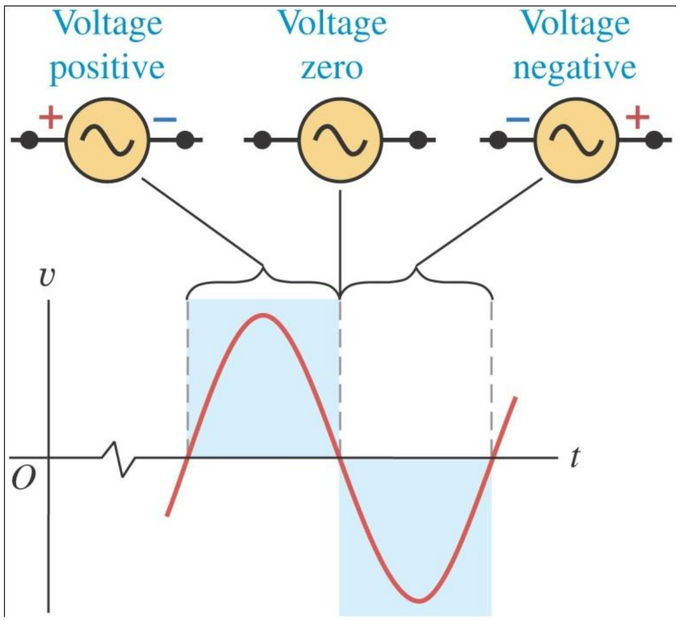
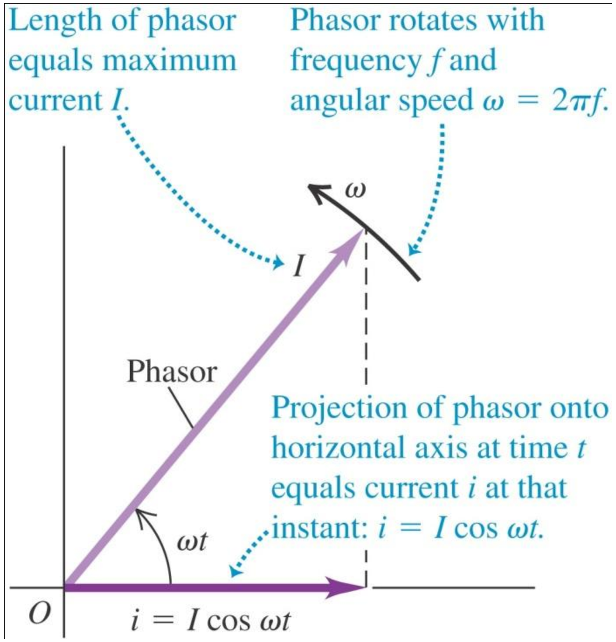
Chapter 21

Ac Circuits

AC current



Voltage positive
Voltage zero
Voltage negative
v
O
t



Length of phasor equals maximum current I.
Phasor rotates with frequency f and angular speed ω = 2πf.
ω
I
Phasor
Projection of phasor onto horizontal axis at time t equals current i at that instant: i = I cos ωt.
ωt
O
i = I cos ωt

