

Chapter 17

Potential and Capacitance

Potential

Voltage (potential) is the analogue of **water pressure** while **current** is the analogue of **flow of water** in say gal/min or Kg/s ...

Think of a potential as the words sounds like – **the potential to do work**
Potential Energy

We define potential as a the work per unit charge in moving a charge from point a to b

Along some path that we choose

In a static field (not time varying) this energy is conservative
No matter what path you take you always get the same energy

In static gravity this is the same – **it is a conservative field**

Potential and Line Integrals in Space

- Mathematically this is a line integral along curve C from a to b .
- Note that in a closed line integral from point a back to point a the total energy is ZERO in a conservative field. Again a static E field is a conservative field

$$\varphi_{\mathbf{E}} = - \int_C \mathbf{E} \cdot d\boldsymbol{\ell} ,$$

Work and Electric Fields

- Work (energy) is then potential (voltage) times charge moved from a to b

$$U_{\mathbf{E}} = q \varphi .$$

For such a field we can show mathematically that we can express the E field as the gradient of the potential. We use a minus as a convention for **work done on the charge** by the field vs **work the charge does on the field**

$$\mathbf{E} = -\nabla \varphi_{\mathbf{E}} .$$

Potential of a Point Charge

$$\varphi_{\mathbf{E}} = \frac{1}{4\pi\epsilon_0} \frac{q}{r},$$

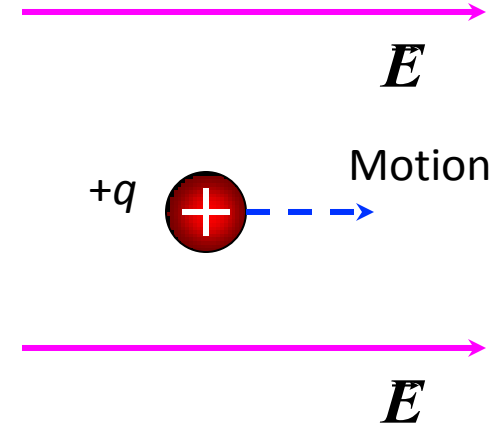
$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{\mathbf{r}} \quad (1)$$

Note that the potential goes as $1/r$ while the E field goes as $1/r^2$

Q23.1

When a positive charge moves in the direction of the electric field,

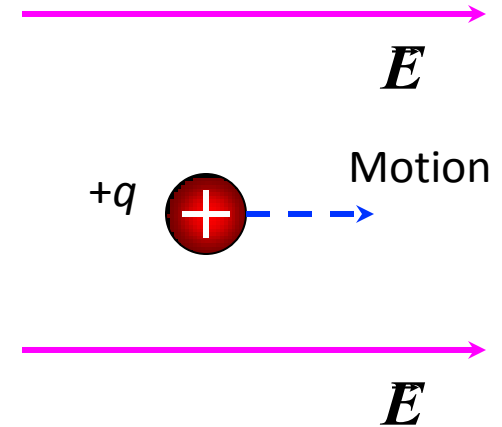
- A. the field does positive work on it and the potential energy increases.
- B. the field does positive work on it and the potential energy decreases.
- C. the field does negative work on it and the potential energy increases.
- D. the field does negative work on it and the potential energy decreases.
- E. The field does zero work on it and the potential energy remains constant.



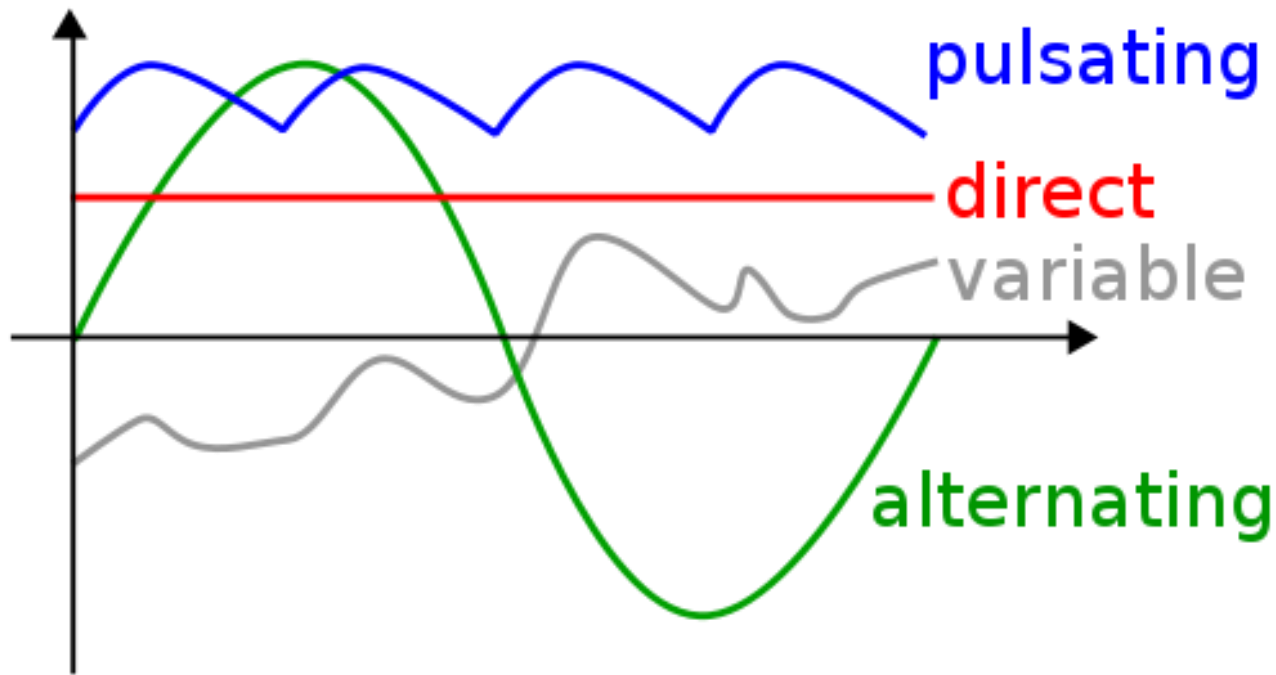
A23.1

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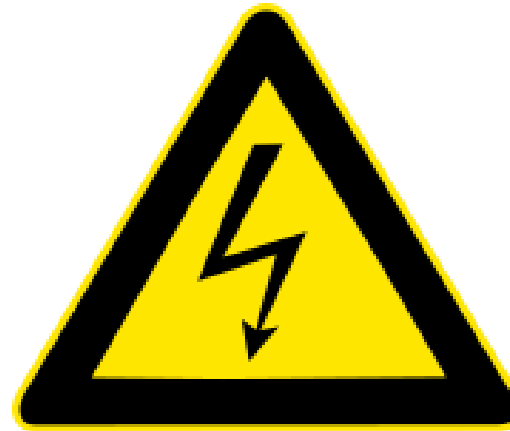
Types of Voltages AC, DC, Pulse, Variable



