

Chapter 13

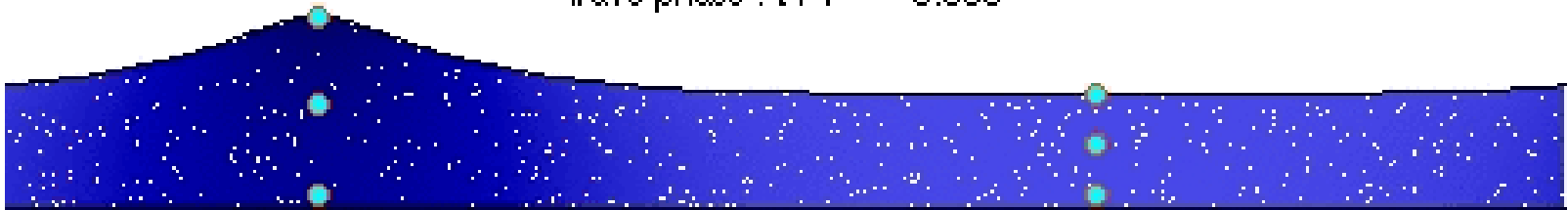
Waves and Acoustics





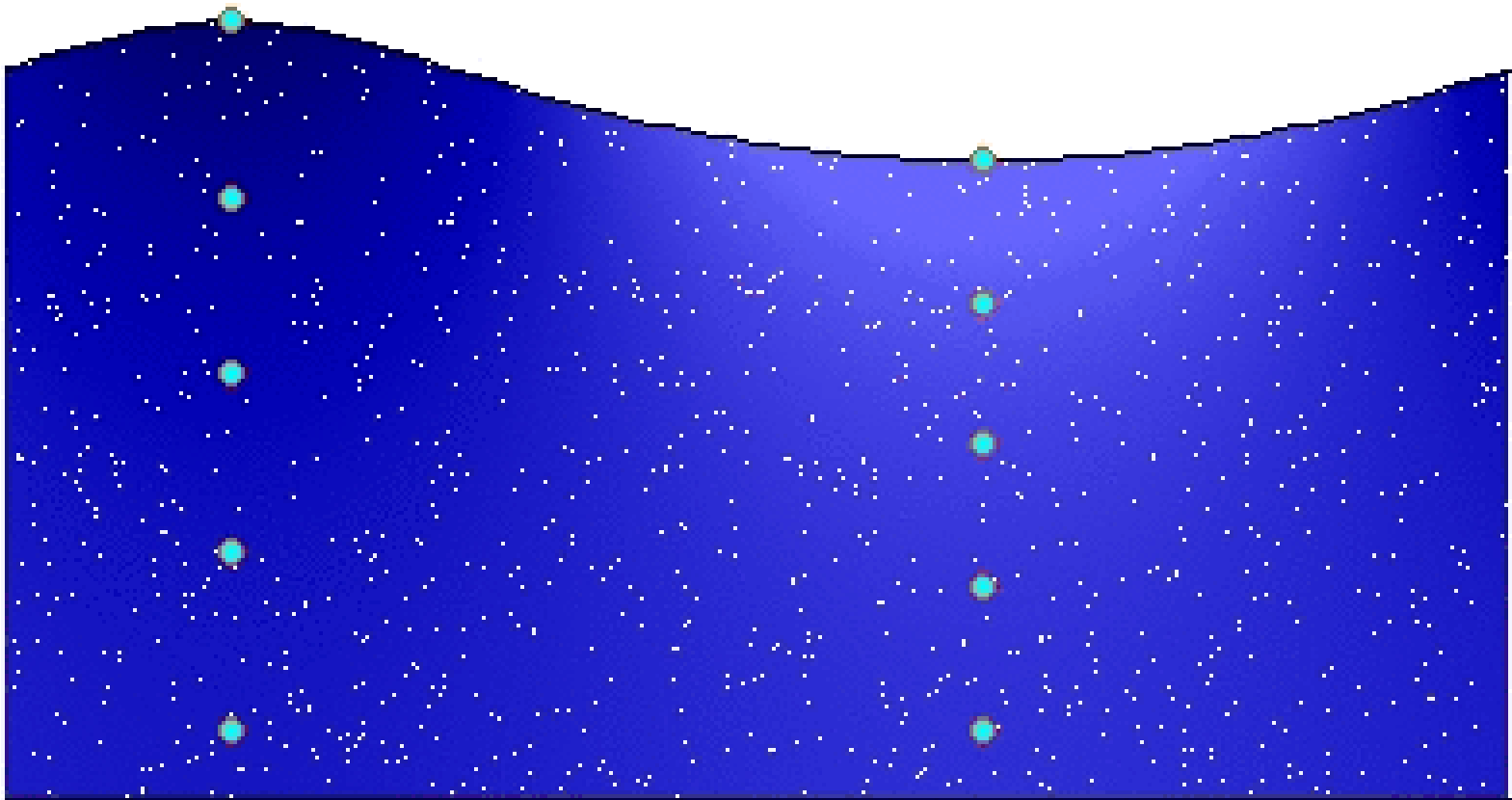
Shallow water “gravity wave”

wave phase : $t/T = 0.000$

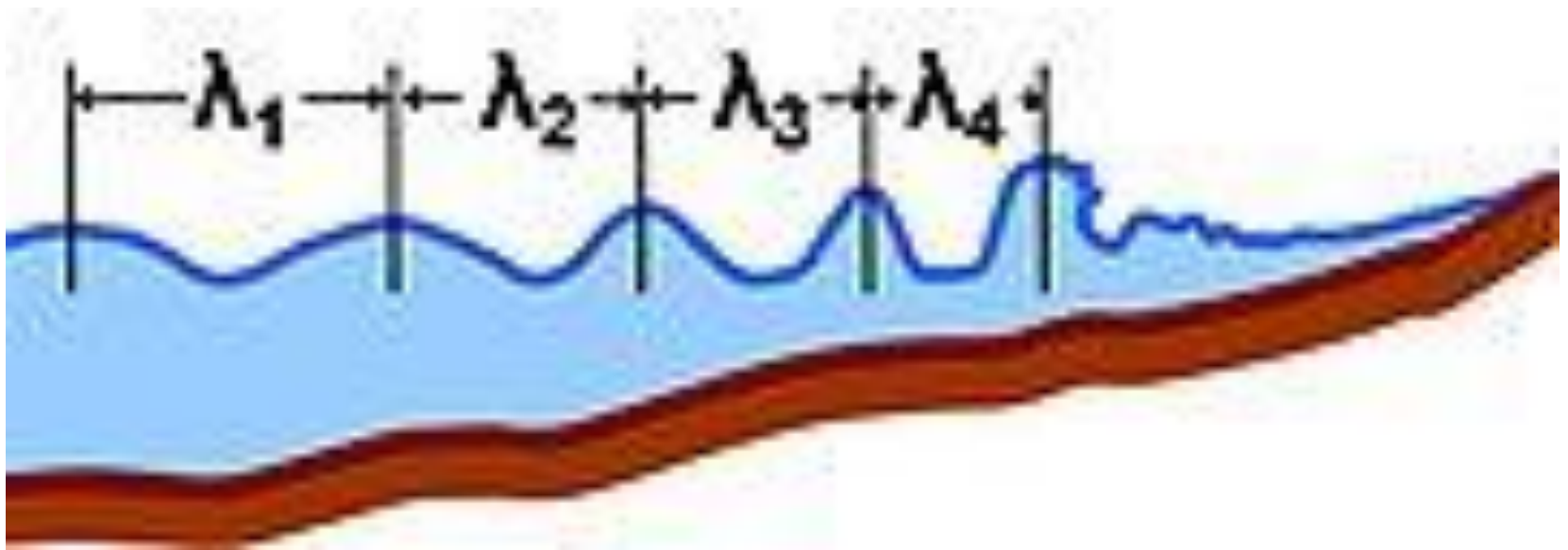


Deep water “gravity wave”

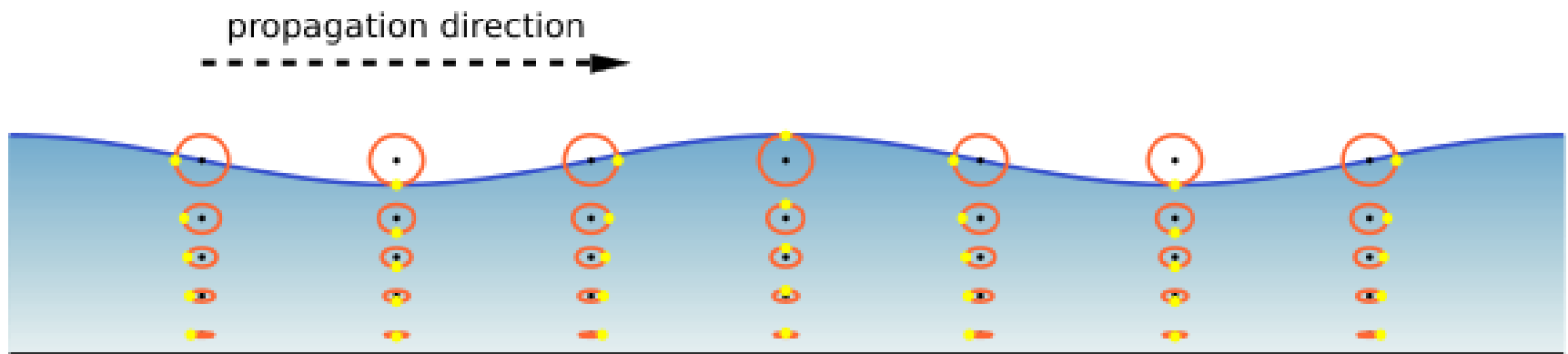
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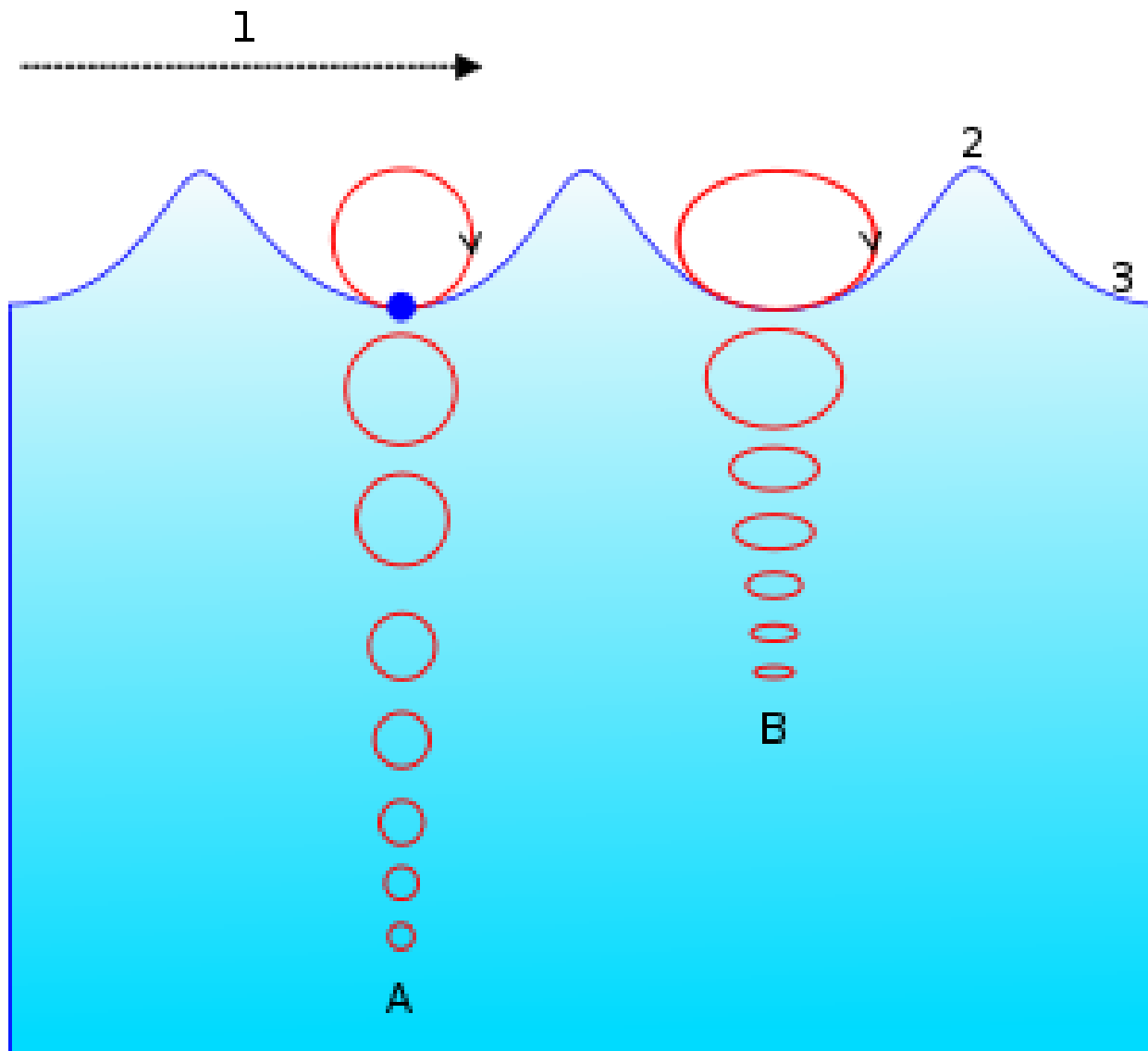
A breaking water wave



