100 ma
- Humans can sense currents of 1 ma (not fatal)
- Recall “skin depth” for “good conductors” like metals were about 1 cm for 60 Hz
- The human body is NOT a good conductor

GFI continued

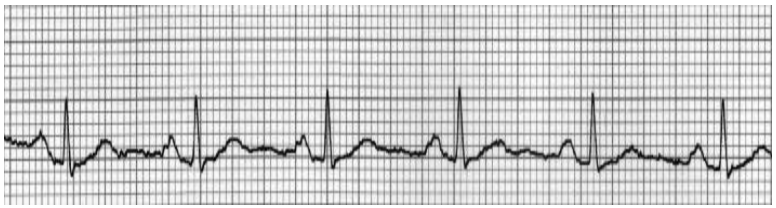
- Typical human resistance (head to toe) is 100KΩ dry
- 1KΩ wet
- Thus 100 Volts when wet => I = V/R ~ 100 ma (lethal)
- 100 Volts dry => 1 ma (not normally lethal – DO NOT TRY
- People vary in shock lethality 30 ma is fatal in some
- Therefore 30 Volts wet can be fatal – be careful please
- ~ 400 deaths per year in US due to shock
- NEC – US National Electric Code set GFI trip limit at 5 ma within 25 ms (milli seconds)
- GFI work by sensing the difference in current between the "hot = live" and "neutral" conductor
- Normally this is done with a differential transformer