Digital Volt Meter and International HV Symbol

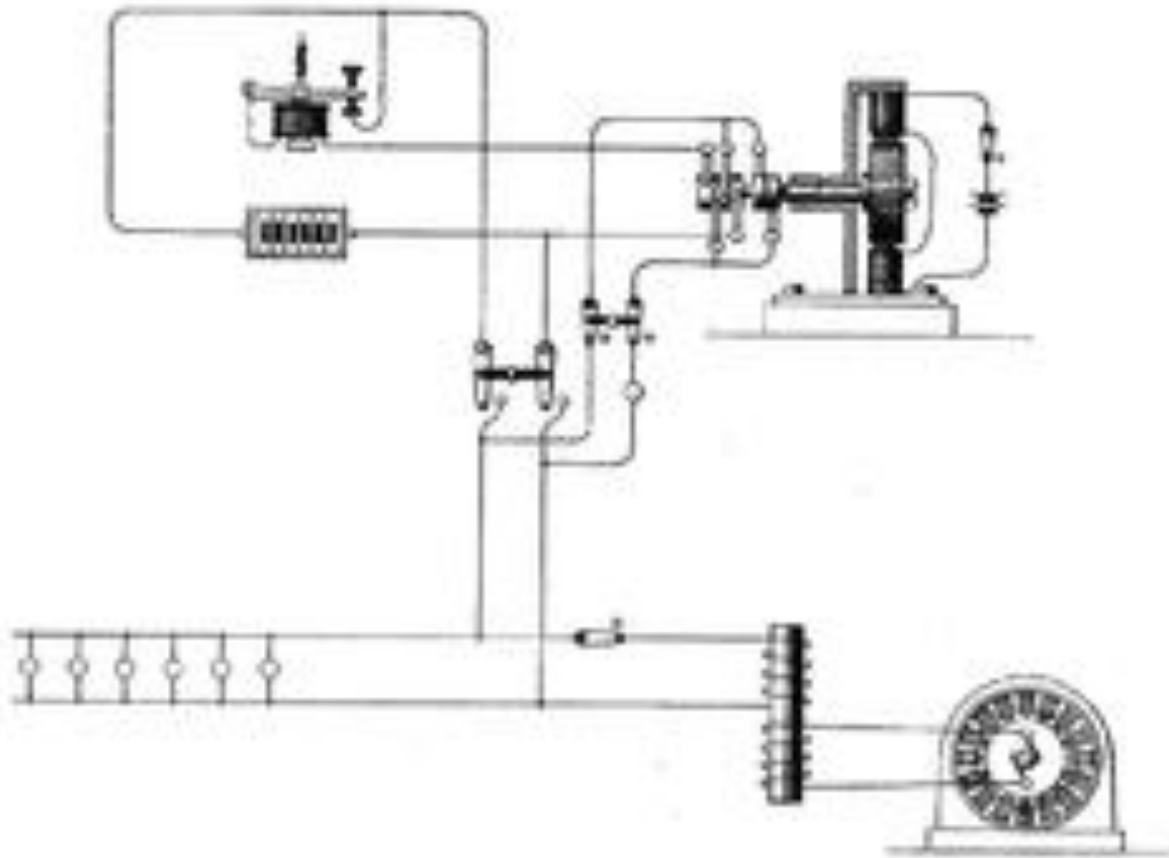


DVM



International Symbol of High Voltage

AC Generators go back ~ 150 years Westinghouse 1887

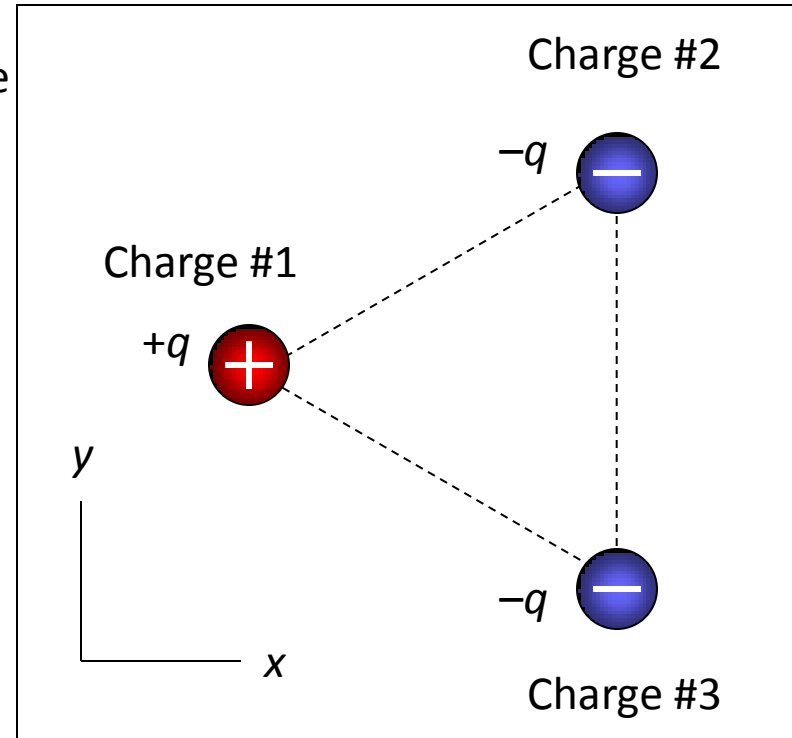


Q23.8

The electric potential due to a point charge approaches zero as you move farther away from the charge.