Transformer

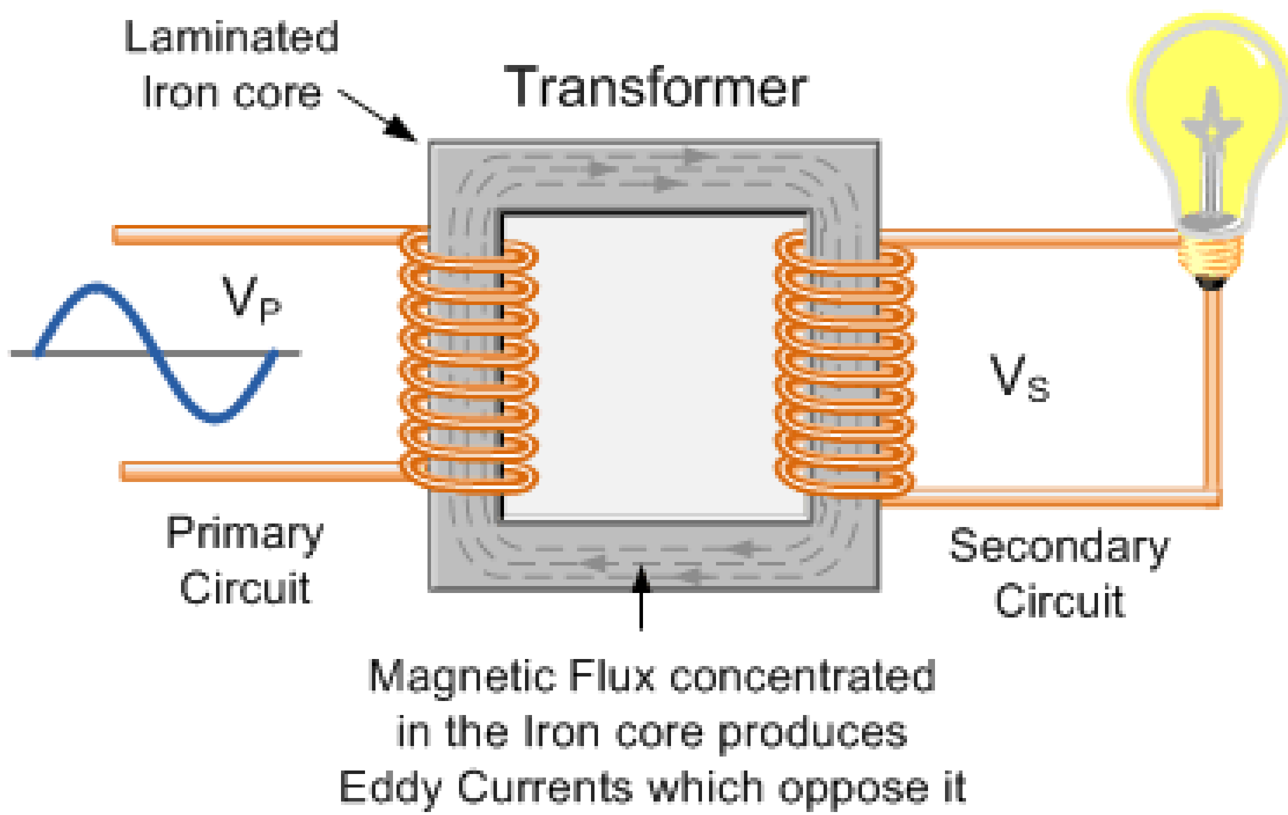
Transforms AC voltage UP or DOWN

Historical basis for AC Grid your use

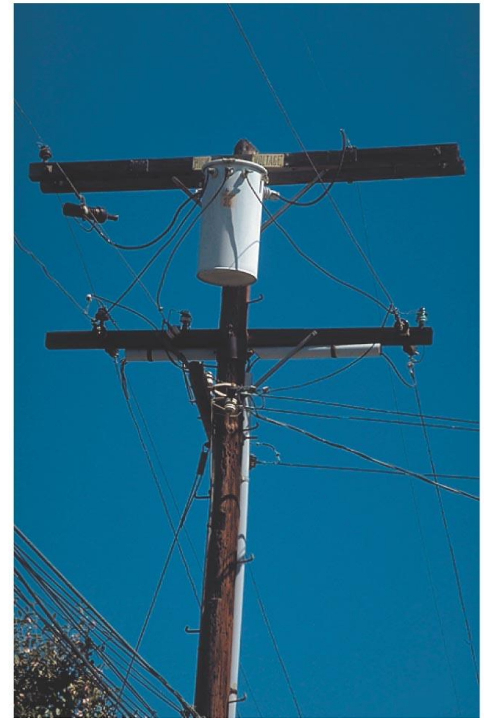
George Westinghouse (AC) vs Edison (DC)