Electrocardiagrams – EKG, ECG

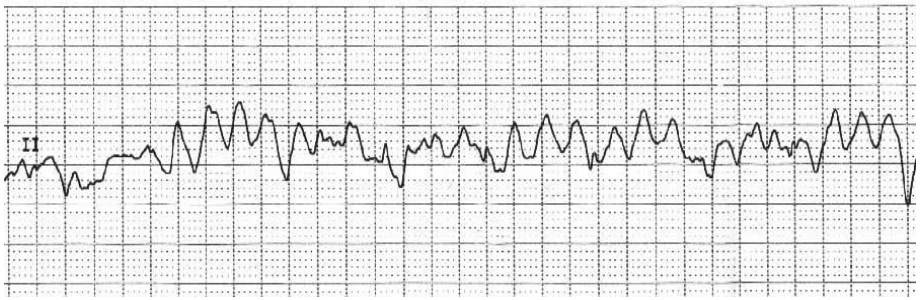




26 year old normal male EKG



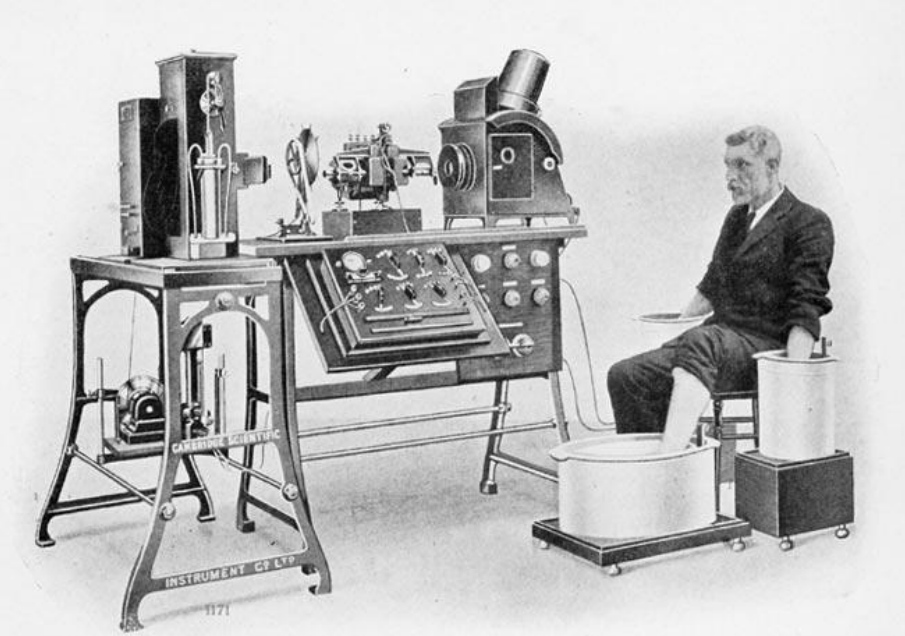
Normal EKG



Ventricular fibrillation

Measuring the Hearts Electrical Activity

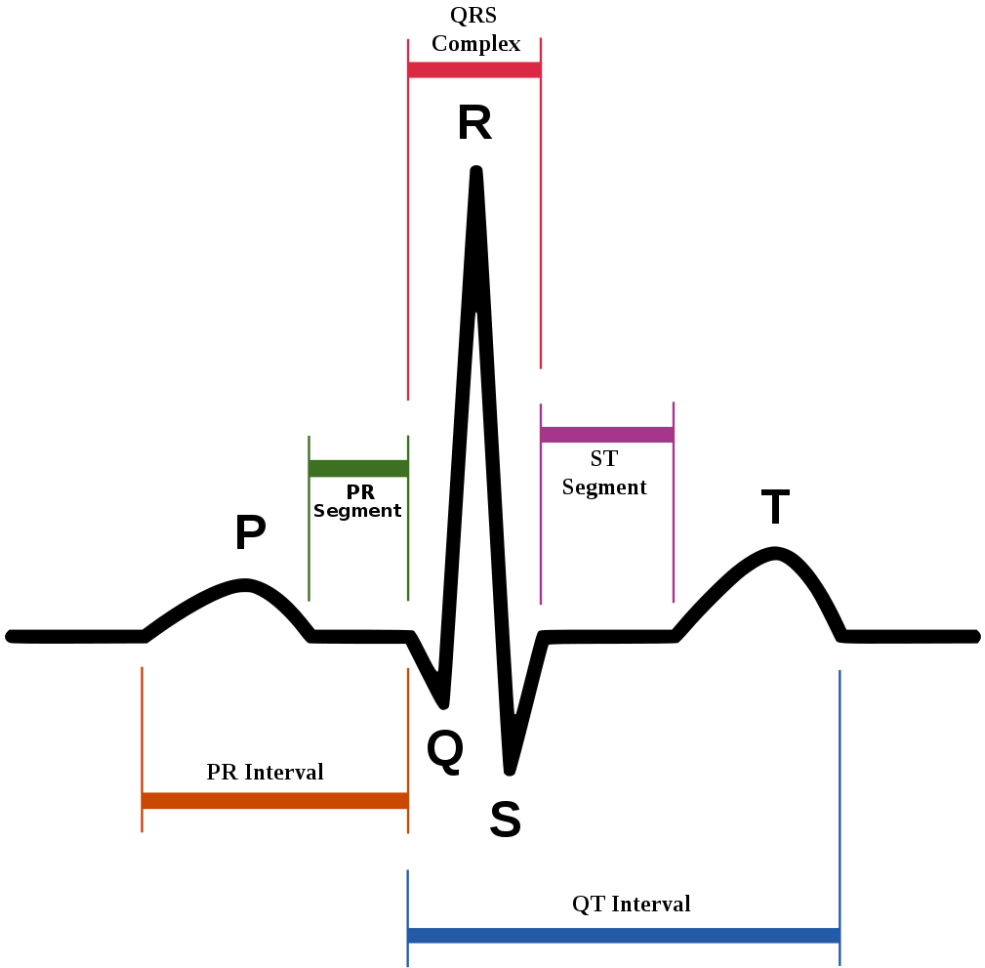
- Alexander Muirhead 1872 measured wrist electrical activity
- Willem Einthoven – Leiden Netherlands 1903 – string galvanometer
- Modern EKG is based on Einthovens work – Nobel 1924

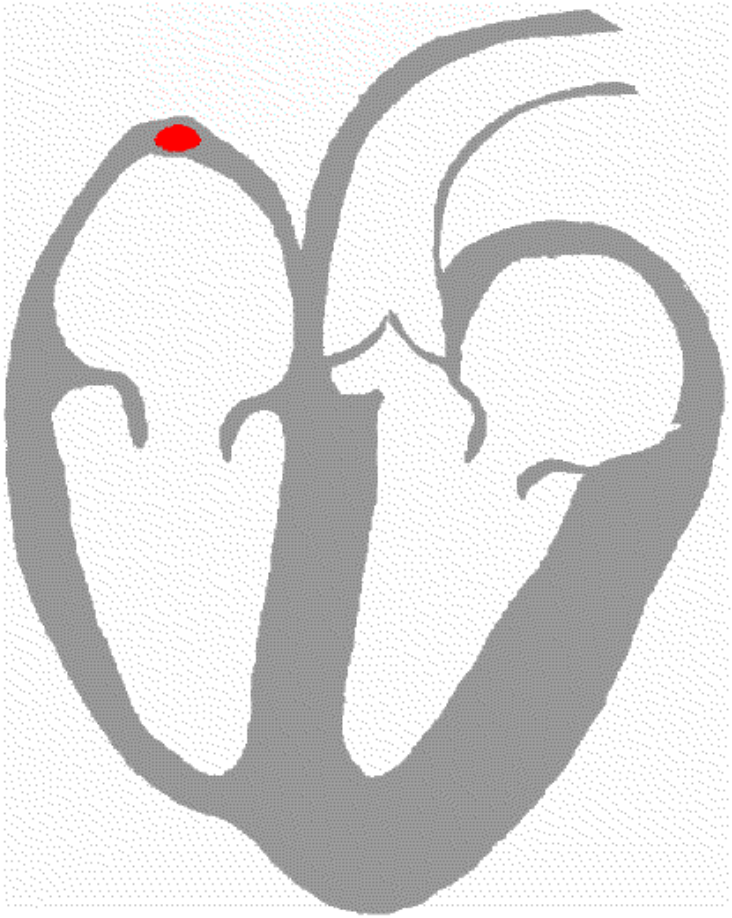


PHOTOGRAPH OF A COMPLETE ELECTROCARDIOGRAPH, SHOWING THE MANNER IN WHICH THE ELECTRODES ARE ATTACHED TO THE PATIENT, IN THIS CASE THE HANDS AND ONE FOOT BEING IMMERSED IN JARS OF SALT SOLUTION

EKG Waveforms

- Einthoven assigned letters P,R,Q,S,T - heart waveform
- Normally 10 leads are used though called 12 lead
- 350,000 cases of SCD – Sudden Cardiac Death