If the three point charges shown here lie at the vertices of an equilateral triangle, the electric potential at the center of the triangle is

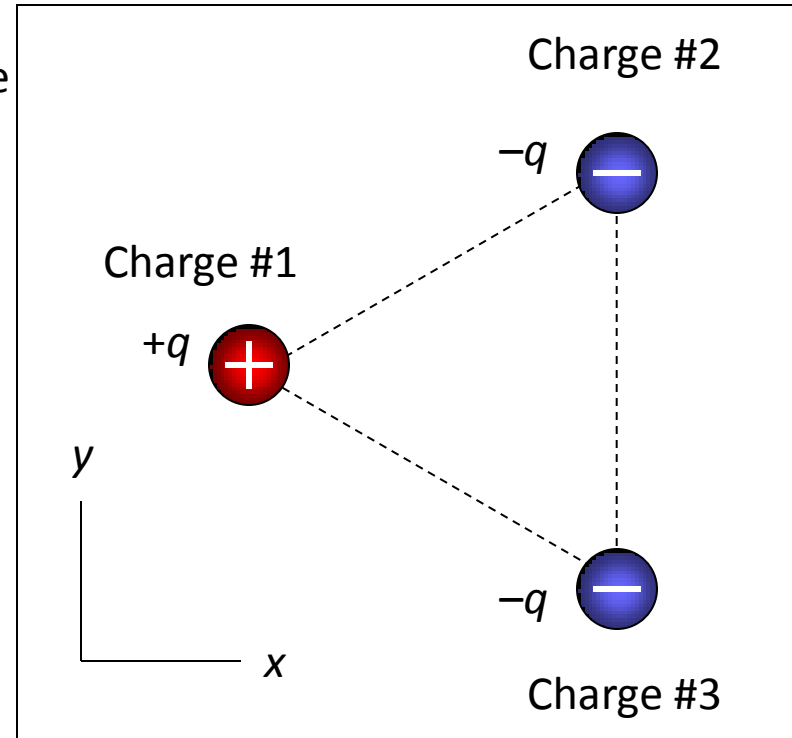
- A. positive.
- B. negative.
- C. zero.
- D. either positive or negative.
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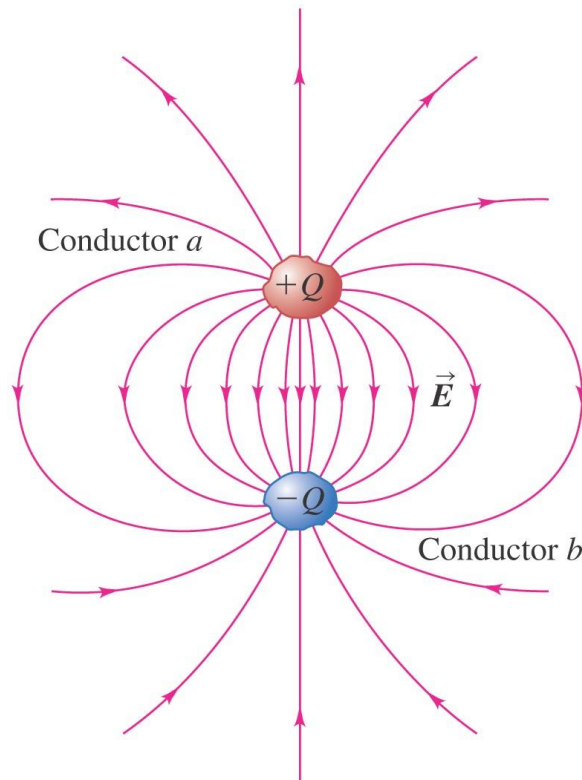
Introduction to Capacitance

- When flash devices made the “big switch” from bulbs and flashcubes to early designs of electronic flash devices, you could use a camera and actually hear a high-pitched whine as the “flash charged up” for your next photo opportunity.
- Just think of all those electrons moving on camera flash capacitors!



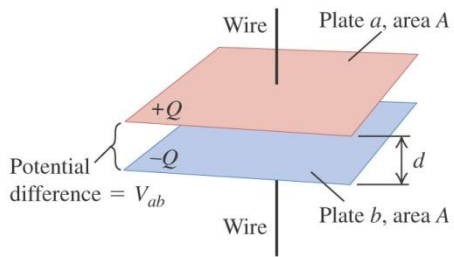
Keep charges apart and you get capacitance

- Define Capacitance $C = Q/V$ where V is the potential (voltage)
- Define unit of Capacitance = Farad = 1 Coulomb/Volt
- Any two charges insulated from each other form a capacitance.

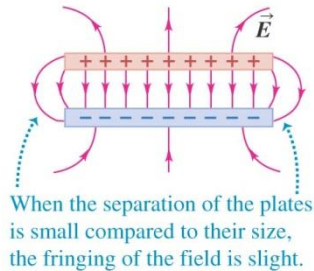


How do we build a capacitor? What's it good for?