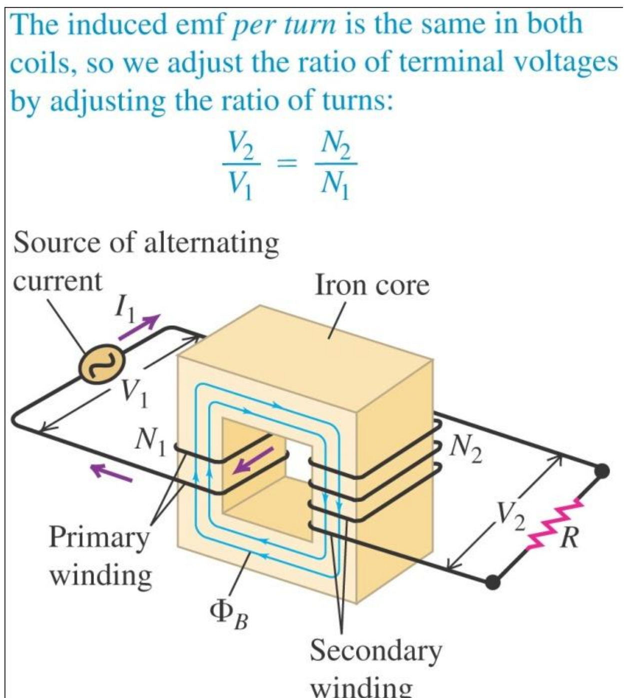
Losses due to resistance in wire and eddy currents in transformer "core"