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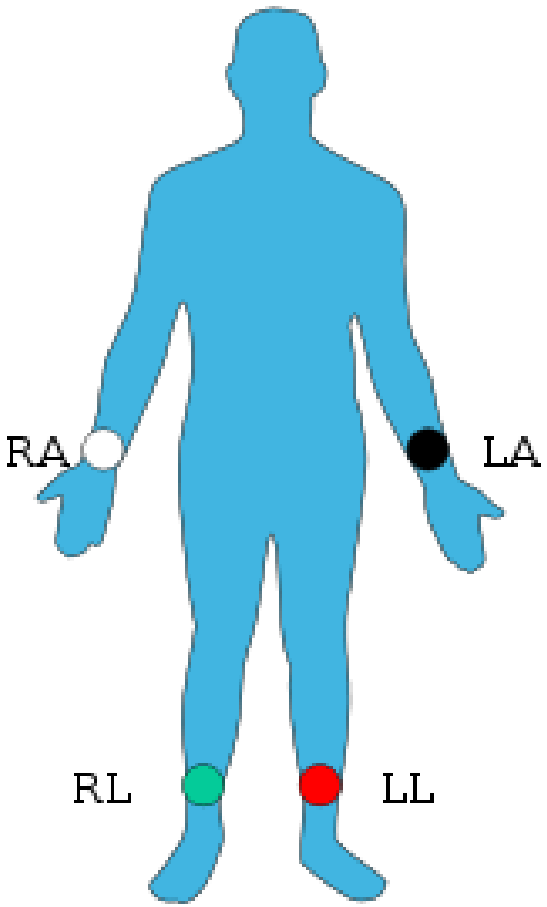
EKG Electrode Placment

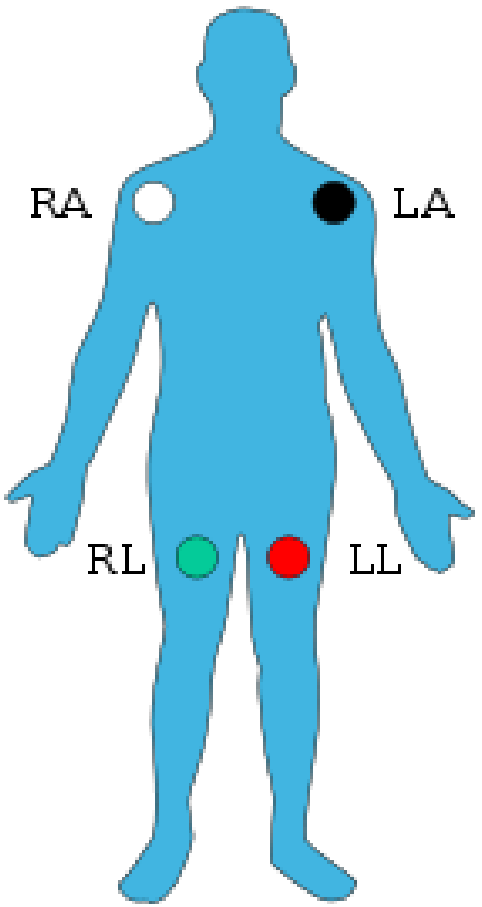
- RA On the right arm, avoiding bony prominences.
- LA In the same location that RA was placed, but on the left arm this time.
- RL On the right leg, avoiding bony prominences.
- LL In the same location that RL was placed, but on the left leg this time.
- V1 In the *fourth* intercostal space (between ribs 4 & 5) just to the *right* of the sternum (breastbone).
- V2 In the *fourth* intercostal space (between ribs 4 & 5) just to the *left* of the sternum.
- V3 Between leads V2 and V4.
- V4 In the fifth intercostal space (between ribs 5 & 6) in the mid-clavicular line (the imaginary line that extends down from the midpoint of the clavicle (collarbone).
- V5 Horizontally even with V4, but in the anterior axillary line. (The anterior axillary line is the imaginary line that runs down from the point midway between the middle of the clavicle and the lateral end of the clavicle; the lateral end of the collarbone is the end closer to the arm.)
- V6 Horizontally even with V4 and V5 in the midaxillary line. (The midaxillary line is the imaginary line that extends down from the middle of the patient's armpit.)

EKG Electrode Placement



RA = Right Arm
LA = Left Arm
RL = Right Leg
LL = Left Leg

RA - White
LA - Black
RL - Green
LL - Red

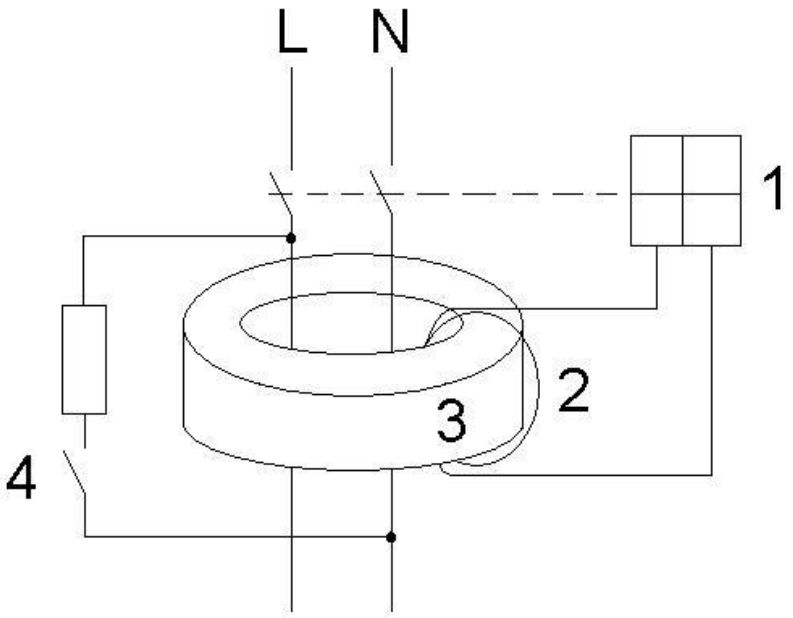


Differential Transformer for GFI

- Works by sensing magnetic field difference in “hot” and “neutral” wire
- Difference in magnetic field is from difference in current flow in these wires
- In a normal circuit the current in the “hot” and “neutral” is equal and opposite
- Thus the magnetic fields should cancel
- If they do not cancel then current is not equal and some of this may be going through your body => shock => **trip (open) circuit immediately** to protect you