(a) Arrangement of the capacitor plates

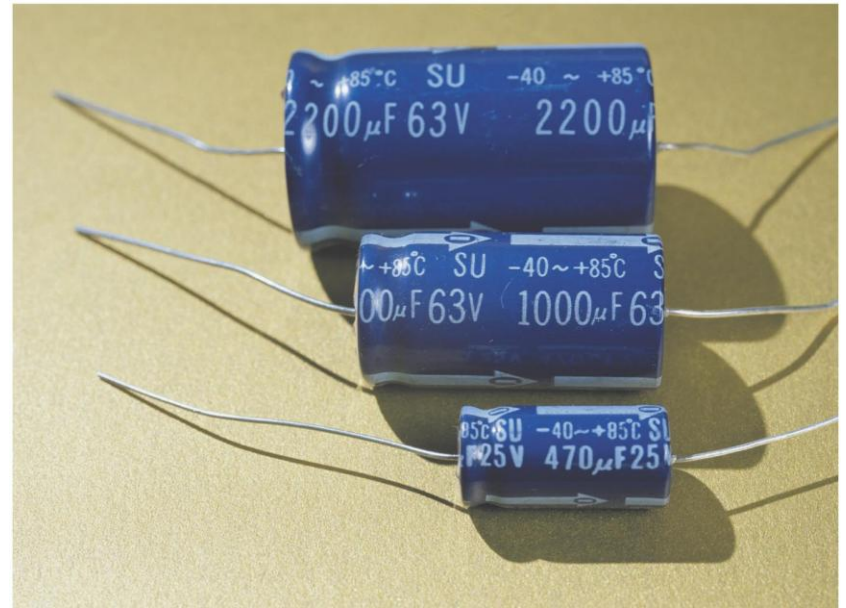


(b) Side view of the electric field \vec{E}

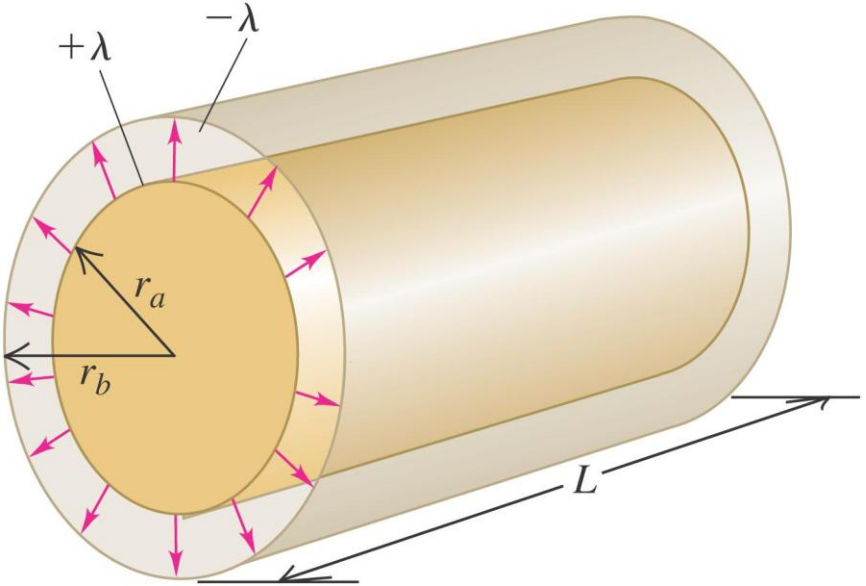
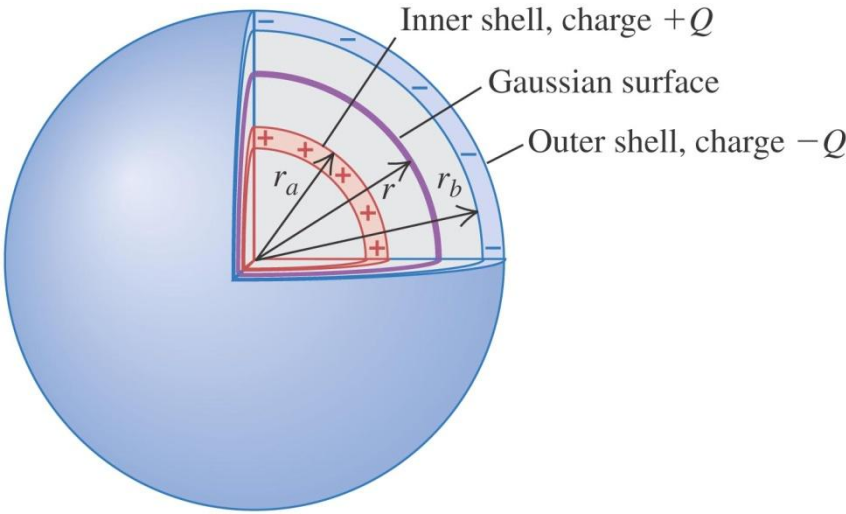


The unit of capacitance, the farad, is very large

- Commercial capacitors for home electronics are often cylindrical, from the size of a grain of rice to that of a large cigar.
- Capacitors like those mentioned above and pictured at right are microfarad capacitors.

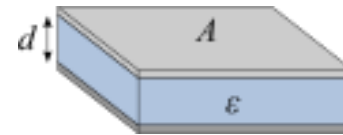


Some examples of flat, cylindrical, and spherical capacitors



Calculation of Capacitance

- Flat plates with dielectric material
 - Dielectric constant $K = \epsilon_R =$ relative permittivity
 - $A =$ Area of plate
 - $d =$ separation of plates
 - $C = K\epsilon_0 A/d$
 - $K = 1$ for vacuum
 - $K = 1.00058986 \pm 0.00000050$ for air at STP DC



Energy Stored in Capacitor

- A capacitor stores energy in its electric field
- $C = q/V$
- $dW = Vdq = q/C dq \rightarrow W = \frac{1}{2} Q^2/C = \frac{1}{2} CV^2$

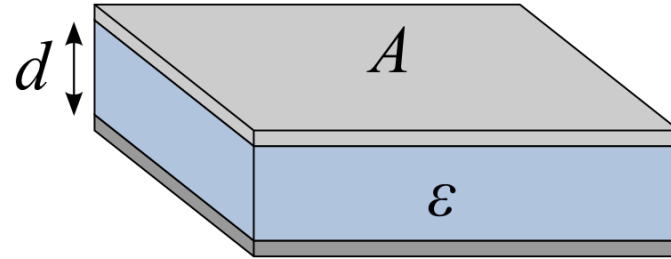
Capacitance and Geometry

Flat plate

(lots of these in your cell phone)

$$C = K\epsilon_0 A/d = \epsilon A/d$$

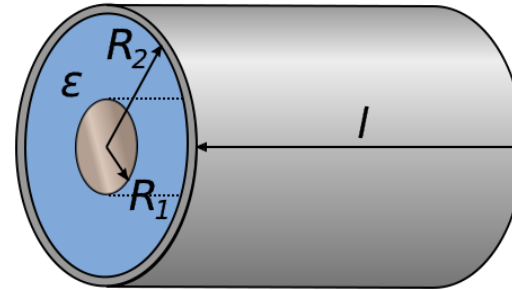
(notation can be confusing $\epsilon = K\epsilon_0$)



Coaxial line (concentric cylinders)

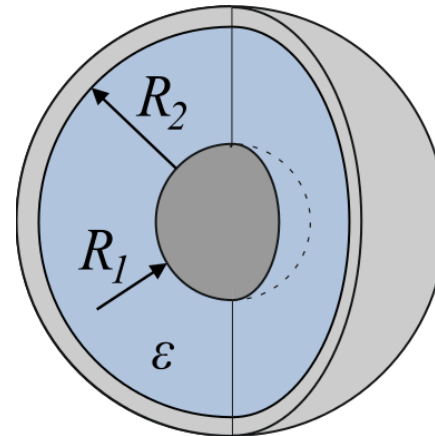
Your "cable" like Cox cable

$$C = 2\pi \epsilon l / (\ln(R_2 / R_1))$$



Concentric Spheres

$$C = 4\pi \epsilon / (1/R_1 - 1/R_2)$$



Capacitance of Single Sphere

- Recall two concentric sphere
- $C = 4\pi \epsilon / (1/R_1 - 1/R_2)$
- Let out sphere go to infinity (R_2)
- $C = 4\pi \epsilon R_1 = 4\pi \epsilon R$
- Example – our van de Graaff generator $R \sim 20\text{cm}$
 $\rightarrow C \sim 20 \text{ pF}$ (p=pico = 10^{-12})
- Earth $R \sim 6.4 \times 10^6 \text{ m}$ $\rightarrow C \sim 710 \mu\text{F}$ (μ =micro= 10^{-6})

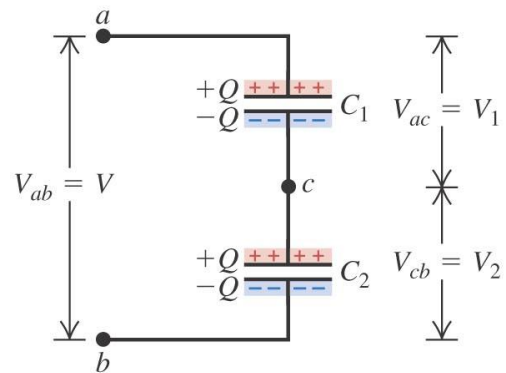
Capacitors may be connected one or many at a time

- Connection “one at a time” in linear fashion is termed “capacitors in series.”
- Multiple connections designed to operate simultaneously is termed “capacitors in parallel.”