Typ eff ~ 95-99%

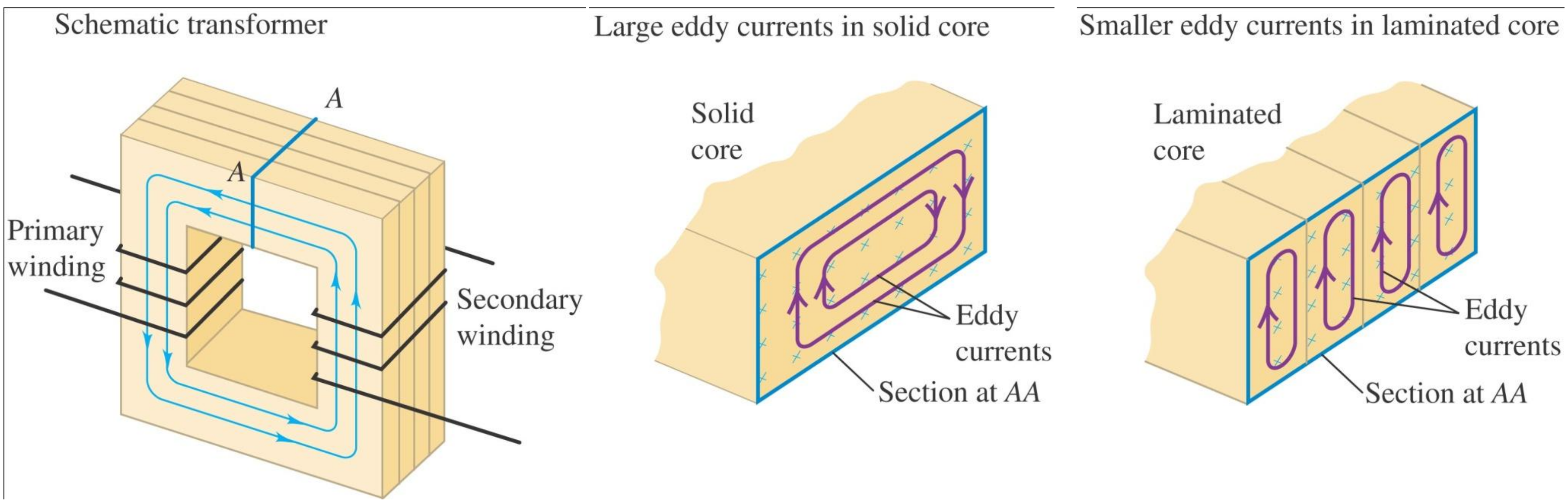


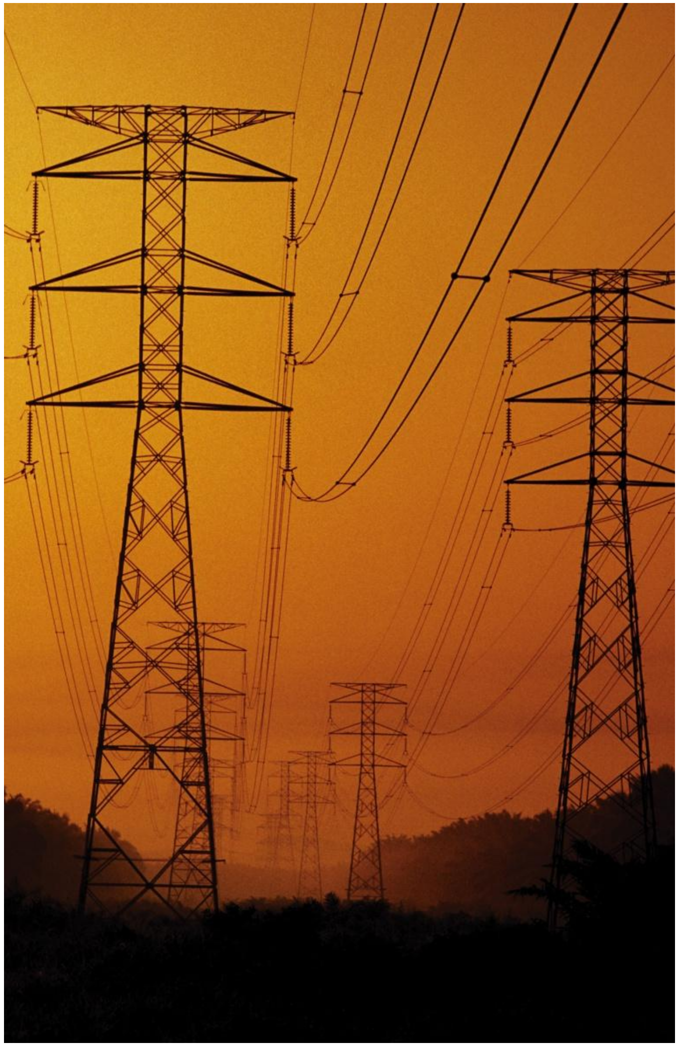
Transformers





Transformers II






U219

Angular speed of the rotating coil

Time

Angle of the coil at $t = 0$

Emf produced by an ac generator

$$\varepsilon = \omega NAB \sin(\omega t + \phi)$$

Number of turns in the coil

Area of the coil

Magnitude of the magnetic field to which the coil is exposed

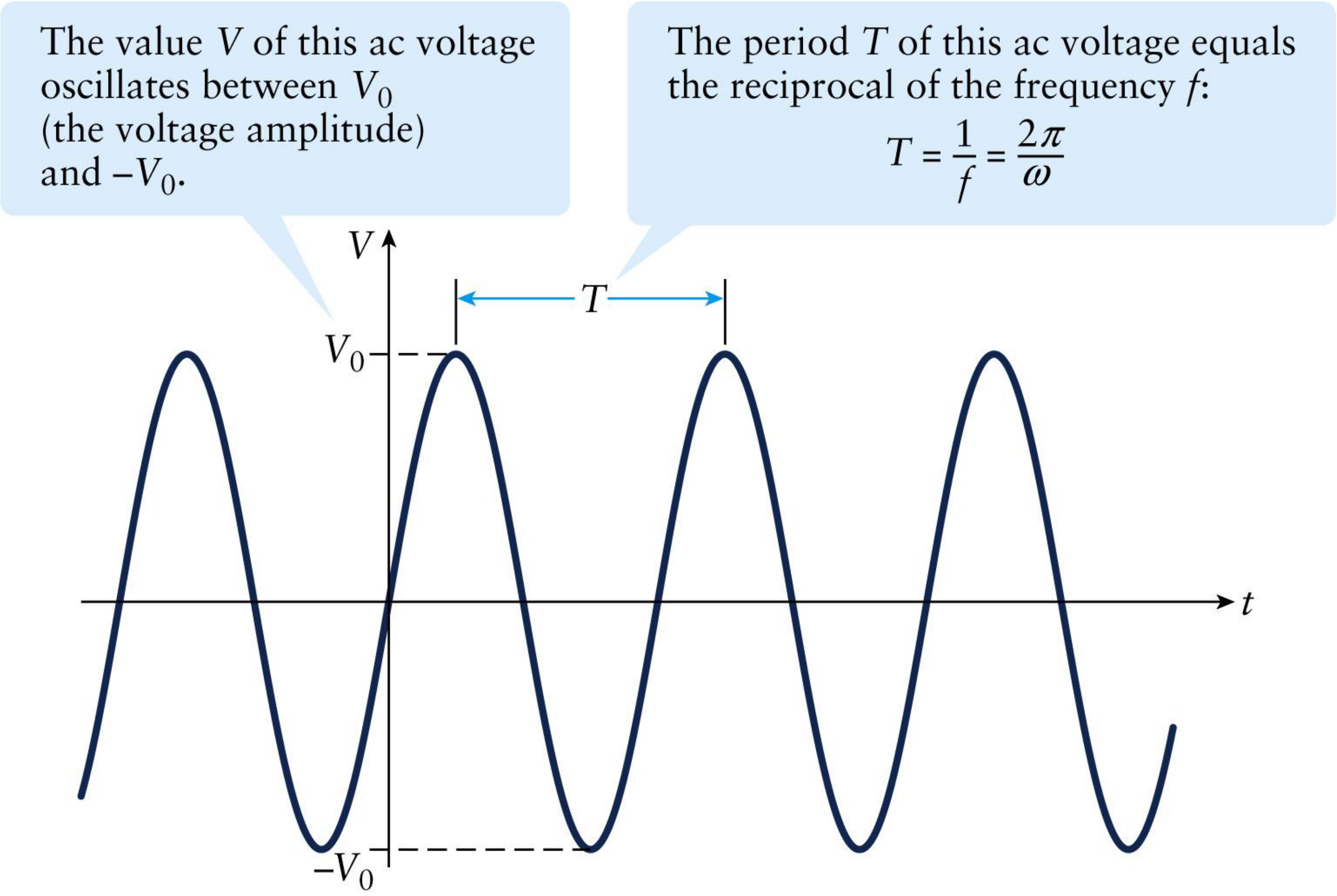
Time-varying voltage provided by an ac source

Angular frequency of the voltage

$$V(t) = V_0 \sin \omega t$$

Time

Voltage amplitude = maximum positive value of $V(t)$


The value V of this ac voltage oscillates between V_0 (the voltage amplitude) and $-V_0$.
The period T of this ac voltage equals the reciprocal of the frequency f:
$T = \frac{1}{f} = \frac{2\pi}{\omega}$
V
T
V_0
$-V_0$
t

Power into a resistor

Current through the resistor

$$P = i^2 R = \frac{V^2}{R}$$

Voltage across the resistor

Resistance of the resistor

The root mean square (rms) value of $V(t)$...

... is the square root of the average value of $V^2(t)$.

$$V_{rms} = \sqrt{(V^2(t))_{average}} = \frac{V_0}{\sqrt{2}}$$

If $V(t)$ is a sinusoidal function, its rms value equals the maximum value of $V(t)$ (the amplitude) divided by $\sqrt{2}$.