Gentle water wave particles execute near circular motion



In shallow larger amplitude waves particle motion becomes elliptical



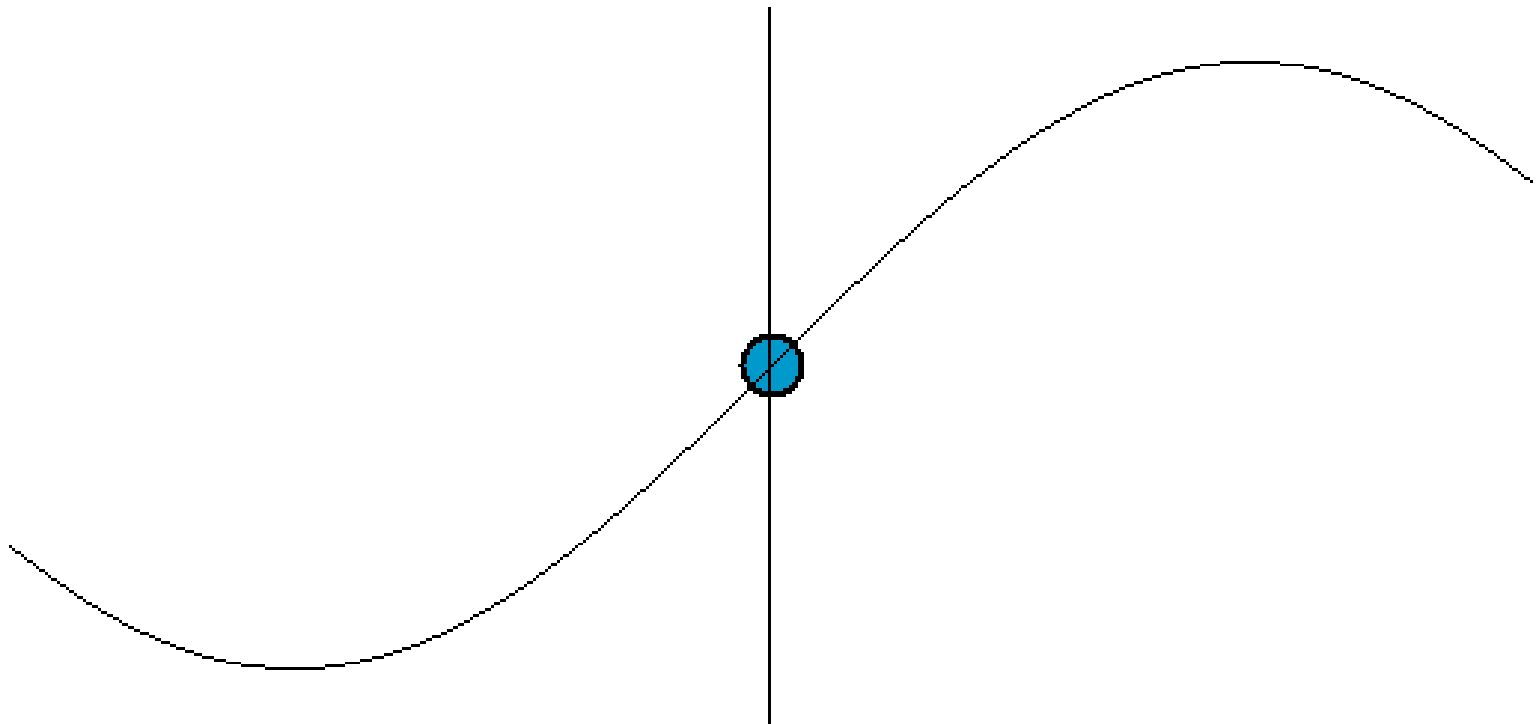
Simple Harmonic Motion (Oscillation) $y(t) = A \sin(\omega t + \phi)$

ω = angular freq (radians/sec)

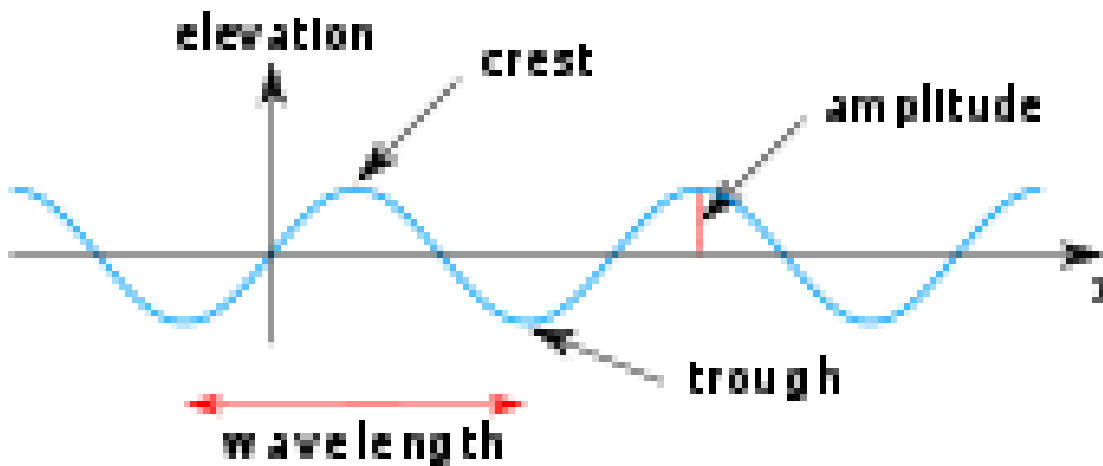
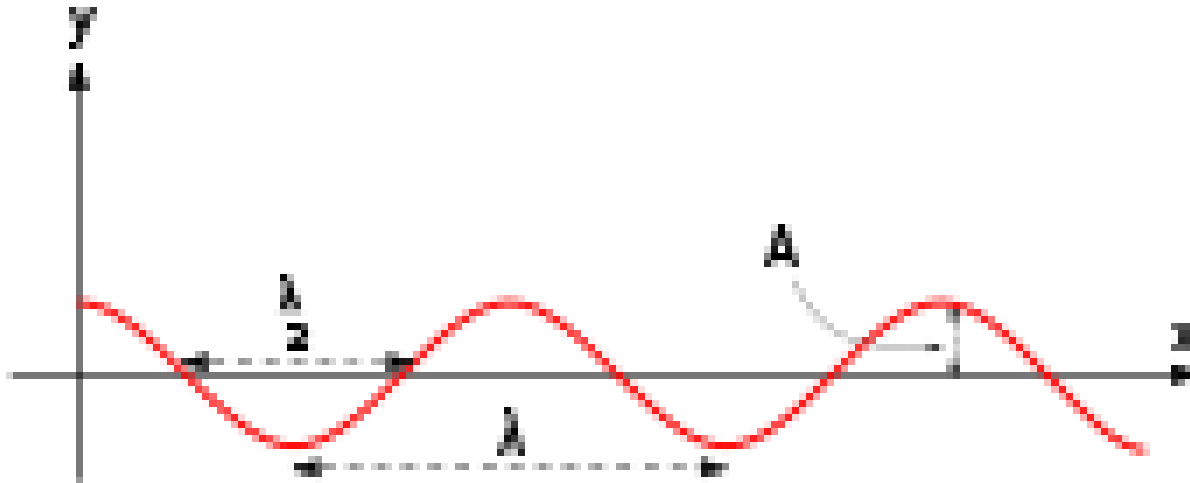
t = time

ϕ = Phase shift (angular – radians)

A = Amplitude



Wave notation



Frequency= f , Period= T , Angular freq= ω

Wavelength= λ Wave number= k

$$f = \frac{1}{T}. \quad \omega = 2\pi f = \frac{2\pi}{T}.$$

$$k = \frac{2\pi}{\lambda}.$$

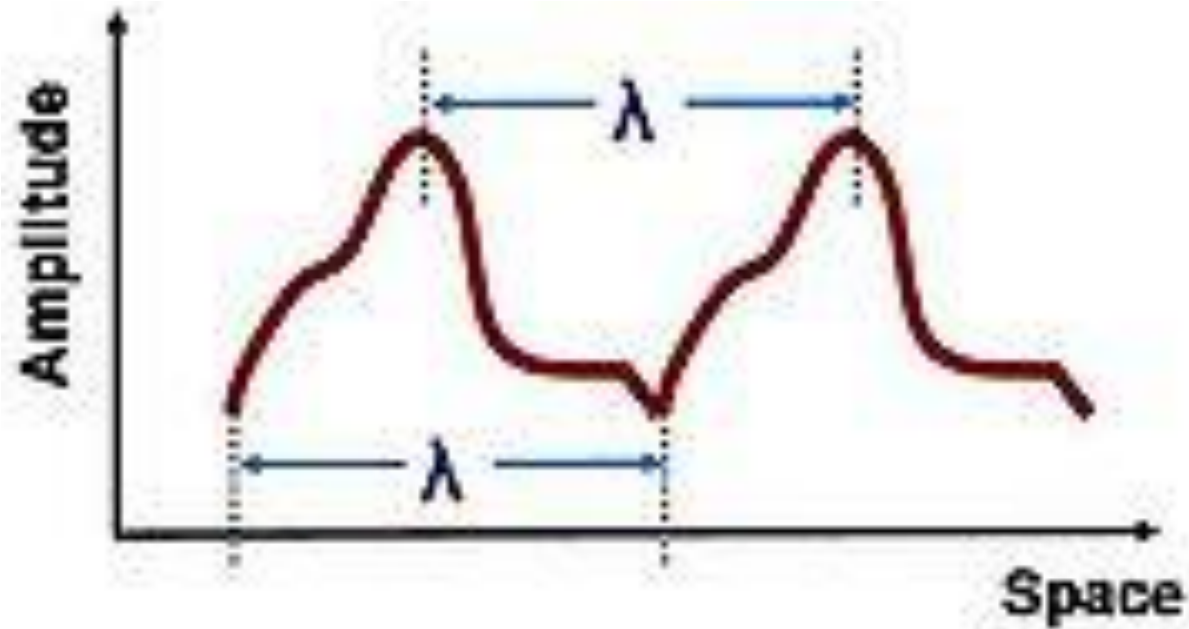
For a purely sinusoidal wave the wavelength and frequency are inversely related

$$\lambda = \frac{v}{f},$$

$$\omega = v * k$$

dispersion relation

Not all waves are sinusoidal – normally they are periodic
hence have a specific wavelength



Q15.1

If you double the wavelength λ of a wave on a string, what happens to the wave speed v and the wave frequency f ?

- A. v is doubled and f is doubled.
- B. v is doubled and f is unchanged.
- C. v is unchanged and f is halved.
- D. v is unchanged and f is doubled.
- E. v is halved and f is unchanged.

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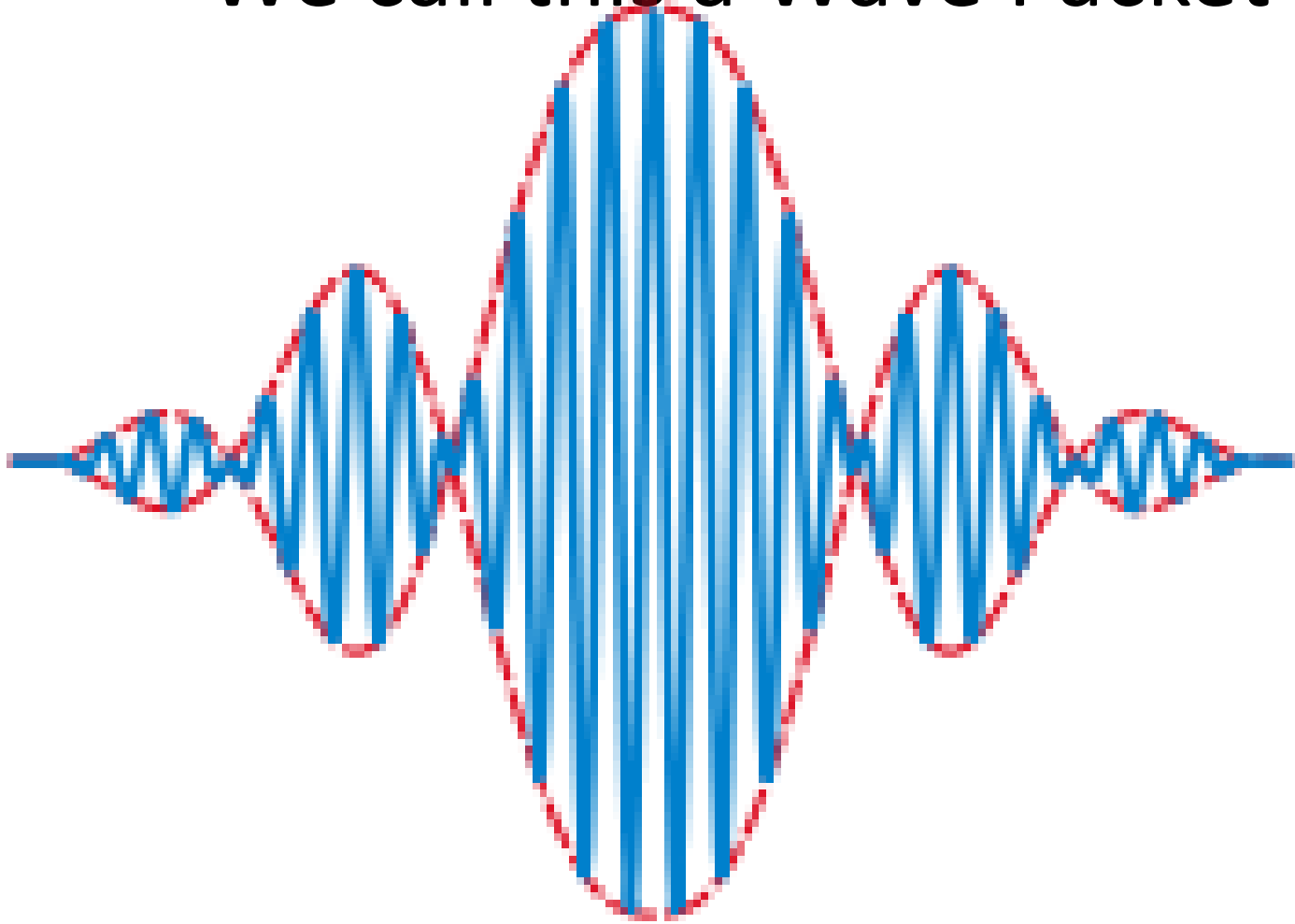
 C. v is unchanged and f is halved.

D. v is unchanged and f is doubled.

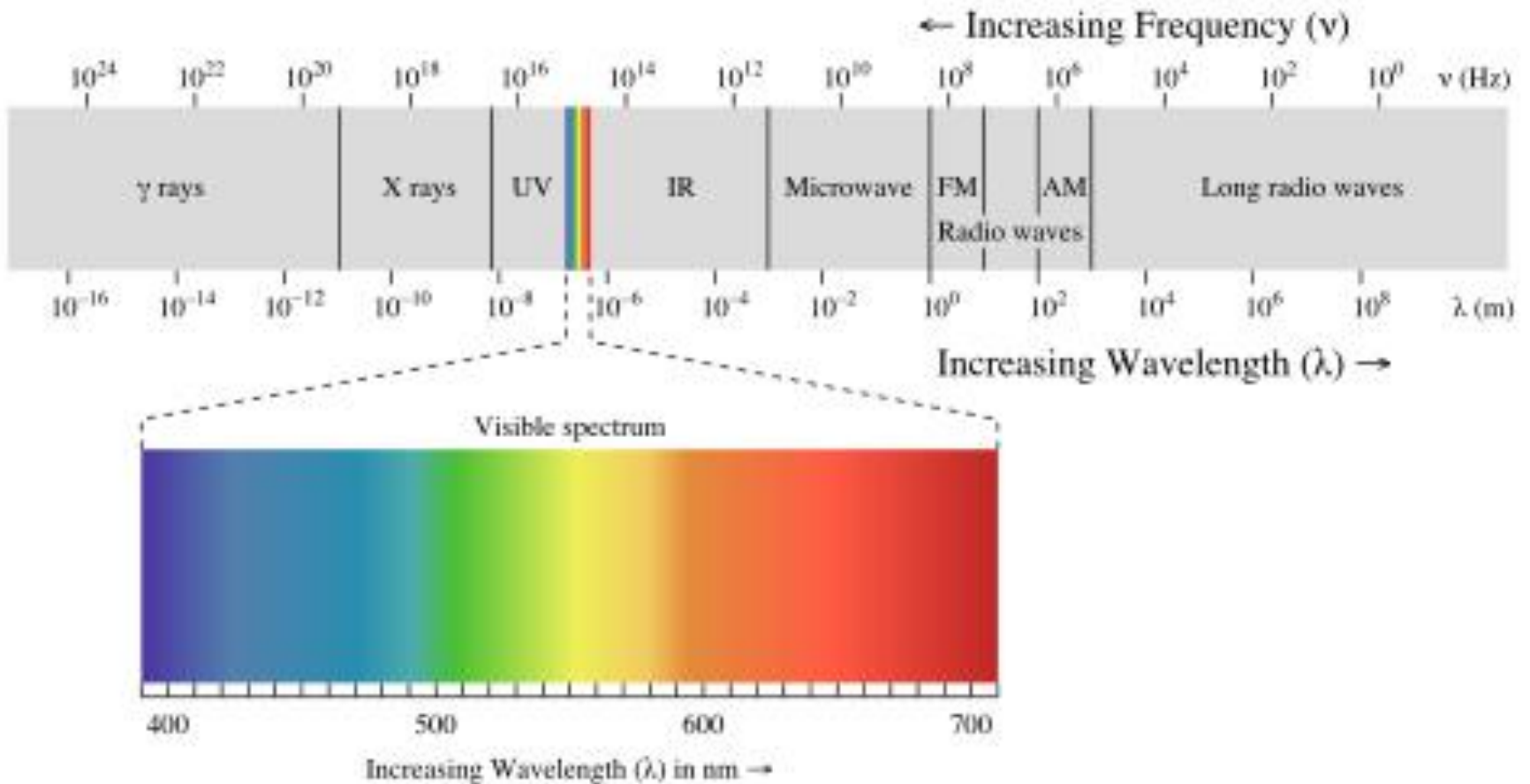
E. v is halved and f is unchanged.