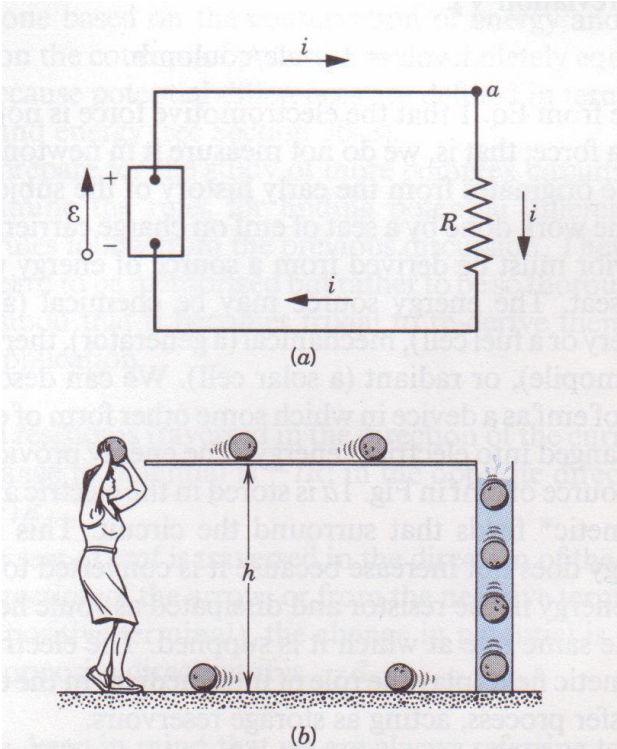
GFI Differential Transformer

- Most GFI's are transformer based – cheaper so far
- They can also be semiconductor based
- L= "live or hot", N= "neutral"
- 1 = relay control to open circuit
- 2= sense winding
- 3=toroid -ferrite or iron core
- 4=Test Switch (test)
- Cost ~ $10



Batteries and EMF

- EMF – ElectroMotive Force – it move the charges in a circuit – source of power
- This can be a battery, generator, solar cell etc
- In a battery the EMF is chemical
- A good analogy is lifting a weight against gravity
- EMF is the “lifter”

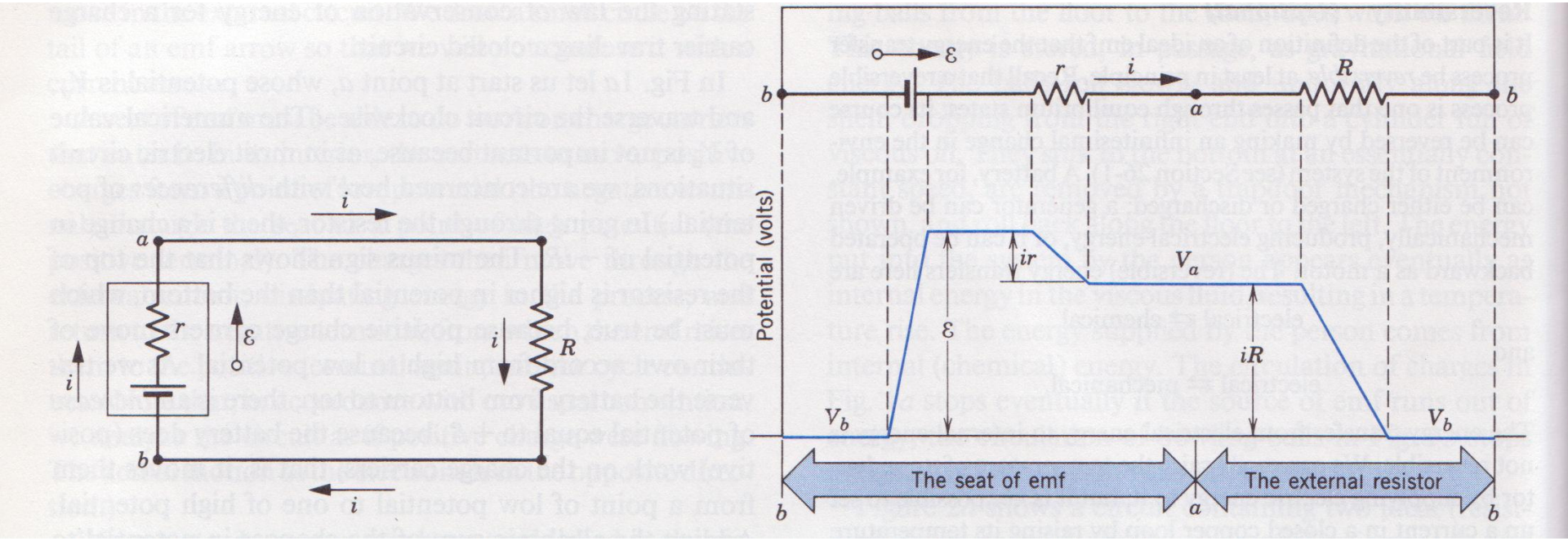


Some EMF rules

- The EMF has a direction and that direction INCREASES energy. The electrical potential is INCREASED.
- The EMF direction is NOT NECESSARILY the direction of (positive) charge flow. In a single battery circuit it is though.
- If you traverse a resistor is traversed IN THE DIRECTION of (positive) current flow the potential is DECREASED by I*R