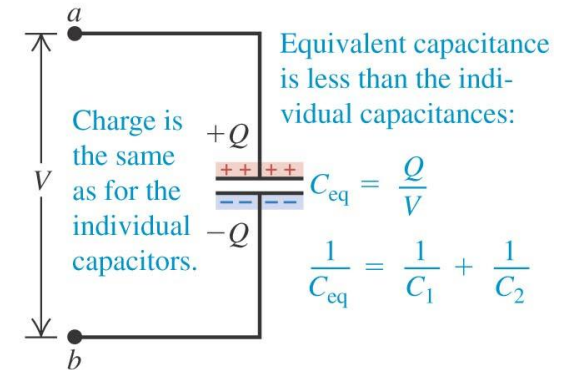
(a) Two capacitors in series

Capacitors in series:

- The capacitors have the same charge Q .
- Their potential differences add:
 $V_{ac} + V_{cb} = V_{ab}$.



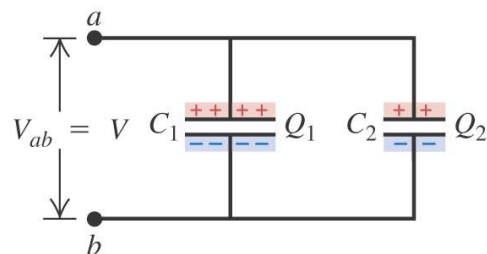
(b) The equivalent single capacitor



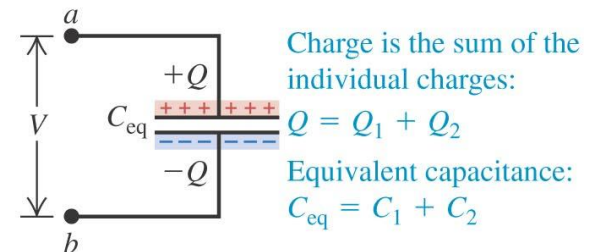
(a) Two capacitors in parallel

Capacitors in parallel:

- The capacitors have the same potential V .
- The charge on each capacitor depends on its capacitance: $Q_1 = C_1V$, $Q_2 = C_2V$.