Wave can add and we get packets of waves

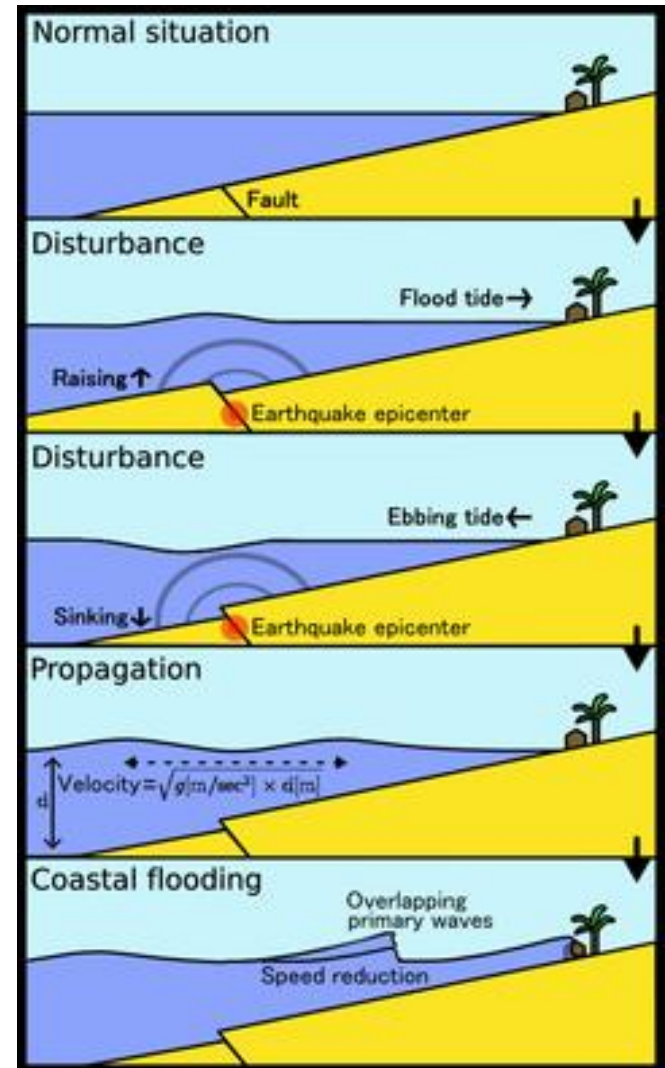
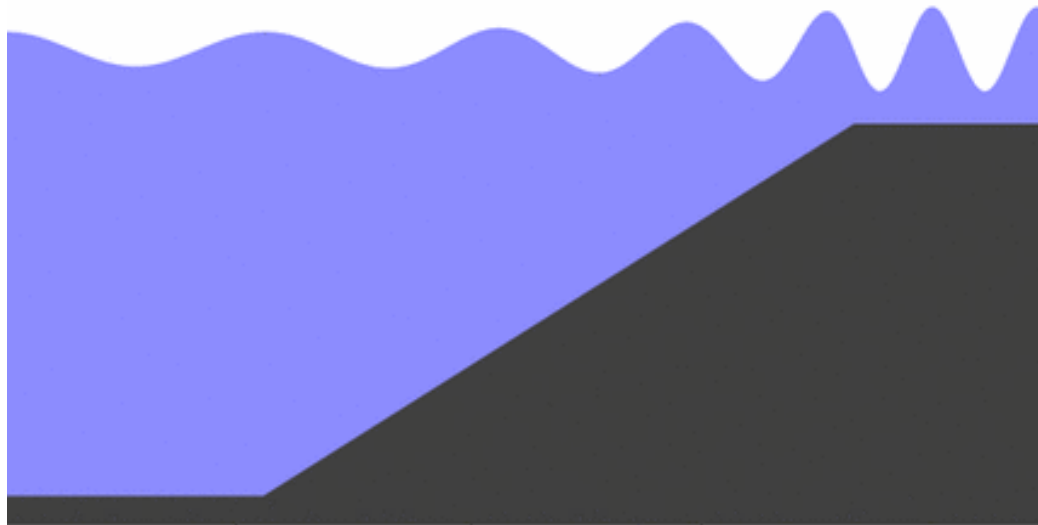
We call this a Wave Packet



Other waves – Electromagnetic Waves



As wave enters shallow bottom is slows and the amplitude increases



Tsunami generated by fault slip and volcano

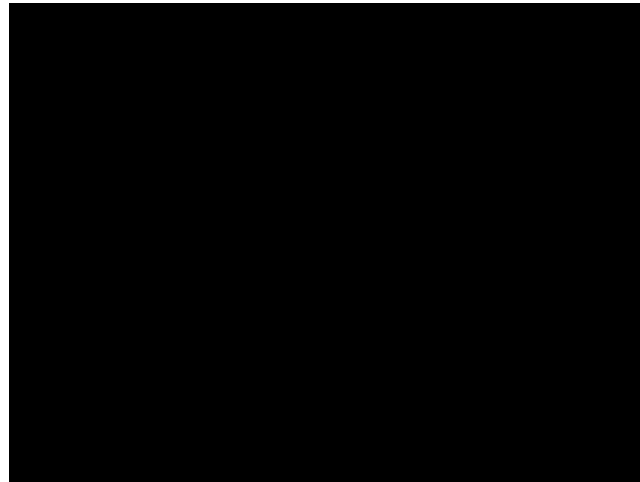


Thailand 12/26/04 – More than 300,000 killed

Large Tsunamis can have speeds exceeding 500 MPH!!
The speed of a tsunami in the open ocean could be supersonic



Samoaan Tsunami Sept 2009

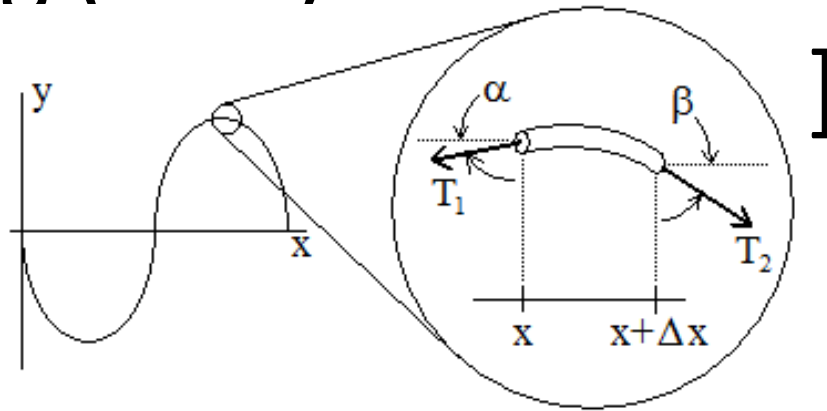


Standing waves have fixed boundary
conditions

A piano and a guitar are examples



String (wire) under tension (T) [μ



$$T_{1x} = T_1 \cos(\alpha) \approx T_{2x} = T_2 \cos(\beta) \approx T.$$

$$\Sigma F_y = T_{2y} - T_{1y} = T_2 \sin(\beta) - T_1 \sin(\alpha) = \Delta m a \approx \mu \Delta x \frac{\partial^2 y}{\partial t^2}.$$

$$\frac{\mu \Delta x}{T} \frac{\partial^2 y}{\partial t^2} = \frac{T_2 \sin(\beta)}{T_2 \cos(\beta)} - \frac{T_1 \sin(\alpha)}{T_1 \cos(\alpha)} = \tan(\beta) - \tan(\alpha)$$

$$\frac{1}{\Delta x} \left(\left. \frac{\partial y}{\partial x} \right|^{x+\Delta x} - \left. \frac{\partial y}{\partial x} \right|^x \right) = \frac{\mu}{T} \frac{\partial^2 y}{\partial t^2}$$

Equation of tensioned wire

2nd order linear diff eq – simple sin, cos solution

$$y=A*\sin (\text{or cos}) (x\pm vt)$$

Use Fourier transform to get any solution

T – tension (N), μ – mass per unit length (kg/m)

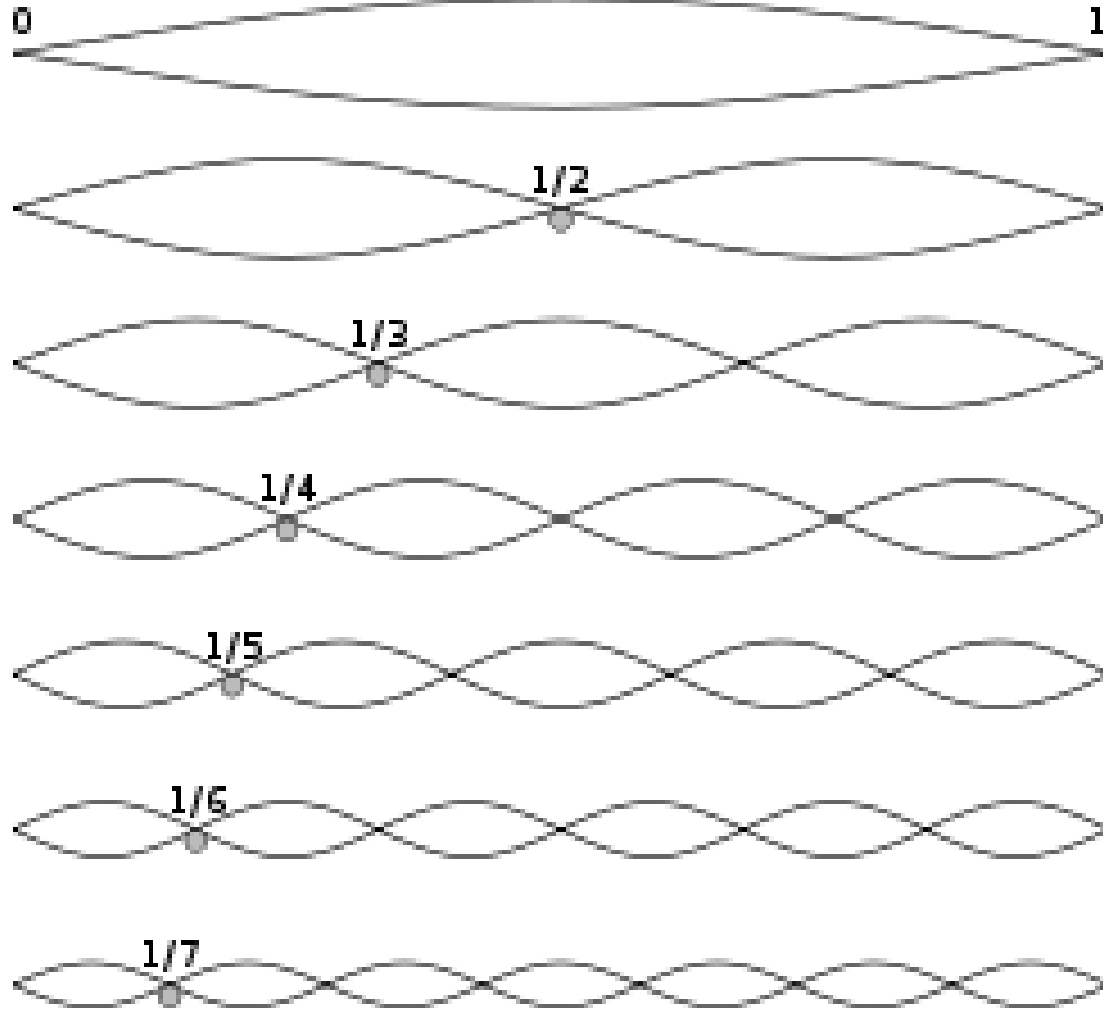
$$\frac{\partial^2 y}{\partial x^2} = \frac{\mu}{T} \frac{\partial^2 y}{\partial t^2} .$$

$$v = \sqrt{\frac{T}{\mu}} ,$$

Standing Waves on a string

Fundamental and 6 harmonics

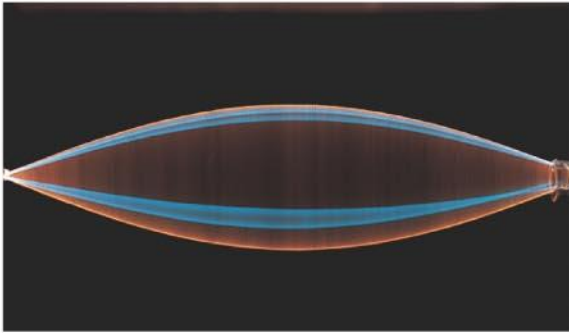
Such studies date back to Ancient Chinese ~3000BC



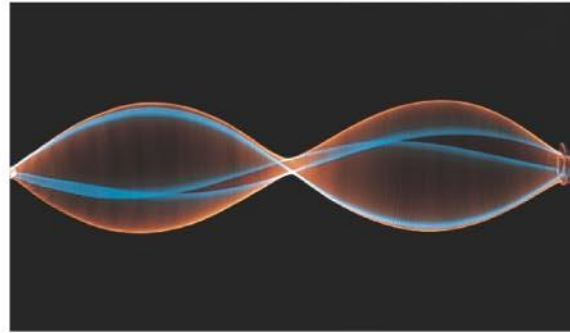
Standing waves on a string

- Fixed at both ends, the resonator must have waveforms that match. In this case, the standing waveform must have nodes at both ends. Differences arise only from increased energy in the waveform.