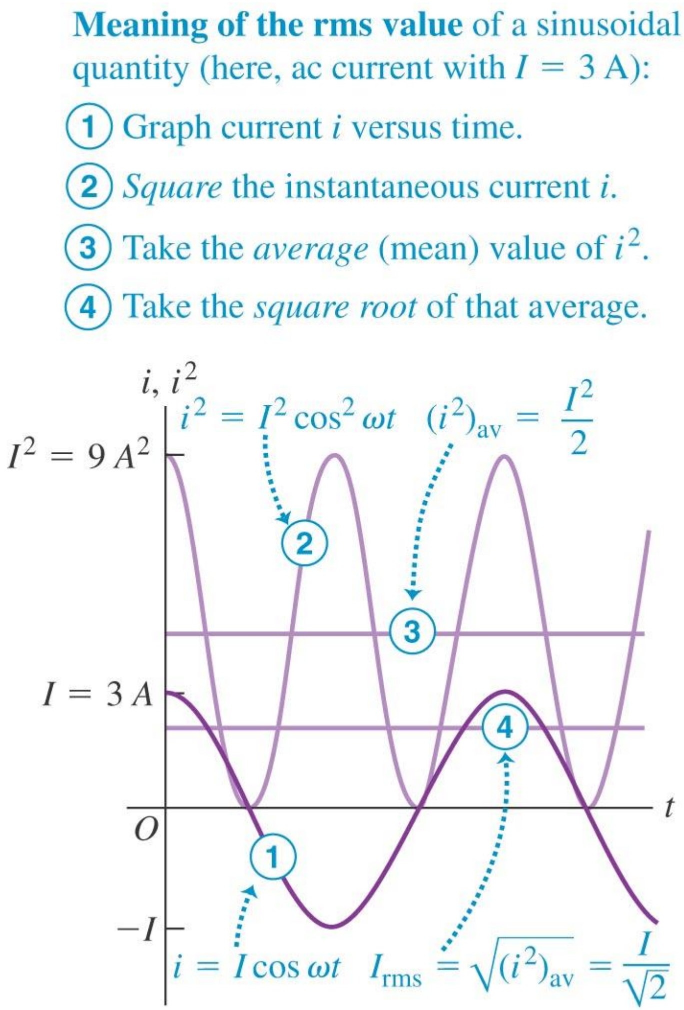
The root mean square (rms) value of $V(t)$...

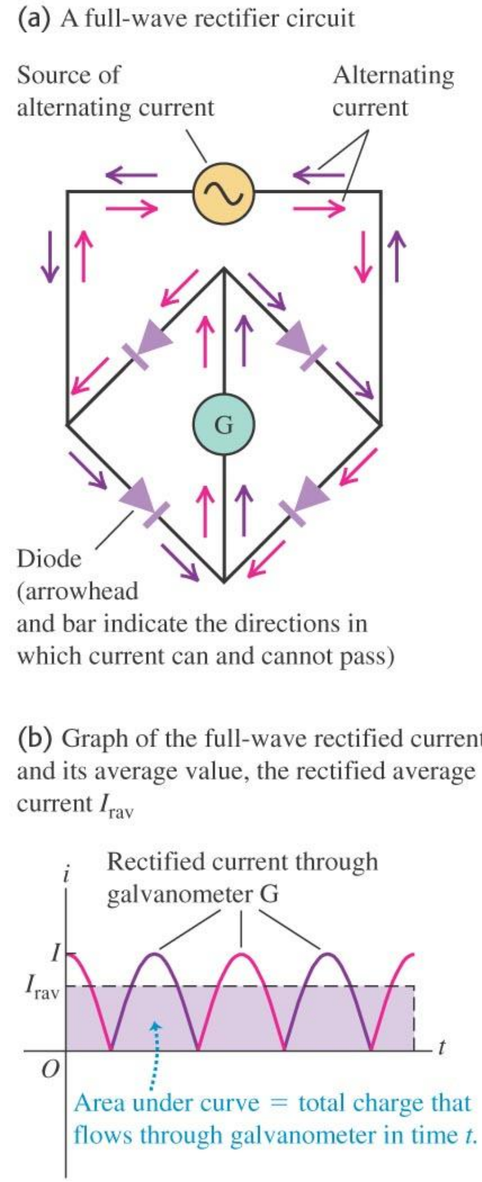
... is the square root of the average value of $V^2(t)$.

$$V_{\text{rms}} = \sqrt{(V^2(t))_{\text{average}}} = \frac{V_0}{\sqrt{2}}$$

If $V(t)$ is a sinusoidal function, its rms value equals the maximum value of $V(t)$ (the amplitude) divided by $\sqrt{2}$.