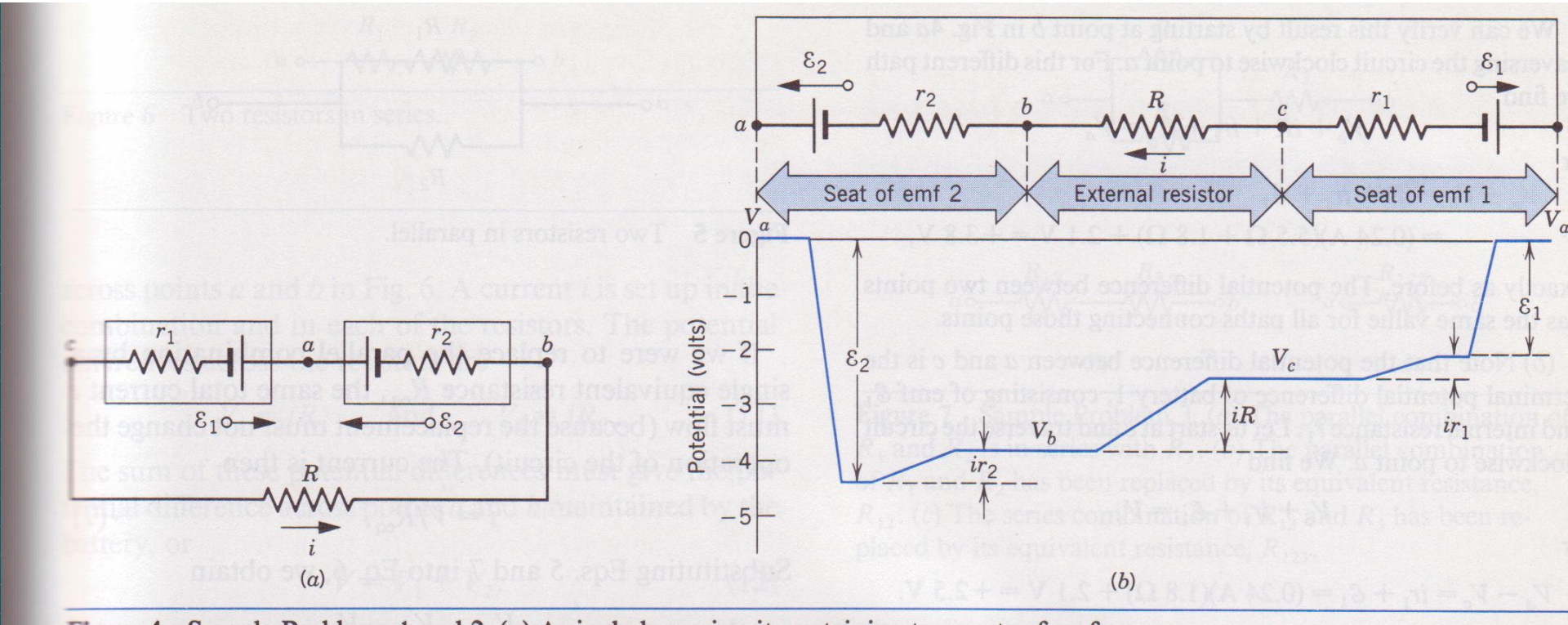
Single battery example

- Recall batteries have an internal resistance r
- In this example we have an external load resistor R
- i= ξ/(R+r)



Double opposing battery example

- In this example we have two batteries with diffferent EMF's and different internal resistances as well as a load resistor.
- Which way will the current flow.
- Your intuition tell you the battery with the higher EMF will force the current in that direction.
- $i = -(\xi_2 - \xi_1)/(R + r_2 + r_1)$