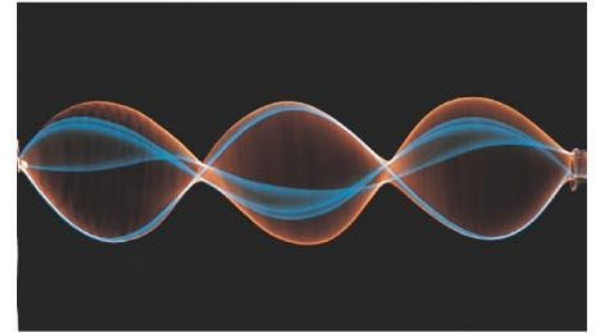
(a) String is one-half wavelength long.



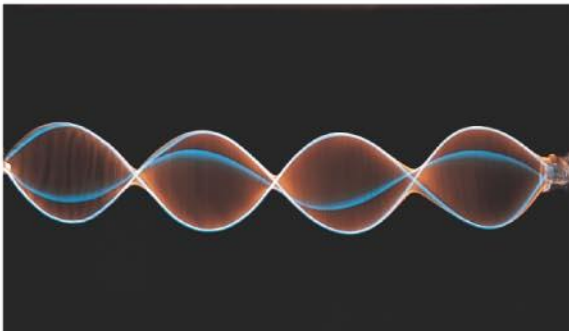
(b) String is one wavelength long.



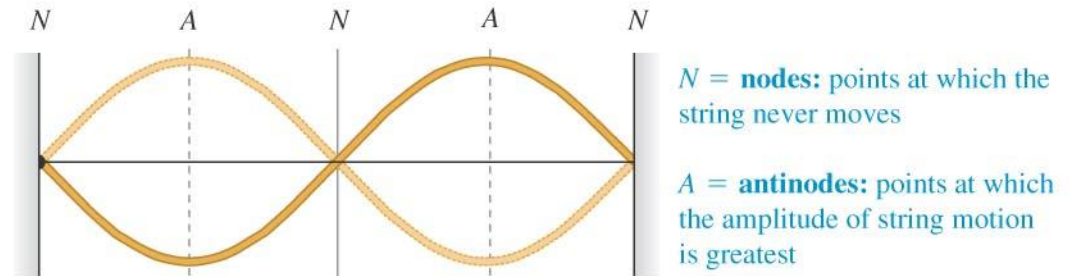
(c) String is one and a half wavelengths long.



(d) String is two wavelengths long.



(e) The shape of the string in (b) at two different instants



Q15.2

Which of the following wave functions describe a wave that moves in the $-x$ -direction?

A. $y(x,t) = A \sin (-kx - \omega t)$

B. $y(x,t) = A \sin (kx + \omega t)$

C. $y(x,t) = A \cos (kx + \omega t)$

D. both B. and C.

E. all of A., B., and C.

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Q15.8

The four strings of a musical instrument are all made of the same material and are under the same tension, but have different thicknesses. Waves travel

- A. fastest on the thickest string.
- B. fastest on the thinnest string.
- C. at the same speed on all strings.
- D. Either A. or B. is possible.
- E. Any of A., B., or C. is possible.

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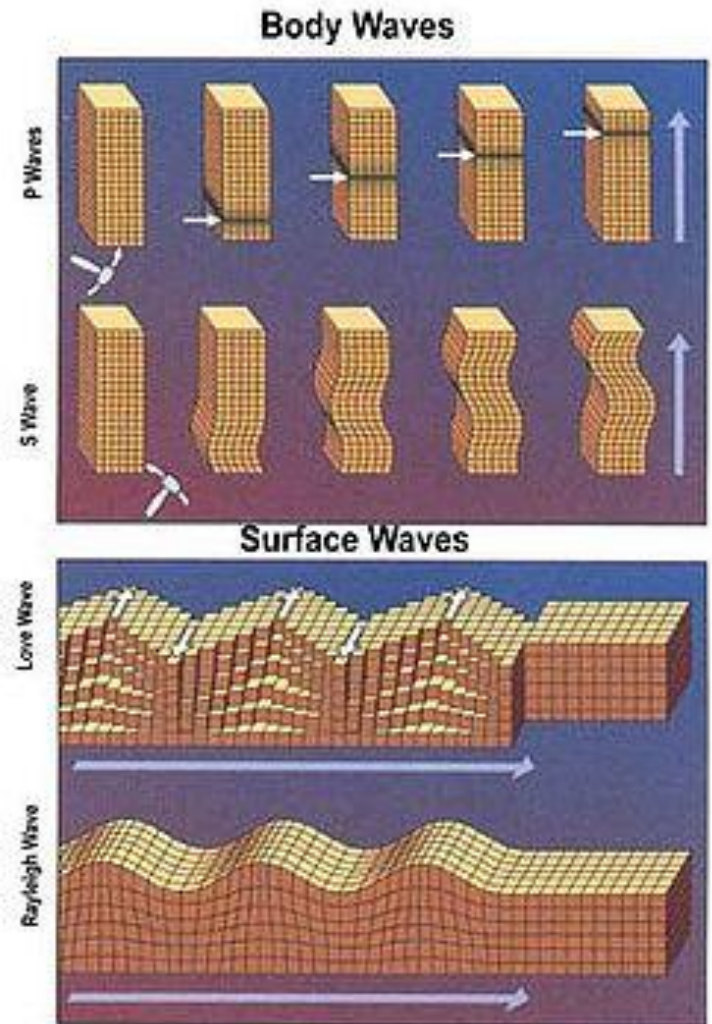
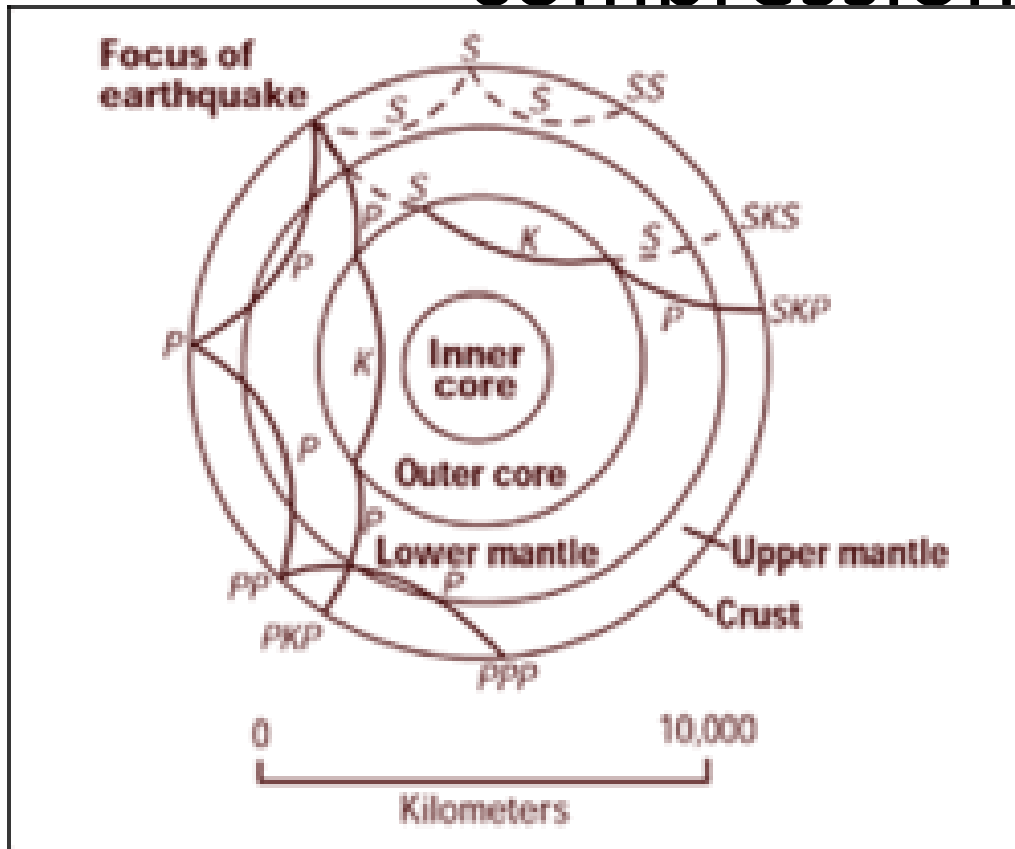
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Body and surface waves in the earth

S – “Secondary” transverse shear wave

P – “Primary” longitudinal compressional

compressional



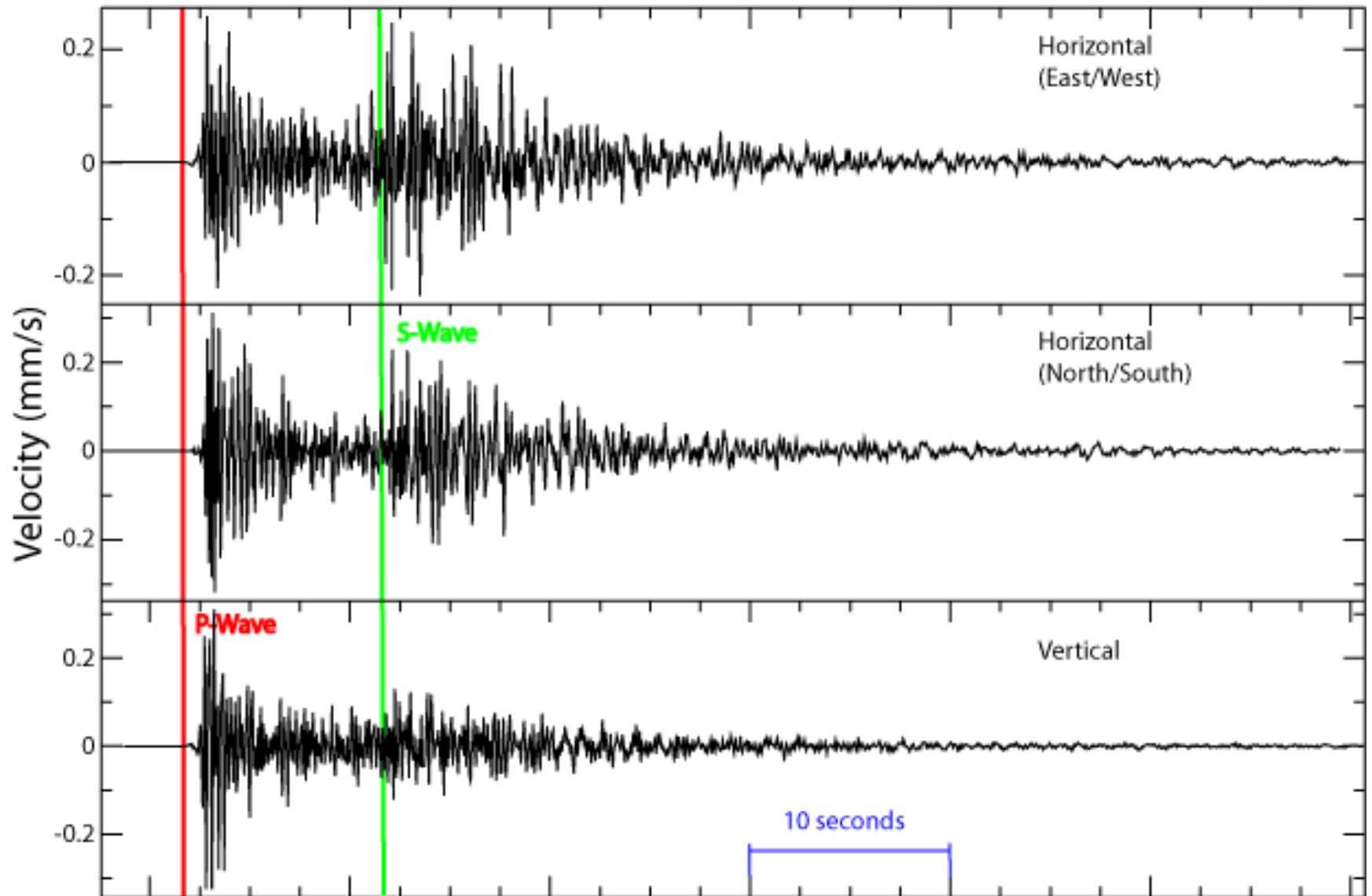
S and P earthquake waves – body waves

- P (longitudinal- compression) waves travel in solids, liquids and gases
- S (transverse) waves travel in solids or gels only
- In earthquake S waves are most destructive
- P waves travel faster hence arrive first
- S waves travel about 50-60% speed of P
- Typ depth of earthquake \sim 40 Km but some >700
- Timing of S and P gives location of epicenter

Surface Waves

- Surface waves generally travel much slower
- But can be extremely destructive
- Rayleigh and Love are the two common types
- Rayleigh – “rolling wave” ~ 90% speed of S
- Love – horizontal shear wave ~ 90% speed of S

Timing of S and P wave



Earthquake metric – Richter scale

Richter magnitudes	Description	Earthquake effects	Frequency of occurrence
Less than 2.0	Micro	Microearthquakes, not felt.	About 8,000 per day
2.0-2.9	Minor	Generally not felt, but recorded.	About 1,000 per day
3.0-3.9		Often felt, but rarely causes damage.	49,000 per year (est.)
4.0-4.9	Light	Noticeable shaking of indoor items, rattling noises. Significant damage unlikely.	6,200 per year (est.)
5.0-5.9	Moderate	Can cause major damage to poorly constructed buildings over small regions. At most slight damage to well-designed buildings.	800 per year
6.0-6.9	Strong	Can be destructive in areas up to about 160 kilometres (100 mi) across in populated areas.	120 per year
7.0-7.9	Major	Can cause serious damage over larger areas.	18 per year
8.0-8.9	Great	Can cause serious damage in areas several hundred miles across.	1 per year
9.0-9.9		Devastating in areas several thousand miles across.	1 per 20 years
10.0+	Epic	Never recorded	Extremely rare (Unknown)

Richter Scale and Moment Magnitude Scale

- Charles Richter and Beno Gutenberg – 1935 Caltech
- Studying S. Cal earthquakes defined a logarithmic scale
- Difference of 1 corresponds to a factor of 10 in displacement
- Energy release scales as $\sim 3/2$ power of displacement
- Difference of 1 = $(10)^{3/2} \sim 32$ in energy, diff of 2 = 1000 energy
- Zero point was set by 1 micron displacement of Wood-Anderson torsion seismometer 100 Km from quake
- Negative values are possible with modern seismometers
- No upper limit
- Mag 10 is extremely destructive

Some historical examples

- Nov 1, 1755 Lisbon M~8.7 80,000 killed – huge Tsunami follows
- Feb 28, 1870 Iran M ~ 7.4 200,000 killed
- Nov 25, 1833 Sumatra M ~ 9.2 – Large number of victims – huge Tsunami
- Jan 23, 1855 Wairapa, New Zealand M~8 4 killed – raised coast by 2 m!
- Mar 26, 1872 Lone Pine, CA M ~ 7.3 27 killed
- Sept 1, 1888 N. Canterbury, NZ M ~ 7.3 – first quake with mainly horizontal fault
- June 15, 1896 Iwate Japan M ~ 8 22,000+ killed
- Apr 18, 1906 San Fransisco M ~ 7.8 3000 killed
- June 29, 1925 Santa Barbara M ~ 6.8 13 killed – mission destroyed
- Mar 28, 1964 Alaska M ~ 9.2 131 killed
- Feb 9, 1971 San Fernando Valley, CA M ~ 6.6 65 killed
- Oct 17, 1989 Loma Prieta – Bay Bridge collapses M ~ 7 63 killed
- Jan 17, 1994 Northridge, CA M ~ 6.7 60 killed

Earthquakes and Nuclear testing

$$M_n = \frac{2}{3} \log_{10} \frac{m_{\text{TNT}}}{\text{Mt}} + 6$$

m_{TNT} is the yield in MegaTons (MT) – TNT equivalent

1 Ton TNT ([trinitrotoluene](#)) $\sim 10^9$ calories $\sim 4 \times 10^9$ Joules

Largest US underground test was ~ 5 MT

Codenamed Cannikin on Nov 6, 1971 in the Aleutian Islands

For 5 MT this would yield about $M \sim 6.5$

Note – 1 KiloTon (1 KT) $M \sim 4$

0.5-1% of yield goes into earthquake energy

1 Kg matter annihilation ~ 1 MT

Do nuclear tests trigger earthquakes?