A full-wave rectifier circuit



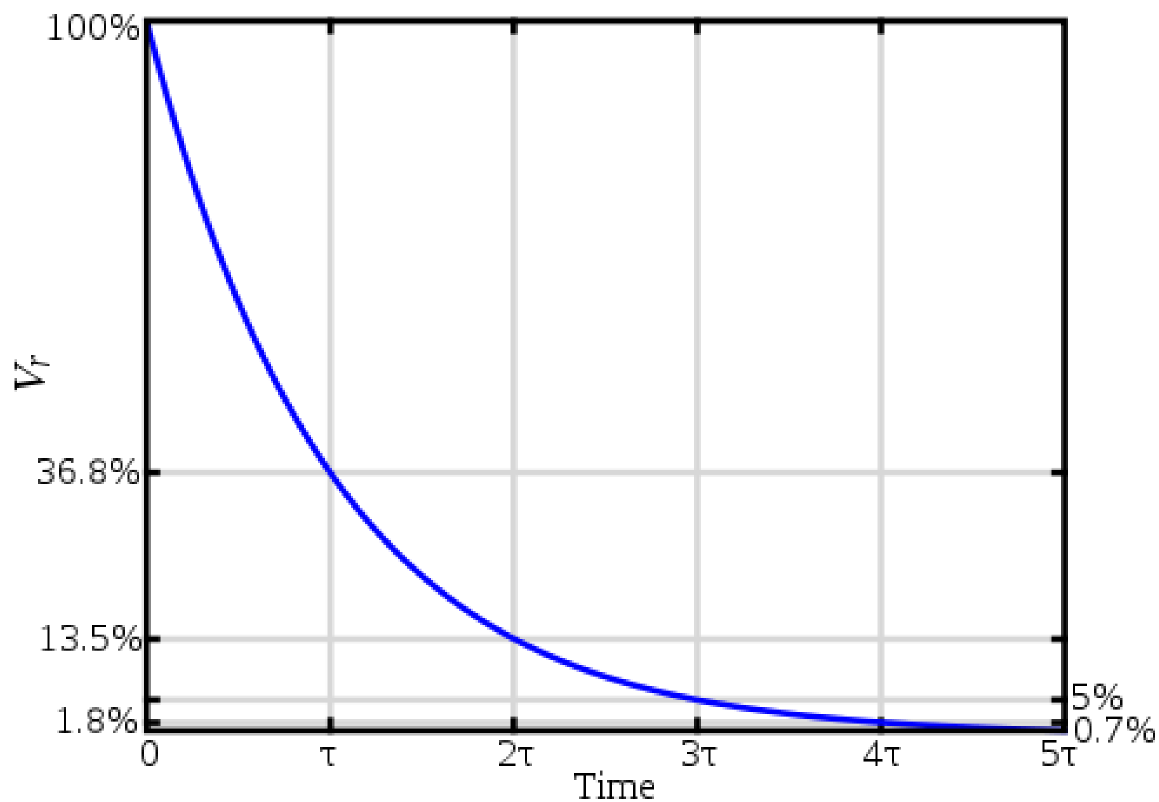


RC Circuit – Exponential Decay

- An RC circuit is a common circuit used in electronic filters
- The basic idea is it take time to charge a capacitor thru a resistor
- Recall that a capacitor C with Voltage V across it has charge Q=CV
- Current I= dQ/dt = C dV/dt
- In a circuit with a capacitor and resistor in parallel the voltage across the resistor must equal opposite that across the capacitor
- Hence $V_c = -V_R$ or Q/C = -IR or Q/C + IR = 0 (note the current I thru the resistor must be responsible for the dQ/dt – Kirchoff or charge conservation
- Now take a time derivative dQ/dt/C + R dI/dt = I/C + R dI/dt =0
- OR dI/dt +I/RC simply first order differential equation
- Solution is $I(t) = I_0\, e^{-t/RC} = I_0\, e^{-t/\tau}$ where τ = RC is the "time constant"
- Voltage across resistor $V_R(t) = IR = I_0\, R\, e^{-t/RC} = V_0\, e^{-t/RC} = -V_c(t)$ voltage across capacitor
- Note the exponential decay
- We can also write the eq as R dQ/dt + Q/C =0