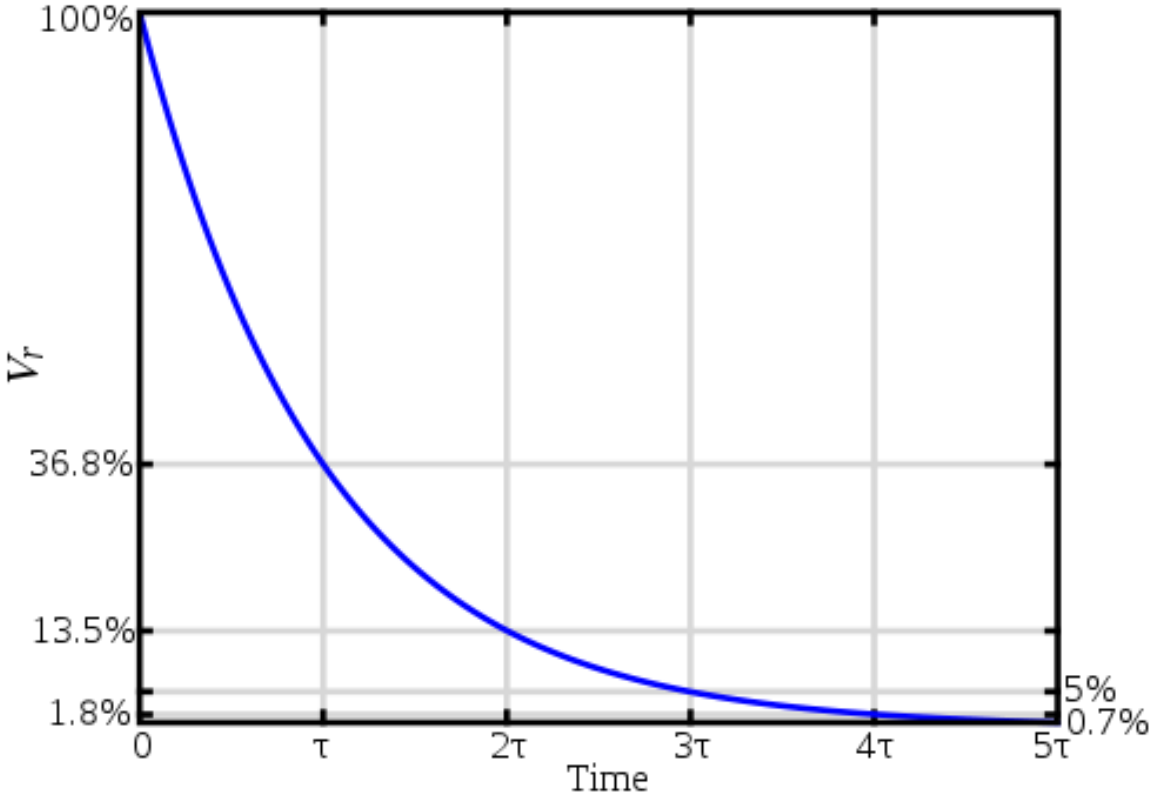


RC Circuit – Exponential Decay

- An RC circuit is a common circuit used in electronic filters
- The basic idea is it take time to charge a capacitor thru a resistor
- Recall that a capacitor C with Voltage V across it has charge Q=CV
- Current I= dQ/dt = C dV/dt
- In a circuit with a capacitor and resistor in parallel the voltage across the resistor must equal opposite that across the capacitor
- Hence $V_c = -V_R$ or Q/C = -IR or Q/C + IR = 0 (note the current I thru the resistor must be responsible for the dQ/dt – Kirchoff or charge conservation
- Now take a time derivative dQ/dt/C + R dI/dt = I/C + R dI/dt =0
- OR dI/dt +I/RC simply first order differential equation
- Solution is $I(t) = I_0 e^{-t/RC} = I_0 e^{-t/\tau}$ where τ = RC is the "time constant"
- Voltage across resistor $V_R(t) = IR = I_0 R e^{-t/RC} = V_0 e^{-t/RC} = - V_c(t)$ voltage across capacitor
- Note the exponential decay
- We can also write the eq as R dQ/dt + Q/C =0

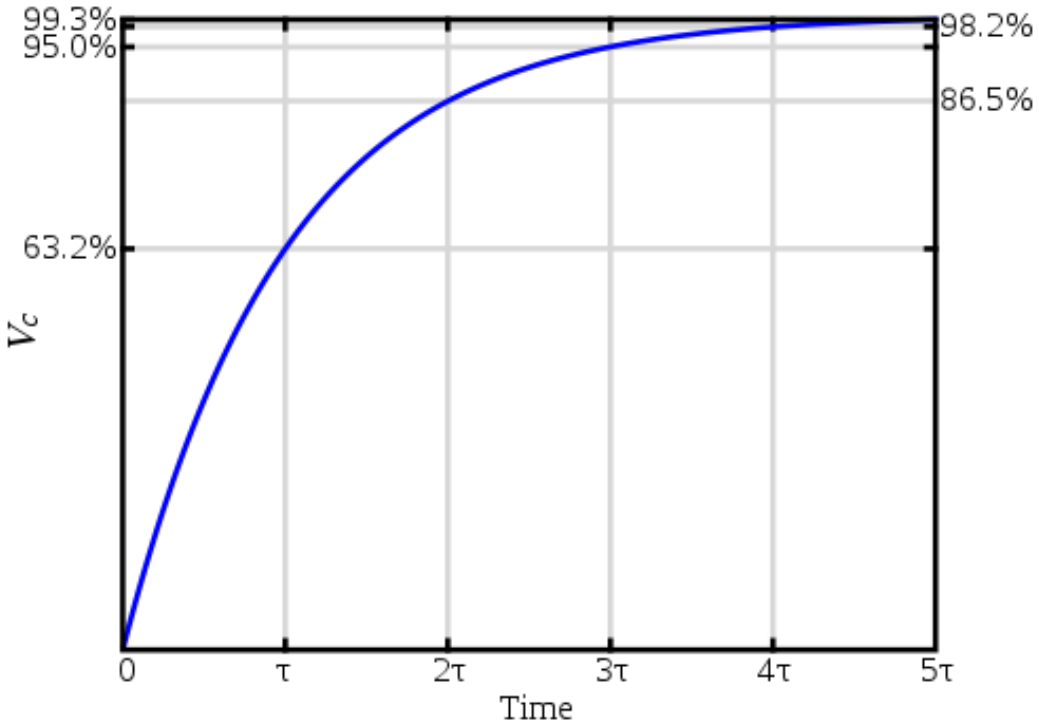
Discharging a capacitor

- Imagine starting with a capacitor C charged to voltage V_0
- Now discharge it starting at t=0 through resistor R
- $V(t) = V_0 e^{-t/RC}$



Charging a Capacitor

- Start with a capacitor C that is discharged (0 volts)
- Now hook up a battery with a resistor R
- Start the charge at t=0
- $V(t) = V_0 (1- e^{-t/RC})$