No evidence to support this

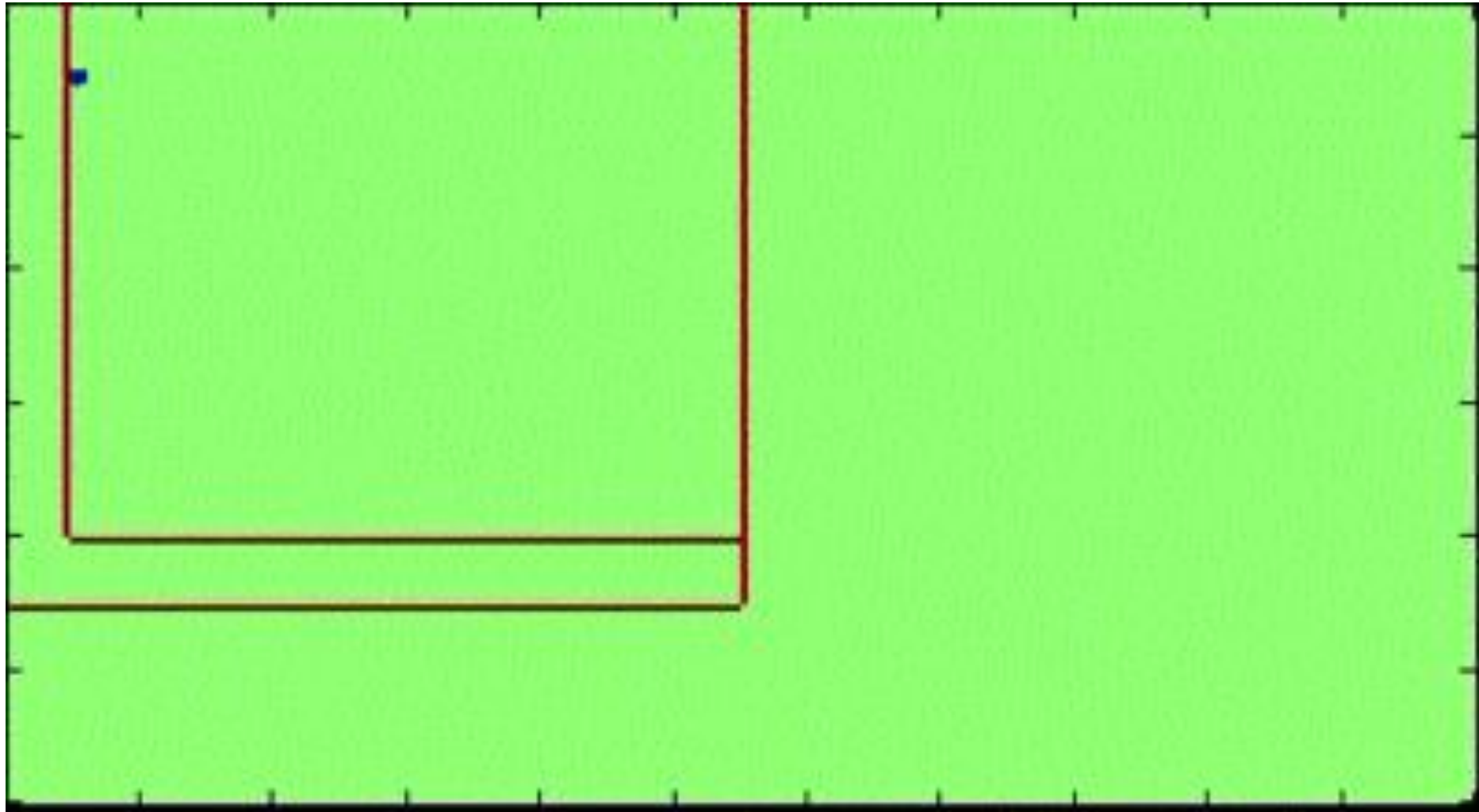
(<http://earthquake.usgs.gov/learn/faq/?categoryID=12&faqID=88>)

Some local earthquakes

- Northridge Earthquake 1971 ->
- Loma Prieta Oct 1989 others

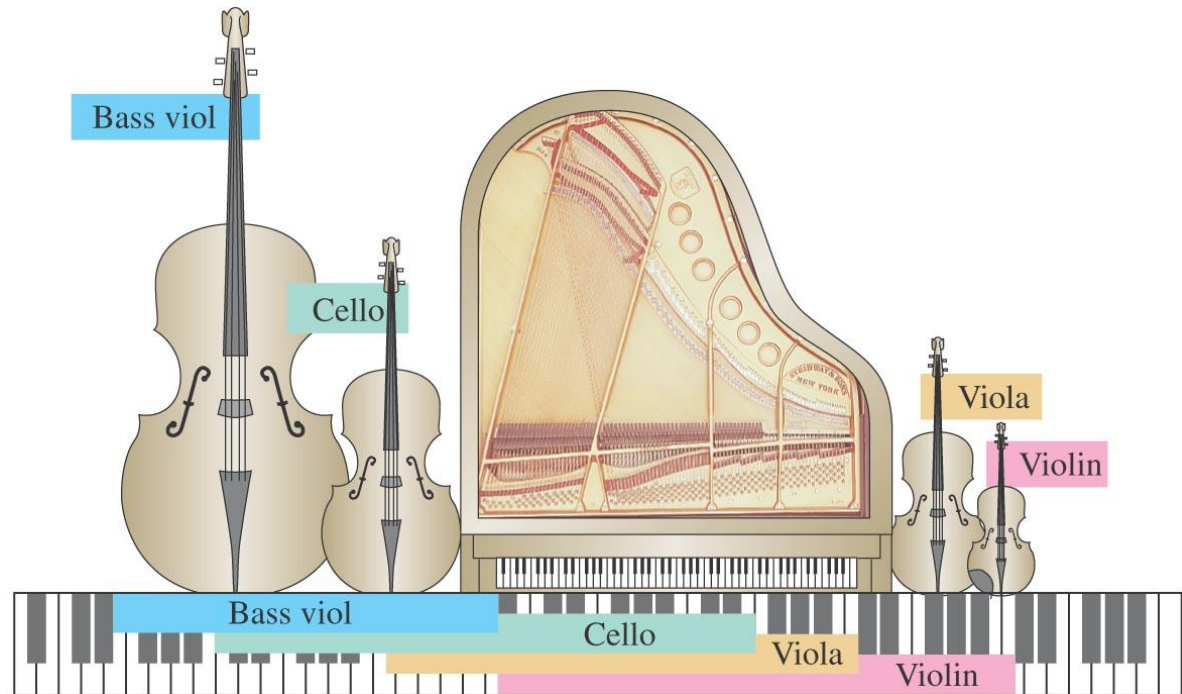
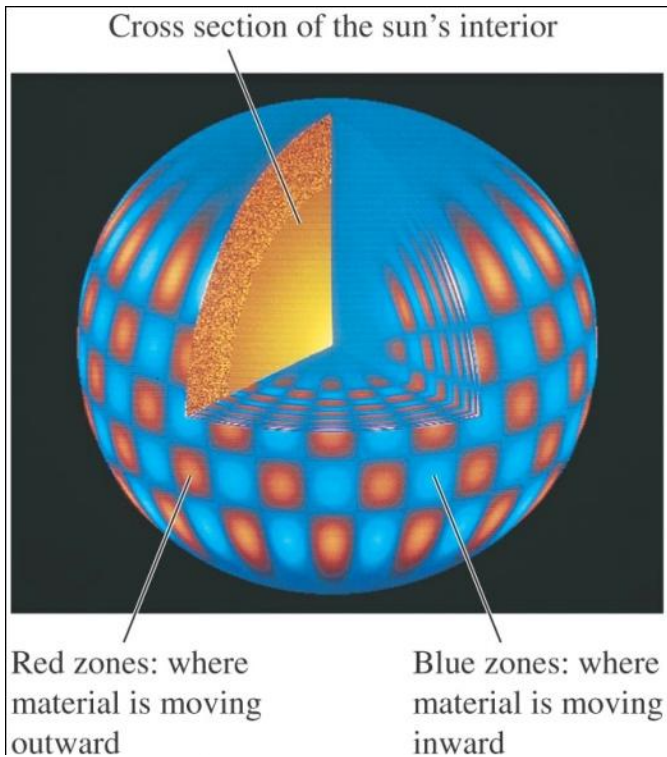


Body waves can be used for exploration – oil, etc
Example below is explosive driven wave to look for land mines



Complex standing waves

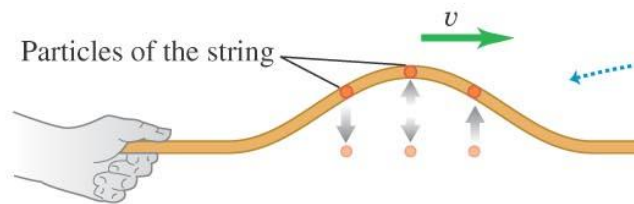
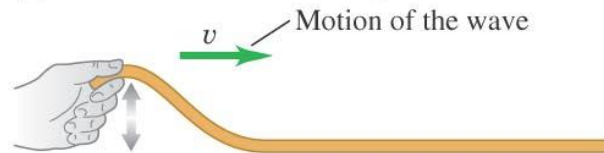
- As the shape and composition of the resonator change, the standing wave changes also.



Types of mechanical waves

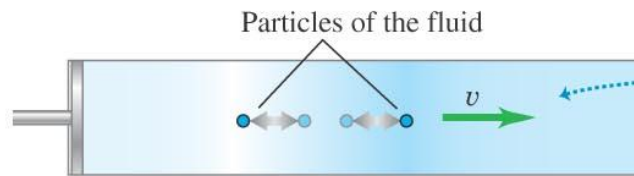
- Waves that have compressions and rarefactions parallel to the direction of wave propagation are longitudinal.
- Waves that have compressions and rarefactions perpendicular to the direction of propagation.

(a) Transverse wave on a string



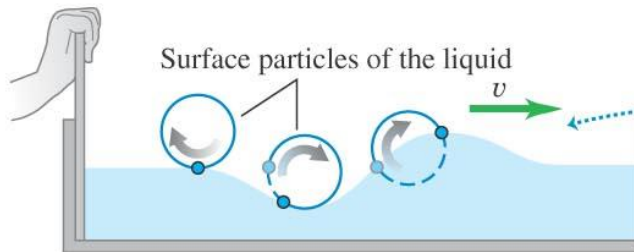
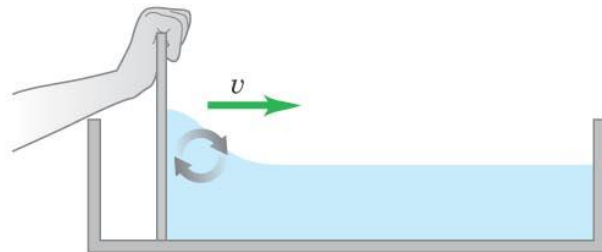
As the wave passes, each particle of the string moves up and then down, *transversely* to the motion of the wave itself.

(b) Longitudinal wave in a fluid



As the wave passes, each particle of the fluid moves forward and then back, *parallel* to the motion of the wave itself.

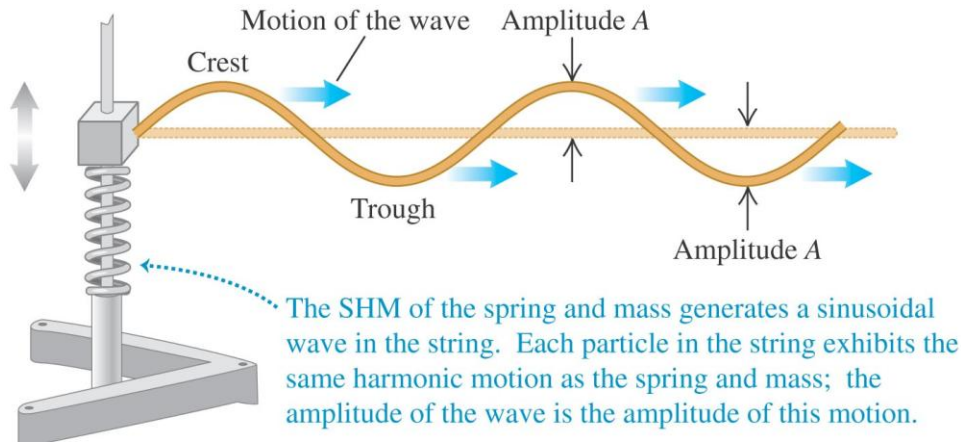
(c) Waves on the surface of a liquid



As the wave passes, each particle of the liquid surface moves in a circle.

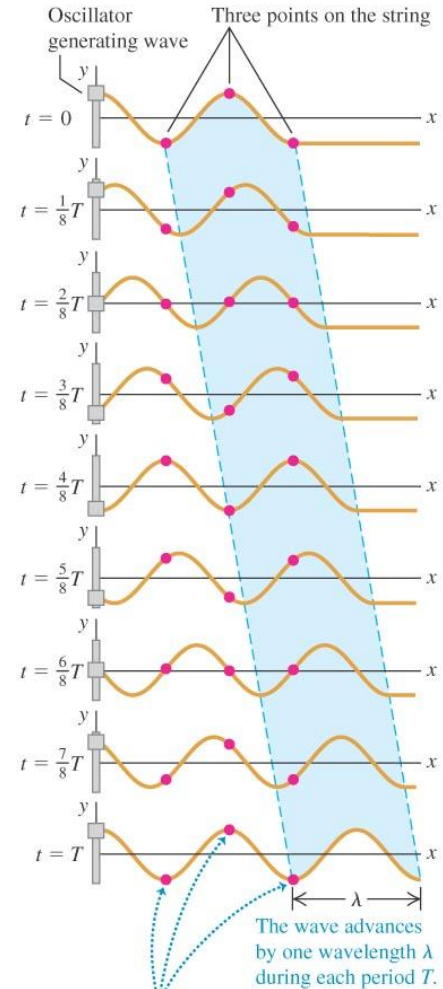
Periodic waves

- A detailed look at periodic transverse waves will allow us to extract parameters.