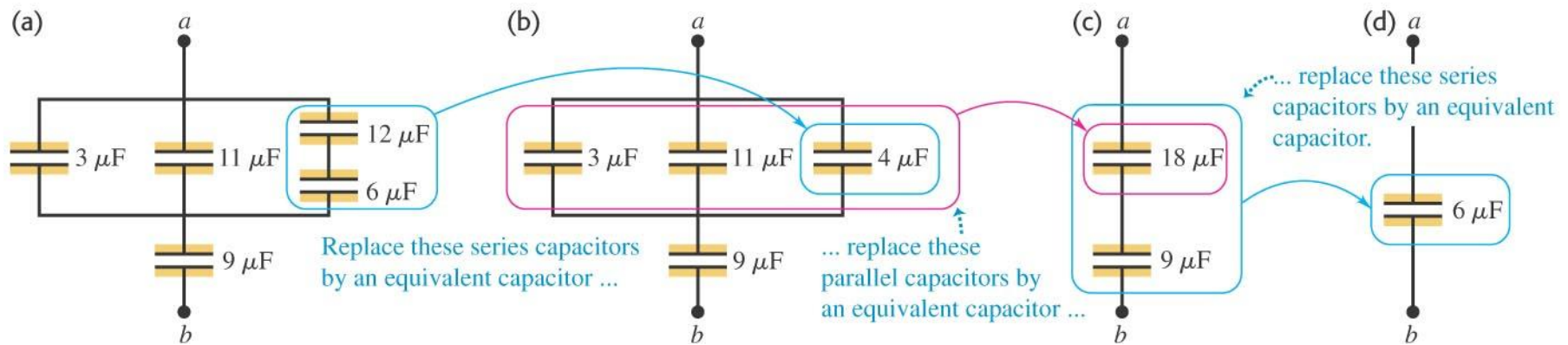


(b) The equivalent single capacitor



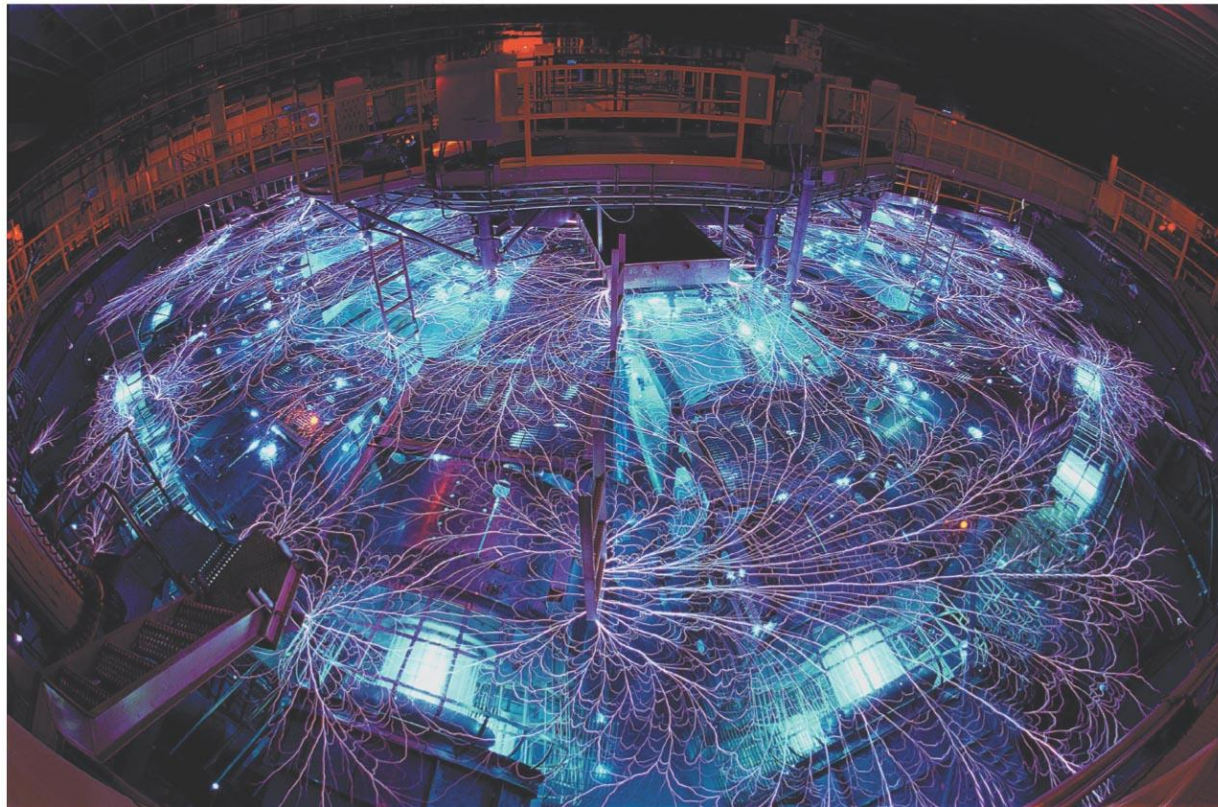
Calculations regarding capacitance

- Series $1/C_T = 1/C_1 + 1/C_2 + 1/C_3 + 1/C_4 + \dots$
- Parallel $C_T = C_1 + C_2 + C_3 + C_4 + \dots$
- We will see the opposite when we get to resistors and inductors



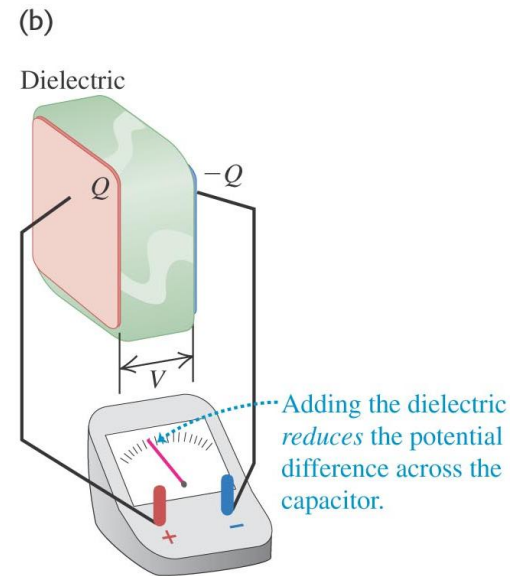
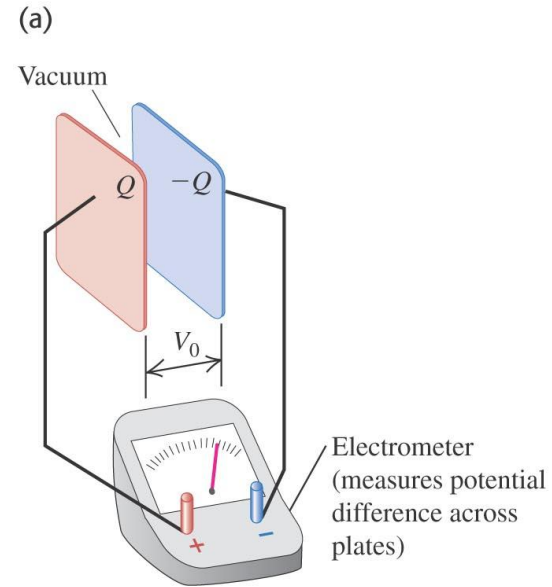
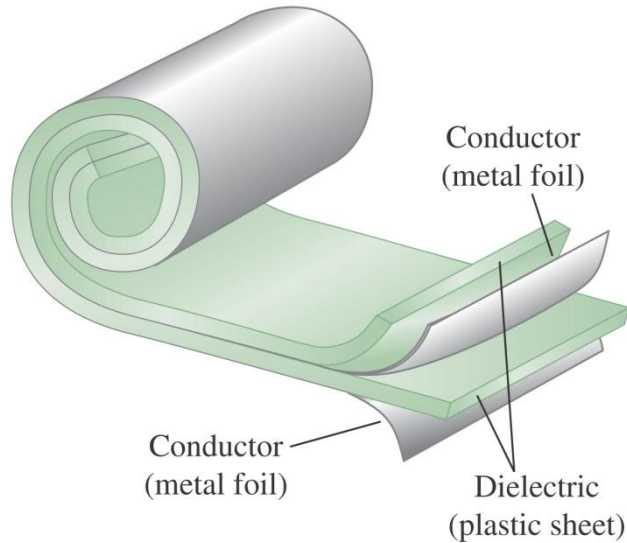
The Z Machine—capacitors storing large amounts of energy

- This large array of capacitors in parallel can store huge amounts of energy. When directed at a target, the discharge of such a device can generate temperatures on the order of 10^9K !



Dielectrics change the potential difference

- The potential between two parallel plates of a capacitor changes when the material between the plates changes. It does not matter if the plates are rolled into a tube or if they are flat