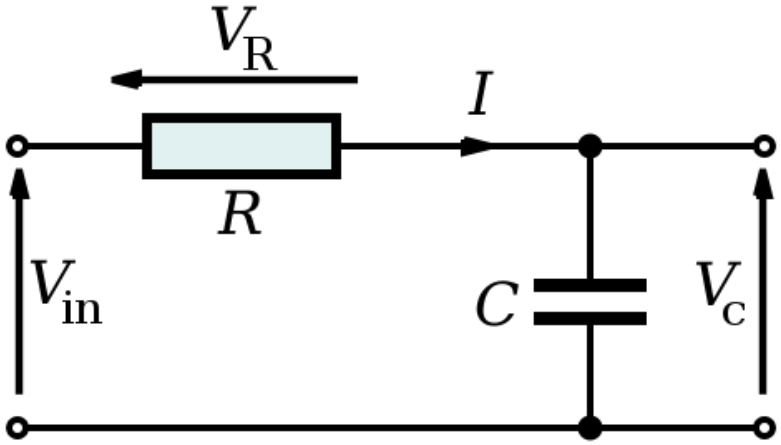


RC Circuits – Another way

- Lets analyze this another way
- In a closed loop the total EMF is zero (must be careful here once we get to induced electric fields from changing magnetic fields)
- In the quasi static case $\int E \bullet dl = 0$ over a closed loop C
- Charge across the capacitor $Q = CV$ $I = dQ/dt = C\, dV/dt$
- But the same $I = -V/R$ (minus as V across cap is minus across R if we go in a loop)
- $C dV/dt = - V/R$ or $C\, dV/dt + V/R = 0$ or $dV/dt + V/RC = 0$
- Solution is $V(t) = V_0\, e^{-t/RC}$
- Same solution as before
- The time required to fall from the initial voltage V_0 to V_0/e is time $\tau = RC$

Complex impedances

- Consider the following series circuit
- If we put an input Voltage V_{in} across the system
- We get a differential eq as before but with V_{in}
- $V_{in} + IR + Q/C = 0$ $\int E \bullet dl = 0$ around the closed loop
- $V_{in} + R\, dQ/dt + Q/C = V_{in} + IR + \int I\, dt/C$ – we can write the solution as a complex solution $I = I_0\, e^{i\omega t}$
- $V_{in} + IR + \int I\, dt/C$
- We can make this more
- General letting $V_{in} = V_0\, e^{i\omega t}$
- This allows a driven osc term – freq ω
- $V_0\, e^{i\omega t} + R\, I_0\, e^{i\omega t} + I_0\, e^{i\omega t} /(i\omega C)$



- $V_0 + R\, I_0 + I_0/(i\omega C)$ - thus we can interpret this as a series of impedances (resistance) Z (general impedance) where $Z_R = R$ is the normal impedance of a resistor and $Z_c = 1/(i\omega C) = -i/(\omega C)$ is the impedance of a capacitor
- Note the impedance of a capacitor is complex and proportional to $1/\omega C$ - the minus i will indicate a 90 degree phase shift

Universal Normailized (Master) Oscillator Eq

No driving (forcing) function equation
System is normalized so undamped resonant freq ω_0 =1. τ = t/t_c t_c = undamped period $\omega = \omega/\omega_0$

$$\frac{\mathrm{d}^2 q}{\mathrm{d}\tau^2} + 2\zeta\frac{\mathrm{d}q}{\mathrm{d}\tau} + q = 0$$

With sinusoidal driving function

$$\frac{\mathrm{d}^2 q}{\mathrm{d}\tau^2} + 2\zeta\frac{\mathrm{d}q}{\mathrm{d}\tau} + q = \cos(\omega\tau).$$

We will consider two general cases
Transient $q_t(t)$ and steady state $q_s(t)$

Transient Solution

$$q_t(\tau) = \begin{cases} e^{-\zeta\tau}\left(c_1 e^{\tau\sqrt{\zeta^2-1}} + c_2 e^{-\tau\sqrt{\zeta^2-1}}\right) & \zeta > 1 \text{ (overdamping)} \\ e^{-\zeta\tau}(c_1 + c_2\tau) = e^{-\tau}(c_1 + c_2\tau) & \zeta = 1 \text{ (critical damping)} \\ e^{-\zeta\tau}\left[c_1 \cos\left(\sqrt{1-\zeta^2}\tau\right) + c_2 \sin\left(\sqrt{1-\zeta^2}\tau\right)\right] & \zeta < 1 \text{ (underdamping)} \end{cases}$$

Steady State Solution

$$\frac{\mathrm{d}^2 q}{\mathrm{d}\tau^2} + 2\zeta \frac{\mathrm{d}q}{\mathrm{d}\tau} + q = \cos(\omega\tau) + i\sin(\omega\tau) = e^{i\omega\tau}.$$

$$q_s(\tau) = Ae^{i(\omega\tau+\phi)}.$$

$$q_s = Ae^{i(\omega\tau+\phi)},\ \frac{\mathrm{d}q_s}{\mathrm{d}\tau} = i\omega Ae^{i(\omega\tau+\phi)},\ \frac{\mathrm{d}^2 q_s}{\mathrm{d}\tau^2} = -\omega^2 Ae^{i(\omega\tau+\phi)}.$$

$$-\omega^2 Ae^{i(\omega\tau+\phi)} + 2\zeta i\omega Ae^{i(\omega\tau+\phi)} + Ae^{i(\omega\tau+\phi)} = (-\omega^2 A + 2\zeta i\omega A + A)e^{i(\omega\tau+\phi)} = e^{i\omega\tau}.$$

$$-\omega^2 A + 2\zeta i\omega A + A = e^{-i\phi} = \cos\phi - i\sin\phi.$$

Steady State Continued

$$A(1-\omega^2) = \cos\phi \qquad 2\zeta\omega A = -\sin\phi.$$

$$\left.\begin{array}{r} A^2(1-\omega^2)^2 = \cos^2\phi \\ (2\zeta\omega A)^2 = \sin^2\phi \end{array}\right\} \Rightarrow A^2[(1-\omega^2)^2 + (2\zeta\omega)^2] = 1.$$

$$A = A(\zeta,\omega) = \frac{1}{\sqrt{(1-\omega^2)^2 + (2\zeta\omega)^2}}.$$

Solve for Phase

$$\tan\phi = -\frac{2\zeta\omega}{1-\omega^2} = \frac{2\zeta\omega}{\omega^2-1} \Rightarrow \phi \equiv \phi(\zeta,\omega) = \arctan\left(\frac{2\zeta\omega}{\omega^2-1}\right).$$