The SHM of the spring and mass generates a sinusoidal wave in the string. Each particle in the string exhibits the same harmonic motion as the spring and mass; the amplitude of the wave is the amplitude of this motion.

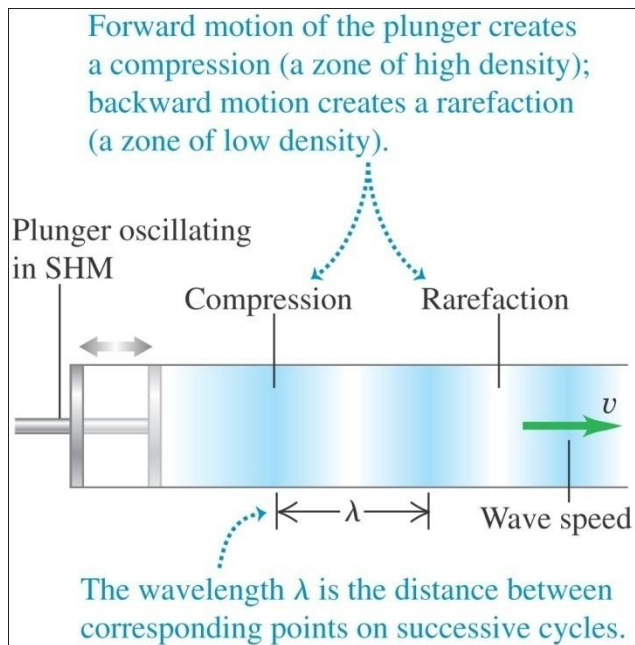
The string is shown at time intervals of $\frac{1}{8}$ period for a total of one period T . The highlighting shows the motion of one wavelength of the wave.



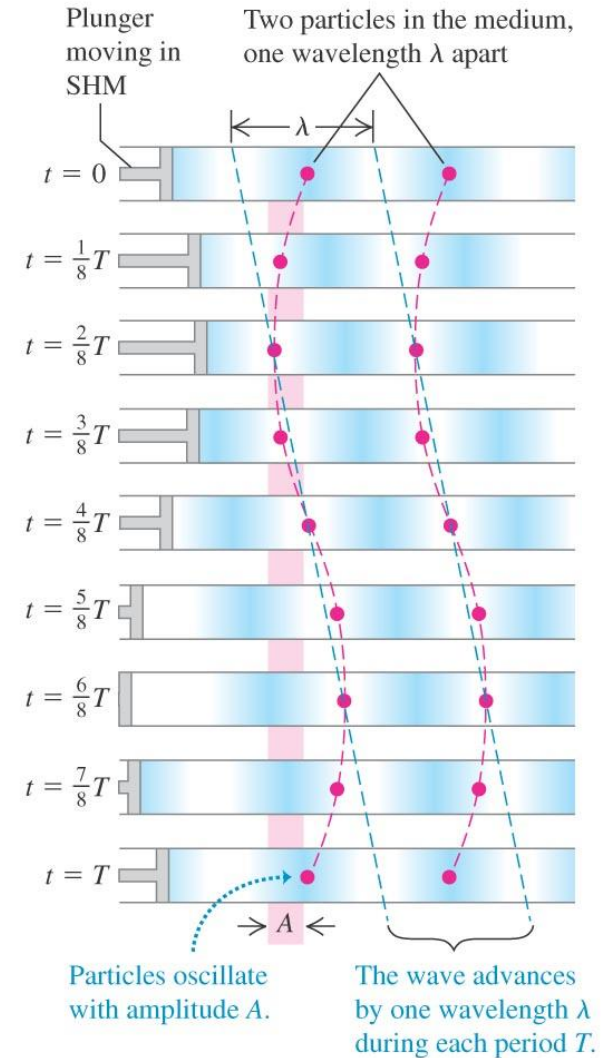
Each point moves up and down in place. Particles one wavelength apart move in phase with each other.

Periodic waves II

- A detailed look at periodic longitudinal waves will allow us to extract parameters just as we did with transverse waves.

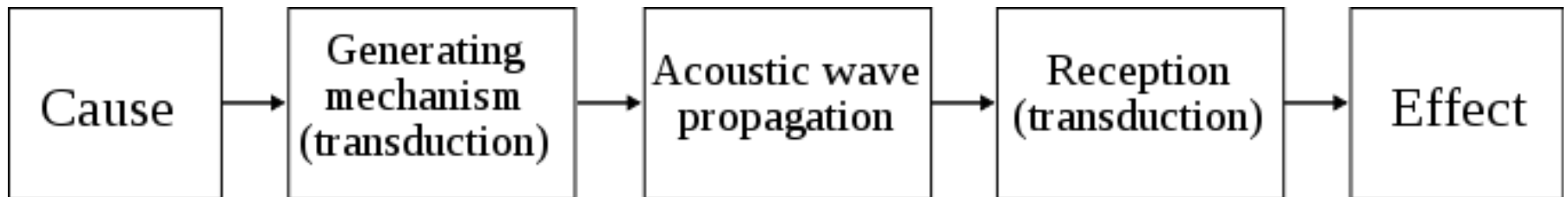


Longitudinal waves are shown at intervals of $\frac{1}{8}T$ for one period T .



Acoustics

- [Greek](#) - *ἀκουστικός (akoustikos)* "of or for hearing, ready to hear"
- *ἀκουστός (akoustos)*, "heard, audible"
- *ἀκούω (akouo)*, "I hear"
- Latin equivalent is "sonic" – sound
- Galileo "Waves produced by [vibrations](#) of sonorous body"
- [Mersenne](#) (1588–1648) – works out harmonics of strings
- Helmholtz and Lord Rayleigh apply rigorous mathematics



What distinguishes Acoustics from Mechanical Waves

- Semantics to some extent
- Usually acoustics refers to propagation in gases not liquids or solid BUT there are liquid and solid acoustics
- So the distinction is really artificial
- We will focus on propagation in gases
- The critical difference is gases ONLY support a compression (longitudinal) wave not a shear (transverse) wave
- Solids can support BOTH transverse and longitudinal waves
- Depending on the viscosity of the liquid generally we think of liquids as supporting only compression (longitudinal) waves

Sound pressure levels

- Sound pressure is defined as the local deviation from the mean – units are Pa (N/m²)
- Effective sound pressure = $\langle \text{RMS}(\text{sound pressure}) \rangle$ = time and space average Root Mean Square (RMS)

$$p = \frac{F}{A} \quad p_{\text{total}} = p_0 + p$$

Sound Intensity

- Sound (Acoustic) Intensity
- $I = \text{Time ave Acoustic Power} / \text{Area} \quad I = P_{ac} / A$
- Let p_{inst} = instantaneous pressure (note the different P's p (pressure), P (power))
- Let v = bulk gas speed (not molecular speed)
- Then
$$I = \frac{1}{T} \int_0^T p_{inst}(t) v(t) dt$$
- Recall Power = Force x speed
- Units of I are watts/m²

How do Intensity and Pressure Scale with distance from a point Source?

- Energy is conserved so $I \times \text{area} = \text{total power emitted } P_{ac}$
- At a distance r from the acoustic point source

$$I_r = \frac{P_{ac}}{A} = \frac{P_{ac}}{4\pi r^2}$$

- What causes the bulk gas flow?
- It is the sound wave
- v (bulk flow) $\sim p$ (pressure)
- **Hence $I_r \sim p(\text{pressure})^2$ p is pressure, P is power**

Scaling of Pressure and Intensity

- Hence $I \sim 1/r^2$ while $p \sim 1/r$
- This is a critical difference
- Power/area $\sim 1/r^2$ while Pressure $\sim 1/r$

Sound Pressure Level - SPL

$$L_p = 10 \log_{10} \left(\frac{p_{\text{rms}}^2}{p_{\text{ref}}^2} \right) = 20 \log_{10} \left(\frac{p_{\text{rms}}}{p_{\text{ref}}} \right) \text{ dB,}$$

The unit is dB (SPL) commonly reduced to just dB (decibel)

This is a relative measure and we need a reference level

Typically for hearing in air we use $P_{\text{ref}} = 20 \mu\text{P}_a$ (RMS) = 2×10^{-5}

This is the typ threshold of human hearing - ~ Mosquito at 3 m

In water we normally use $P_{\text{ref}} = 1 \mu\text{P}_a$ (RMS)

94 dB ~ 1 P_a

Note – 10 times the pressure = 20 dB increase in SPL

Notice that $L_p \sim \text{Log}(p^2)$.

This is because Intensity $I \sim p^2$

Q16.2

Increasing the pressure amplitude of a sound wave by a factor of 4 (while leaving the frequency unchanged)

- A. causes the intensity to increase by a factor of 16.
- B. causes the intensity to increase by a factor of 4.
- C. causes the intensity to increase by a factor of 2.
- D. has no effect on the wave intensity.
- E. The answer depends on the frequency of the sound wave.

A16.2

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Symbol	Units	Meaning
p	<u>pascals</u>	RMS <u>sound pressure</u>
f	<u>hertz</u>	<u>frequency</u>
ξ	m, <u>metres</u>	<u>particle displacement</u>
c	<u>m/s</u>	<u>speed of sound</u>
v	<u>m/s</u>	<u>particle velocity</u>
$\omega = 2\pi f$	<u>radians/s</u>	<u>angular frequency</u>
ρ	<u>kg/m³</u>	<u>density of air</u>
$Z = c \cdot \rho$	<u>N·s/m³</u>	characteristic <u>acoustic impedance</u>
a	<u>m/s²</u>	<u>particle acceleration</u>
I	<u>W/m²</u>	sound intensity
E	<u>W·s/m³</u>	<u>sound energy density</u>
P_{ac}	W, <u>watts</u>	<u>sound power</u> or <u>acoustic power</u>
A	<u>m²</u>	<u>area</u>

Sound Intensity level and Reference Level

$$L_I = 10 \log_{10} \frac{|I|}{I_0}$$

We define the reference level to be about the threshold of human hearing

$$I_0 = 10^{-12} \text{ w/m}^2$$

For some interesting comparisons see

<http://www.sengpielaudio.com/calculator-levelchange.htm>

Source of sound	Sound pressure	Sound pressure level
Sound in air	pascal	dB re 20 μPa
Shockwave (distorted sound waves > 1 atm ; waveform valleys are clipped at zero pressure)	>101,325 Pa (peak-to-peak)	>194 dB
Krakatoa explosion at 100 miles (160 km) in air ^[dubious – discuss]	20,000 Pa (RMS)	180 dB
Simple open-ended thermoacoustic device ^[6]	12,619 Pa	176 dB
.30-06 rifle being fired 1 m to shooter's side	7,265 Pa	171 dB (peak)
M1 Garand rifle being fired at 1 m	5,023 Pa	168 dB
Jet engine at 30 m	632 Pa	150 dB
Threshold of pain	63.2 Pa	130 dB
Hearing damage (possible)	20 Pa	approx. 120 dB
Jet at 100 m	6.32 – 200 Pa	110 – 140 dB
Jack hammer at 1 m	2 Pa	approx. 100 dB
Traffic on a busy roadway at 10 m	2×10^{-1} – 6.32×10^{-1} Pa	80 – 90 dB
Hearing damage (over long-term exposure, need not be continuous)	0.356 Pa	78 dB
Passenger car at 10 m	2×10^{-2} – 2×10^{-1} Pa	60 – 80 dB
TV (set at home level) at 1 m	2×10^{-2} Pa	approx. 60 dB
Normal conversation at 1 m	2×10^{-3} – 2×10^{-2} Pa	40 – 60 dB
Very calm room	2×10^{-4} – 6.32×10^{-4} Pa	20 – 30 dB
Light leaf rustling, calm breathing	6.32×10^{-5} Pa	10 dB
Auditory threshold at 1 kHz	2×10^{-5} Pa (RMS)	0 dB

Acoustics In water

Source of sound	Sound pressure	Sound pressure level
Sound under water	pascal	dB re 1 μ Pa
Pistol shrimp	79,432 Pa	218 dB ^[7]
Sperm Whale	141-1,000 Pa	163-180 dB ^[8]
Fin Whale	100-1,995 Pa	160-186 dB ^[9]
Humpback Whale	16-501 Pa	144-174 dB ^[10]
Bowhead Whale	2-2,818 Pa	128-189 dB ^[11]
Blue Whale	56-2,511 Pa	155-188 dB ^[12]
Southern Right Whale	398-2238 Pa	172-187 dB ^[13]
Gray Whale	12-1,778 Pa	142-185 dB ^[14]
Auditory threshold of a diver at 1 kHz	2.2×10^{-3} Pa	67 dB ^[15]

Adding sources of **incoherent** sound

Total intensity = sum of intensities

Coherent waves have beats - different

$$L_{\Sigma} = 10 \cdot \log_{10} \left(\frac{p_1^2 + p_2^2 + \dots + p_n^2}{p_{\text{ref}}^2} \right) = 10 \cdot \log_{10} \left(\left(\frac{p_1}{p_{\text{ref}}} \right)^2 + \left(\frac{p_2}{p_{\text{ref}}} \right)^2 + \dots + \left(\frac{p_n}{p_{\text{ref}}} \right)^2 \right)$$

$$L_{\Sigma} = 10 \cdot \log_{10} \left(10^{\frac{L_1}{10}} + 10^{\frac{L_2}{10}} + \dots + 10^{\frac{L_n}{10}} \right) \text{ dB}$$

$$\left(\frac{p_i}{p_{\text{ref}}} \right)^2 = 10^{\frac{L_i}{10}}, \quad i = 1, 2, \dots, n$$

RT16.1

Four sinusoidal sound waves propagate in the same region of our atmosphere. **Rank** the waves in order of their *displacement amplitude*, from largest to smallest