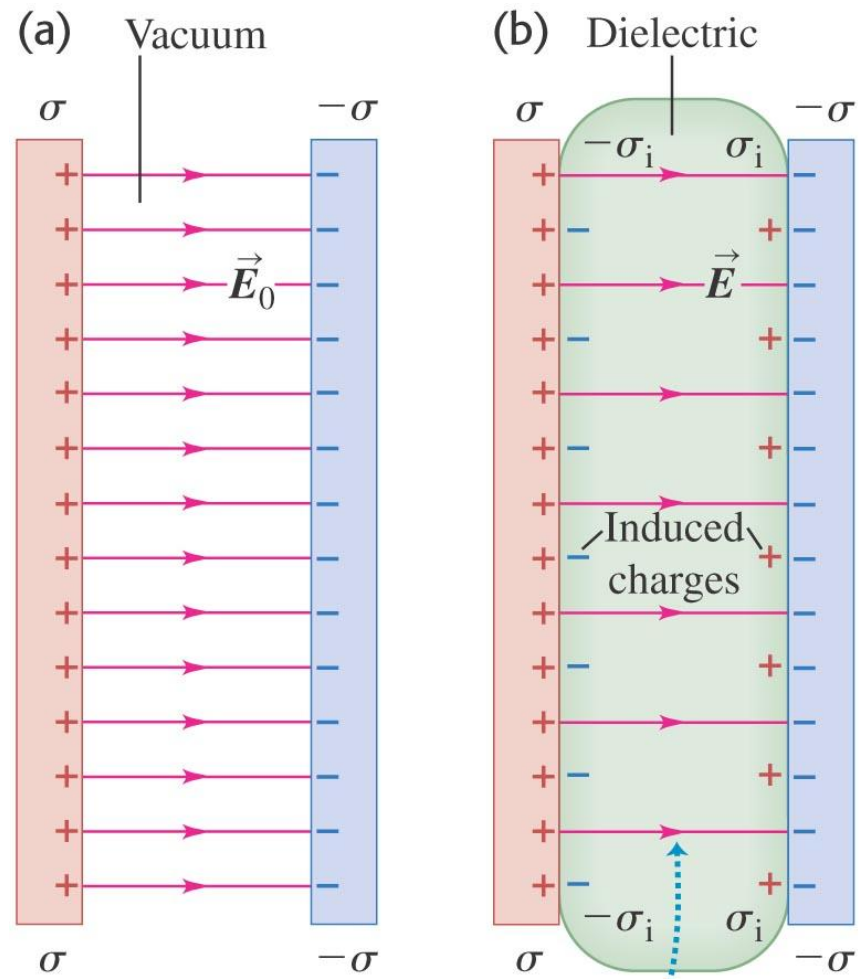


Dielectric constants

Table 24.1 Values of Dielectric Constant K at 20°C

Material	K	Material	K
Vacuum	1	Polyvinyl chloride	3.18
Air (1 atm)	1.00059	Plexiglas	3.40
Air (100 atm)	1.0548	Glass	5–10
Teflon	2.1	Neoprene	6.70
Polyethylene	2.25	Germanium	16
Benzene	2.28	Glycerin	42.5
Mica	3–6	Water	80.4
Mylar	3.1	Strontium titanate	310

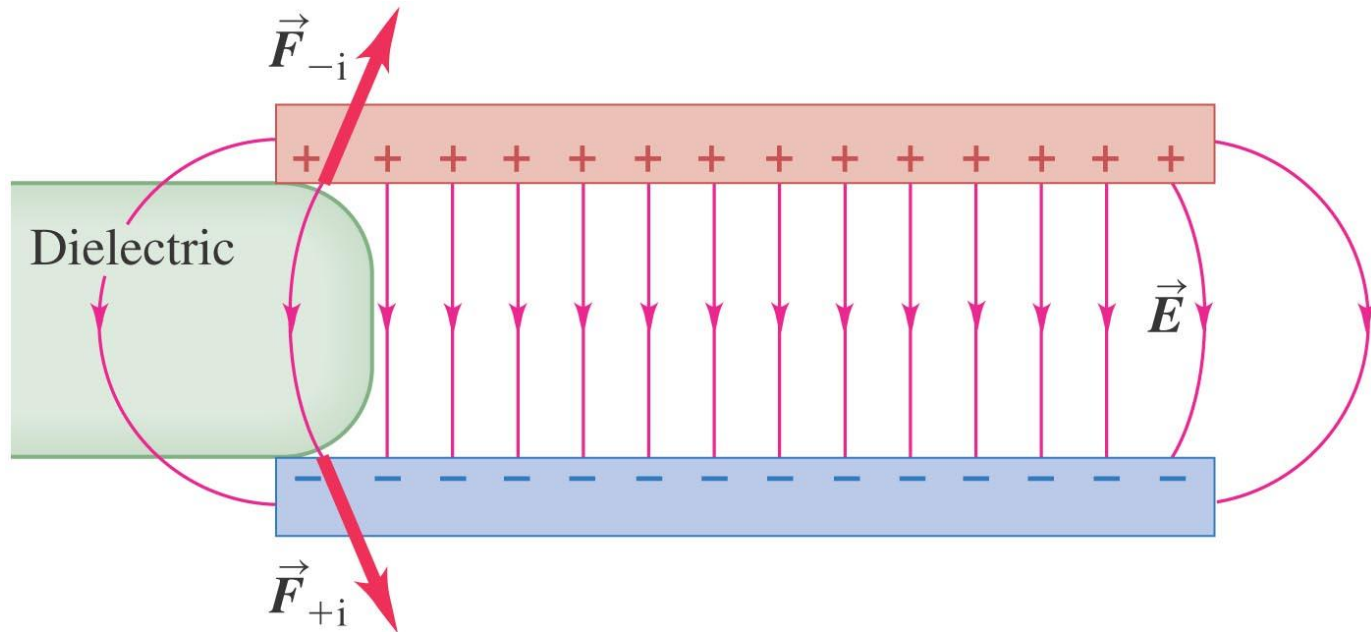
Field lines as dielectrics change



For a given charge density σ , the induced charges on the dielectric's surfaces reduce the electric field between the plates.

Capacitors with and without dielectrics

- Dielectric is polarized
- Dielectric increases capacitance
- Dielectric increases energy
- Dielectric is pulled into plates



Dielectric breakdown

- A very strong electrical field can exceed the strength of the dielectric and then there will be ionization and breakdown.

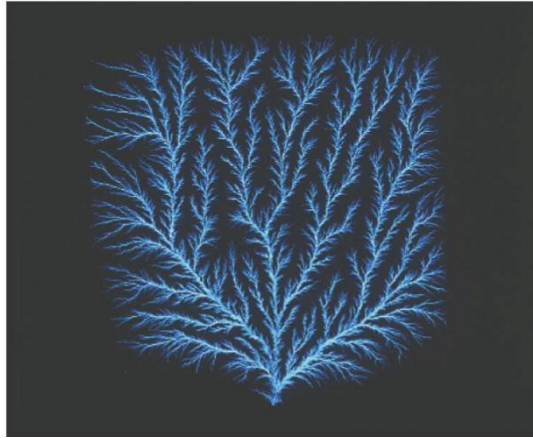
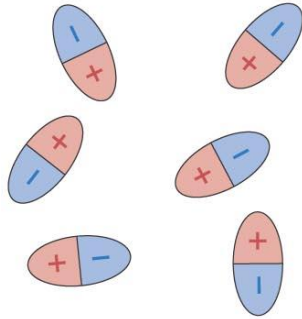


Table 24.2 Dielectric Constant and Dielectric Strength of Some Insulating Materials

Material	Constant, K	E_m (V/m)
Polycarbonate	2.8	3×10^7
Polyester	3.3	6×10^7
Polypropylene	2.2	7×10^7
Polystyrene	2.6	2×10^7
Pyrex glass	4.7	1×10^7