Note the phase shift is frequency dependent
At low freq ϕ -> 0
At high freq ϕ -> 180 degrees
Remember $\omega = \omega/\omega_0$

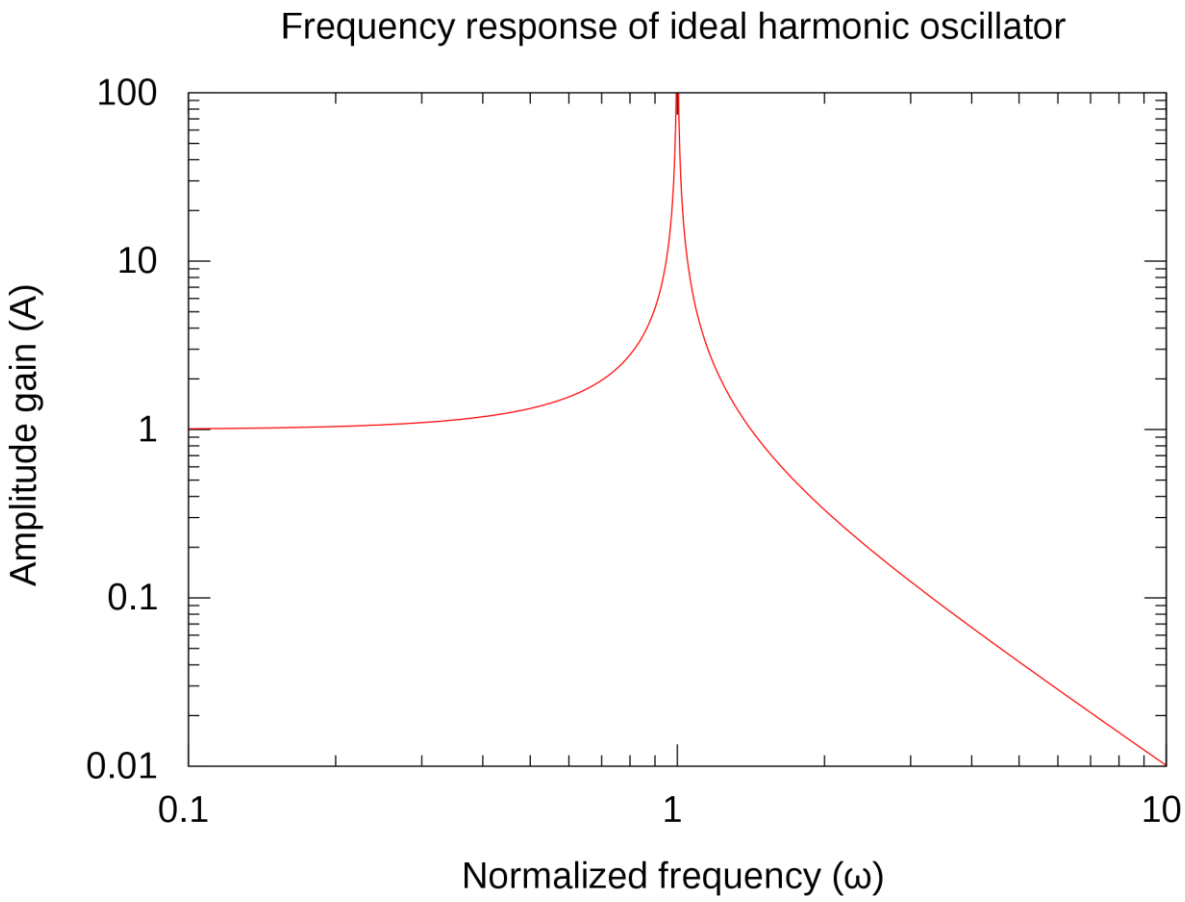
Full solution

$$q_s(\tau) = A(\zeta, \omega)\cos(\omega\tau + \phi(\zeta, \omega)) = A\cos(\omega\tau + \phi).$$

$$q(\tau) = q_t(\tau) + q_s(\tau).$$

Amplitude vs freq – Bode Plot

$$A = A(\zeta, \omega) = \frac{1}{\sqrt{(1-\omega^2)^2 + (2\zeta\omega)^2}}.$$



Various Damped Osc Systems

Translational Mechanical	Torsional Mechanical	Series RLC Circuit	Parallel RLC Circuit
Position x	Angle θ	Charge q	Voltage e
Velocity dx/dt	Angular velocity $d\theta/dt$	Current dq/dt	de/dt
Mass m	Moment of inertia I	Inductance L	Capacitance C
Spring constant K	Torsion constant μ	Elastance 1/C	Susceptance 1/L
Friction γ	Rotational friction Γ	Resistance R	Conductance 1/R
Drive force F(t)	Drive torque $\tau(t)$	e(t)	di/dt

Undamped resonant frequency : f_n

$\frac{1}{2\pi}\sqrt{\frac{K}{M}}$	$\frac{1}{2\pi}\sqrt{\frac{\mu}{I}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$	$\frac{1}{2\pi}\sqrt{\frac{1}{LC}}$

Differential equation:

$M\ddot{x}+\gamma\dot{x}+Kx=F$	$I\ddot{\theta}+\Gamma\dot{\theta}+\mu\theta=\tau$	$L\ddot{q}+R\dot{q}+q/C=e$	$C\ddot{e}+\dot{e}/R+e/L=\dot{i}$