A. intensity = $2.0 \times 10^{-5} \text{ W/m}^2$, frequency = 100 Hz

B. intensity = $2.0 \times 10^{-5} \text{ W/m}^2$, frequency = 200 Hz

C. intensity = $4.0 \times 10^{-5} \text{ W/m}^2$, frequency = 200 Hz

D. intensity = $8.0 \times 10^{-5} \text{ W/m}^2$, frequency = 800 Hz

A-RT16.1

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Answer: ACBD

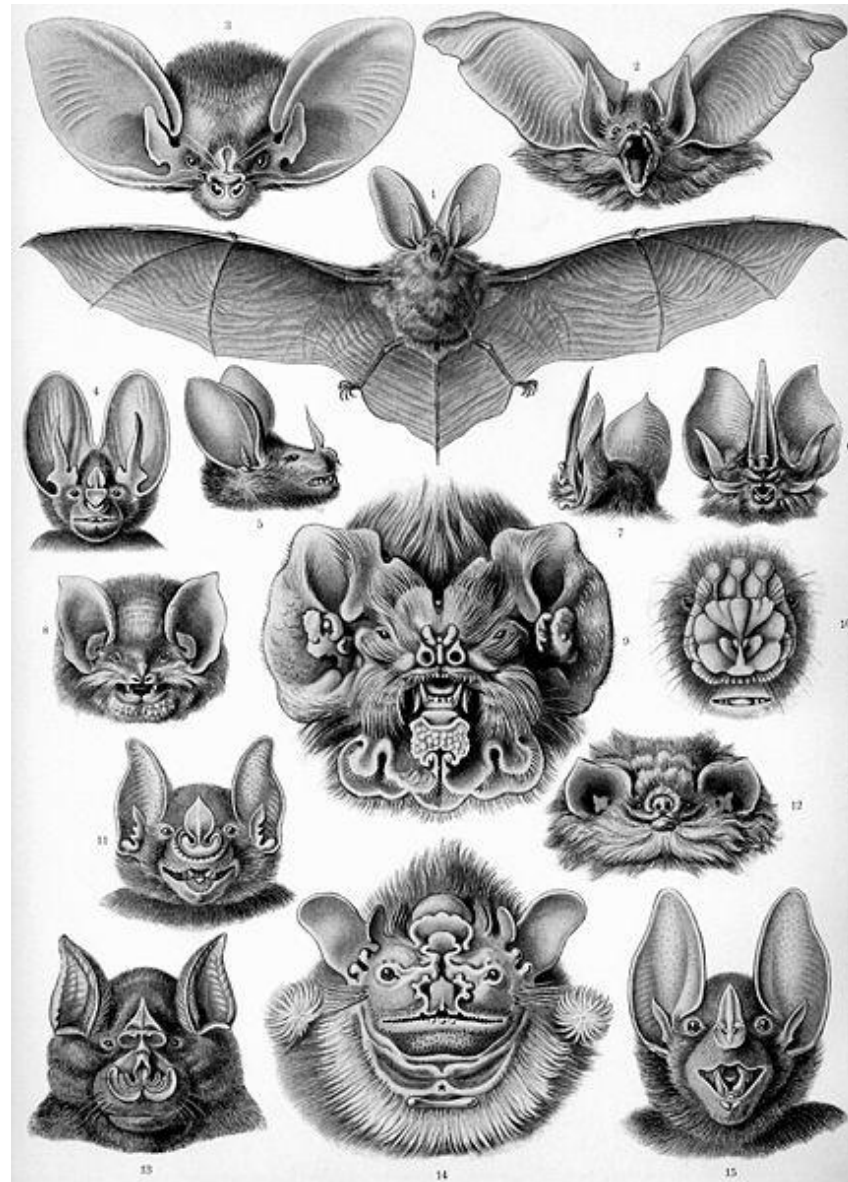
Frequency Response

- Humans typically hear 12Hz -20 KHz under ideal conditions
- BUT too many ear buds (this Buds for You)
- Too many rock concerts == loss of hearing
- High freq response goes first (after age 8)
- Woman have better high freq response
- Humans can feel infrasonic 4-16 Hz

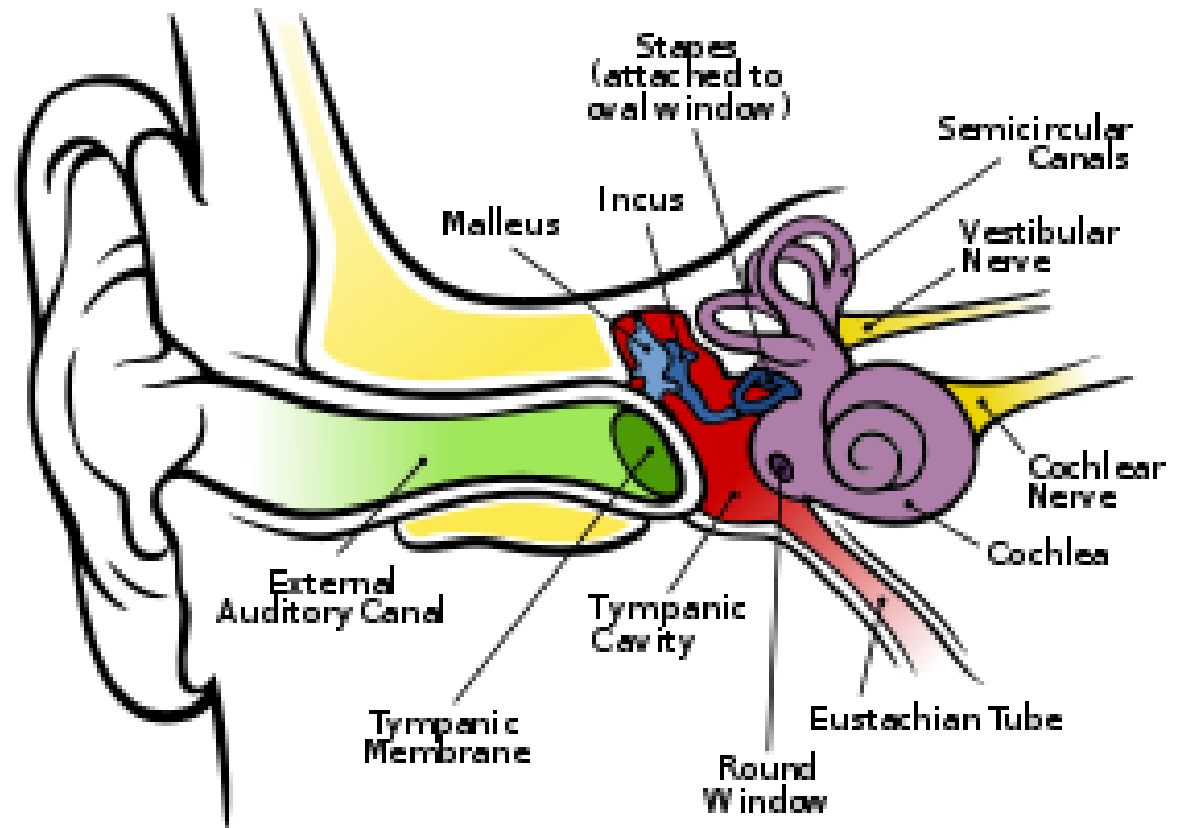
Other animals

- Dogs can hear 40 Hz to 60 KHz
- Bats 20 Hz to 120 KHz – Use Freq Modulation to get Doppler shift of target
- Mice 1 KHz to 90 KHz – communicate above our hearing – we do not hear them
- Bottlenose Dolphins 0.25 to 150 KHz
- Harbour Porpoise typ emit at 2 KHz and 110 KHz
- Bats and many sea animals use echolocation for hunting (sonar)

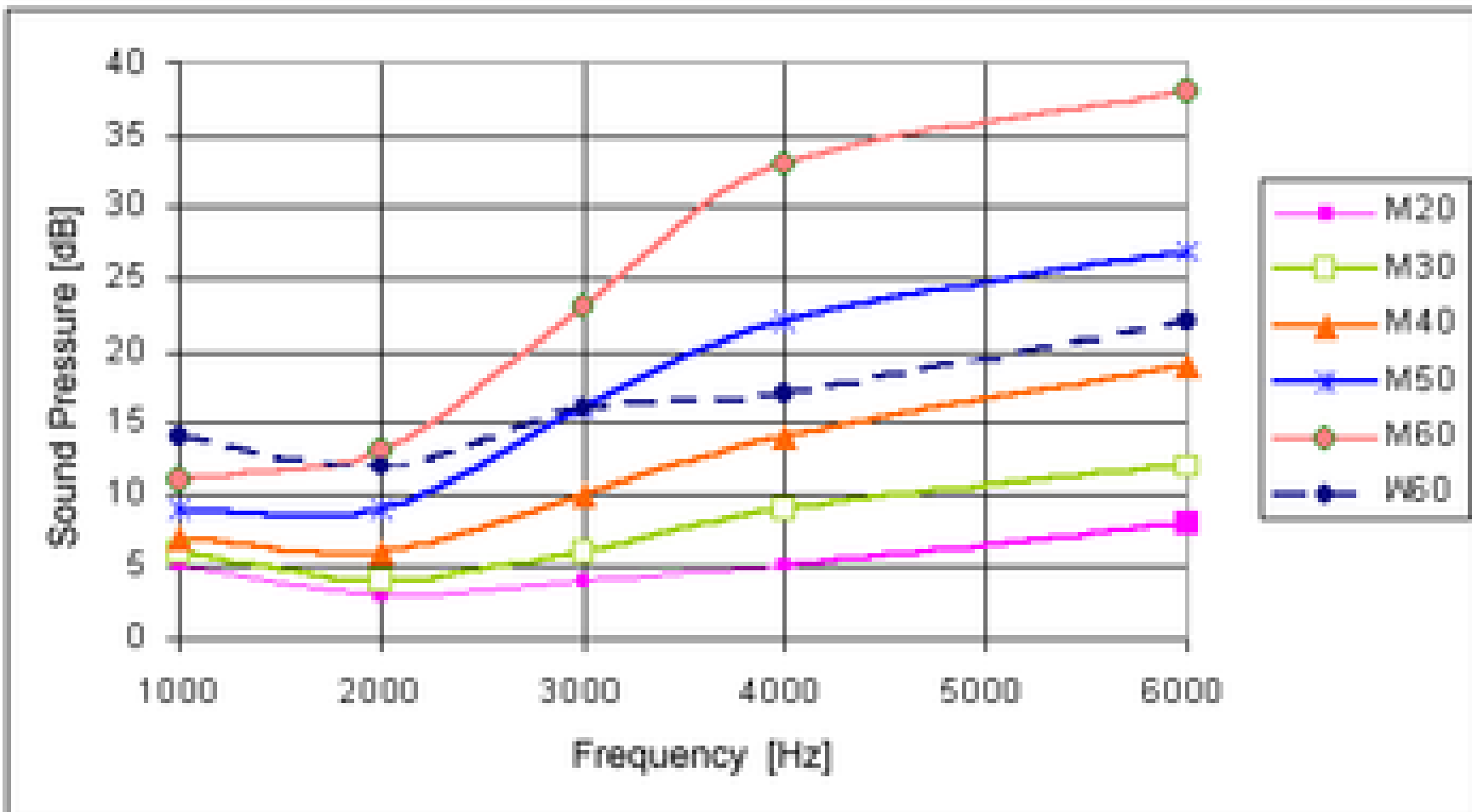
External Ear



Human Ear

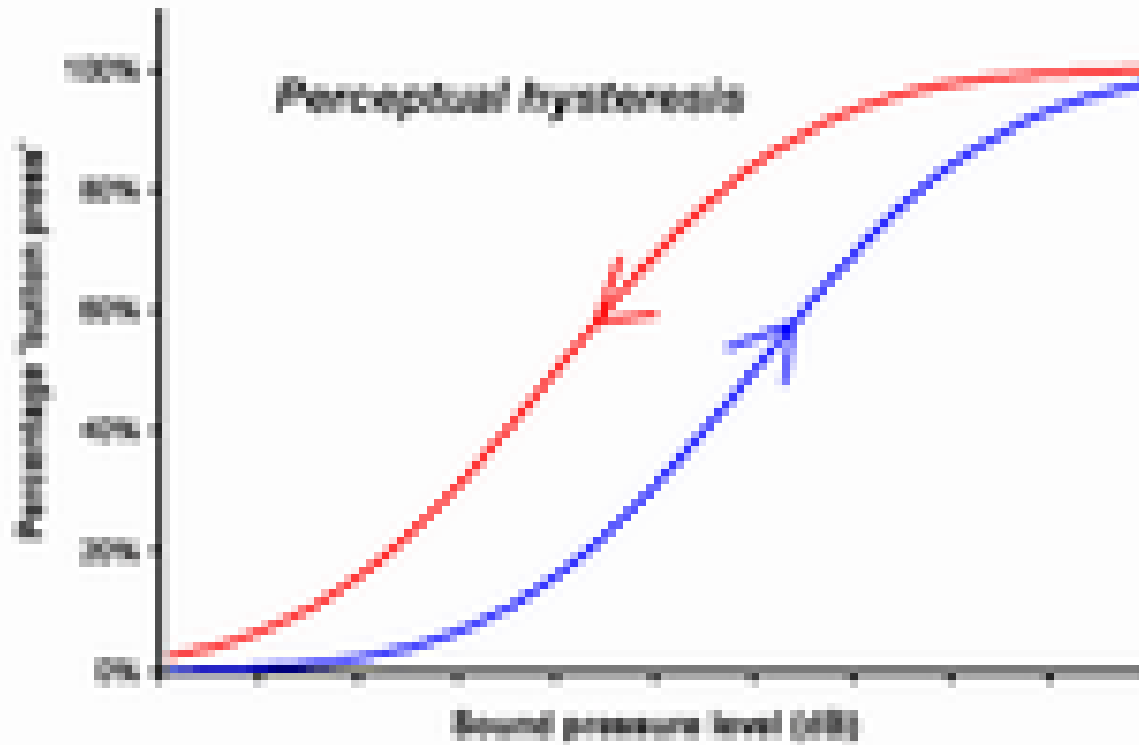


Frequency Range and Variation with Age and Gender
Degradation is very dependent on exposure
Protect your ears – DO NOT blast iPods