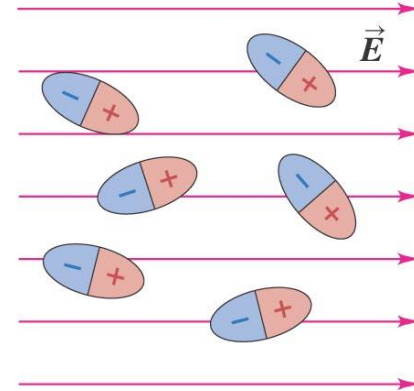
Molecular models

(a)



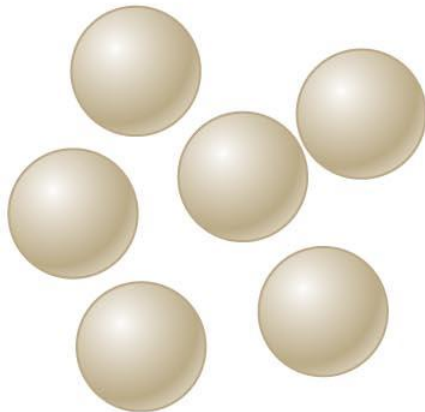
In the absence of an electric field, polar molecules orient randomly.

(b)



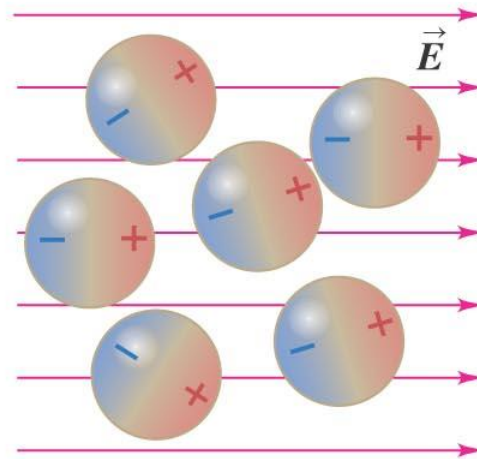
When an electric field is applied, the molecules tend to align with it.

(a)



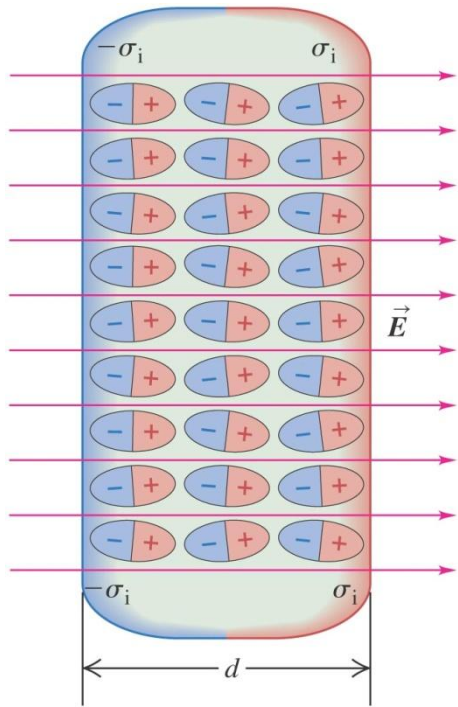
In the absence of an electric field, nonpolar molecules are not electric dipoles.

(b)

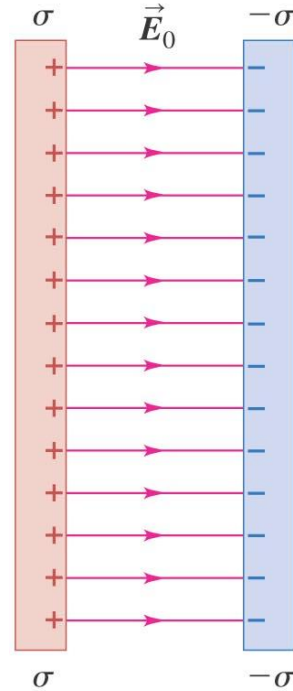


An electric field causes the molecules' positive and negative charges to separate slightly, making the molecule effectively polar.

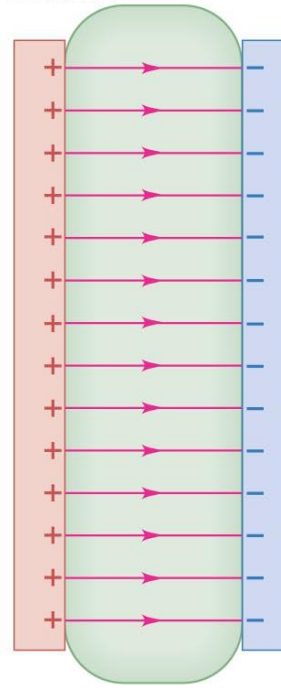
Polarization and electric field lines



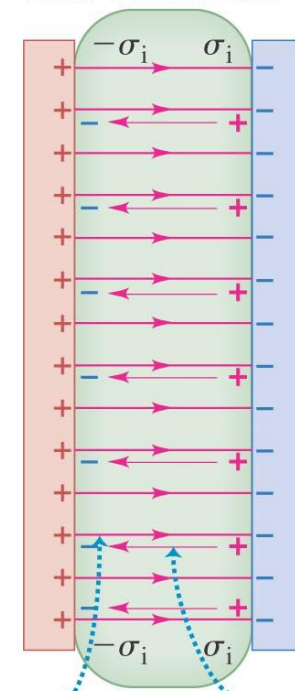
(a) No dielectric



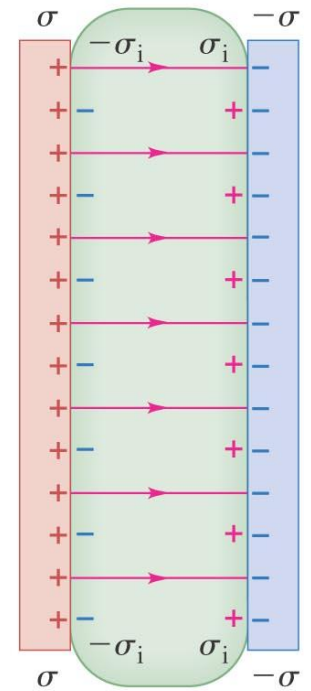
(b) Dielectric just inserted



(c) Induced charges create electric field



(d) Resultant field



Original electric field

Weaker field in dielectric due to induced (bound) charges

Gauss's Law in dielectrics