Hearing Threshold and Hysteresis

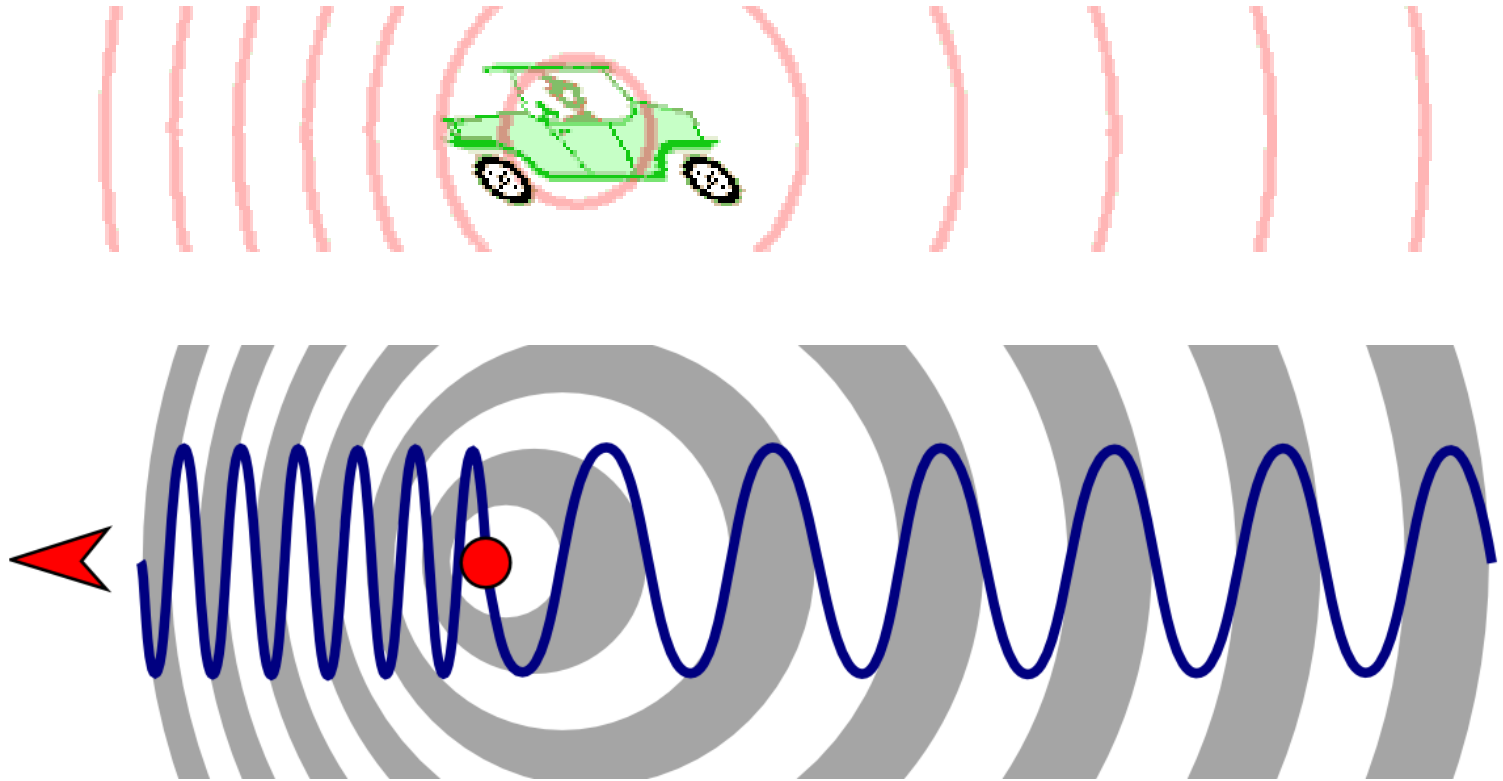
Threshold is lower going from higher to lower intensity



Doppler Shift – Frequency Change with Motion

- Named after [Austrian](#) physicist [Christian Doppler](#) 1842
- The effect is simple to understand if you think of the total number of waves emitted being constant
- Motion of you (the receiver) or the emitter (the source) causes the waves to “bunch up” or “spread out” in time
- Hence you measure a larger or smaller frequency
- This is true in acoustics and electromagnetic waves
- Examples are:
 - Radar guns for speed control or baseball speed
 - Ultrasound for heart monitoring or imaging
 - Measurement of redshift in astronomy
 - Blood flow monitoring and testing of arteries
 - Sonar – detecting fish, submarines, bats detecting bug motion

Doppler shift

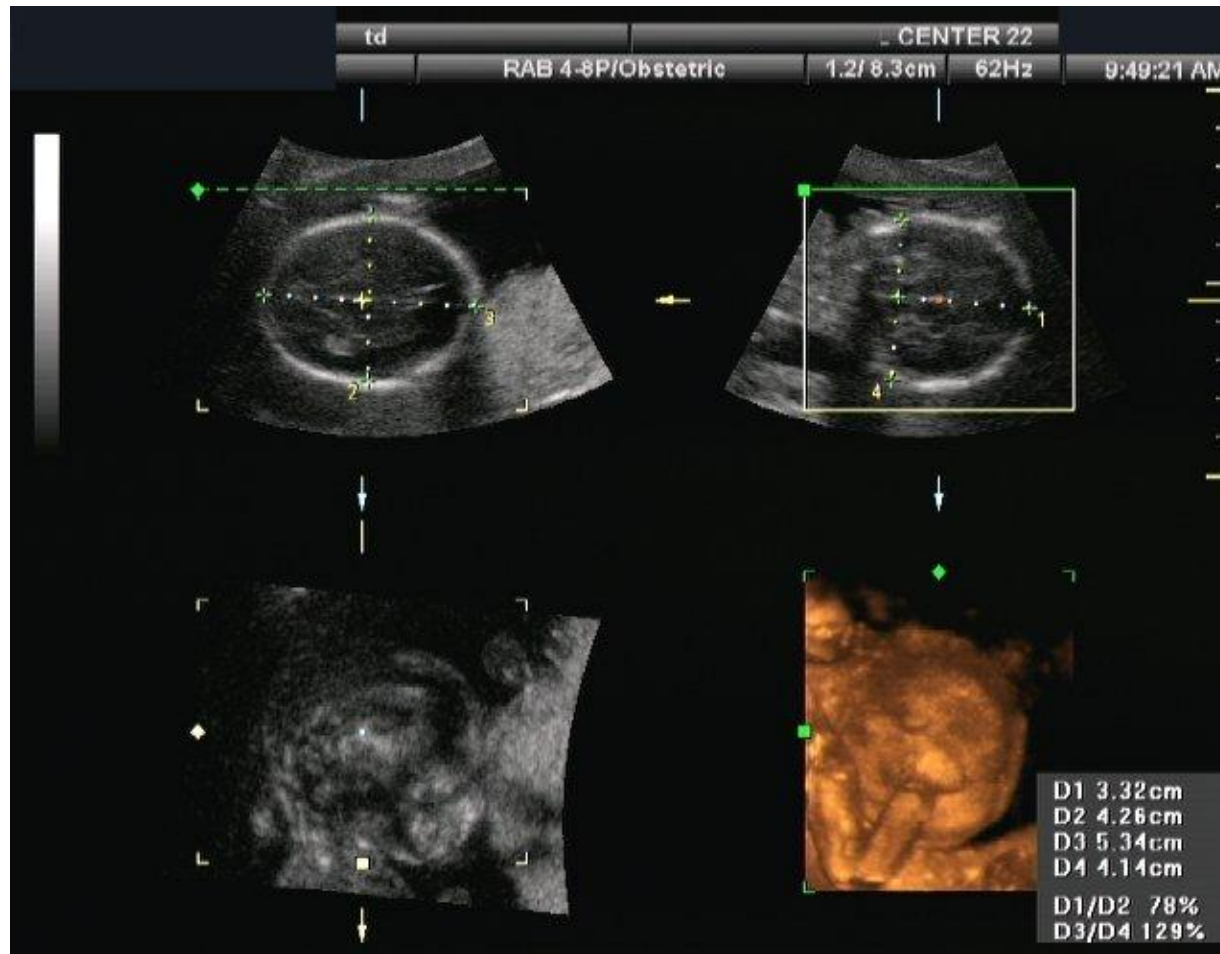


Doppler shift calculations

- In a medium such as water, air etc
- Let v = speed of sound in the medium
- V_r = speed of receiver relative to medium
 - V_r positive if receiver is moving toward emitter, negative if moving away
- V_s = speed of emitter relative to medium
 - V_s positive if source moving towards receiver, negative if away
- f_0 = frequency emitted
- f = frequency detected (received)

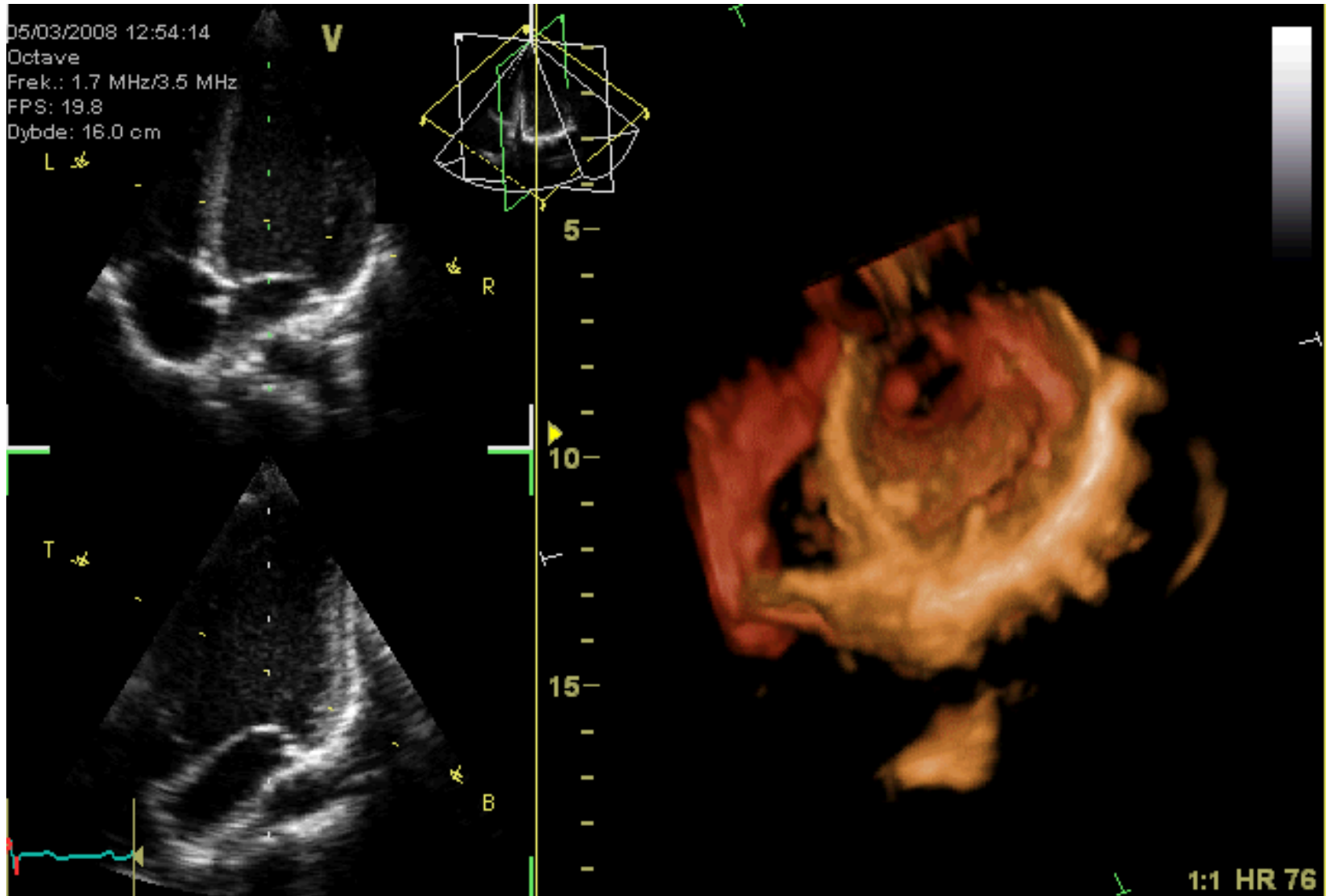
$$f = \left(\frac{v + v_r}{v - v_s} \right) f_0$$

Ultrasound imaging – typ 2-20 MHz

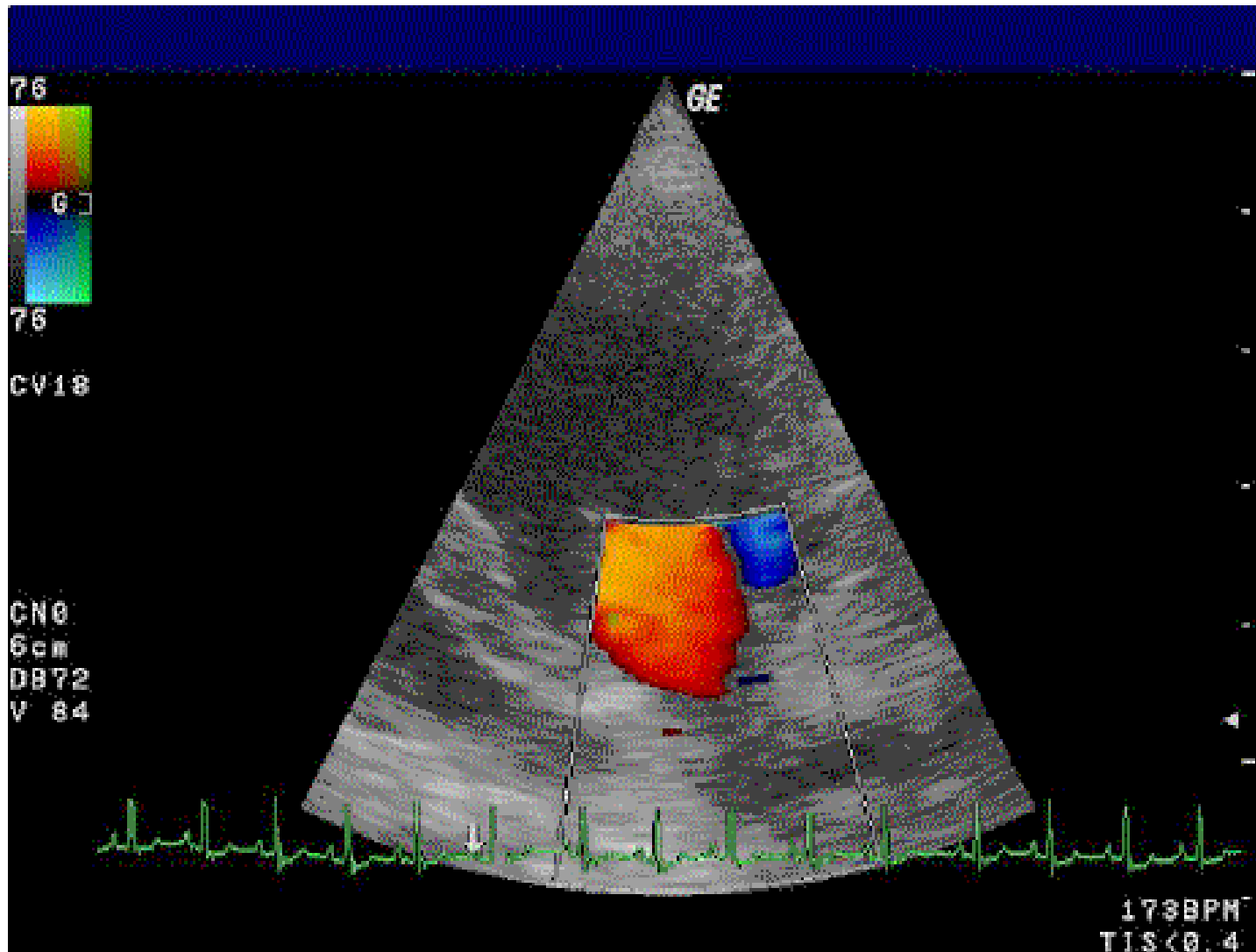


Heart ultrasound imaging

Echocardiogram



Doppler Echocardiogram



Sonoluminescence – Light emission from sound waves – bubble implosion
Still largely unexplained – some claim fusion