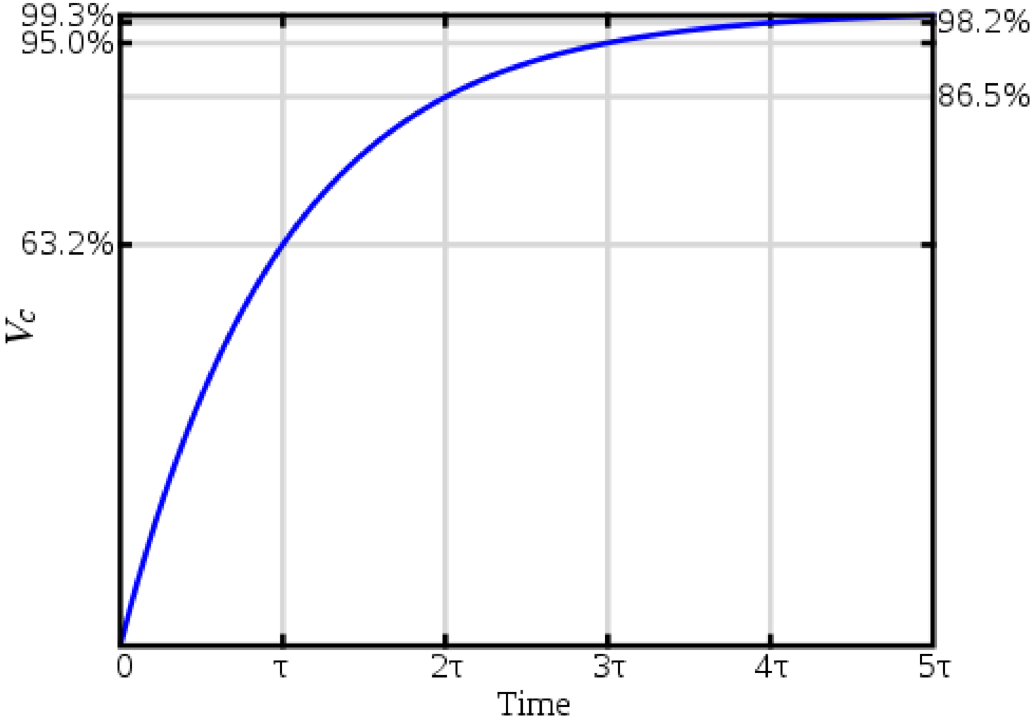
Discharging a capacitor

- Imagine starting with a capacitor C charged to voltage V_0
- Now discharge it starting at t=0 through resistor R
- $V(t) = V_0\, e^{-t/RC}$



Charging a Capacitor

- Start with a capacitor C that is discharged (0 volts)
- Now hook up a battery with a resistor R
- Start the charge at t=0
- $V(t) = V_0\,(1 - e^{-t/RC})$

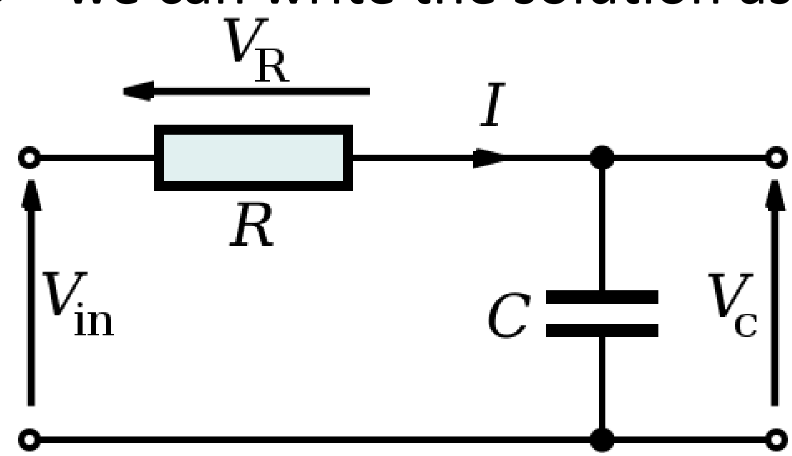


RC Circuits – Another way

- Lets analyze this another way
- In a closed loop the total EMF is zero (**must be careful here once we get to induced electric fields from changing magnetic fields)**
- In quasi static case $\int E \bullet dl = 0$ (voltage sum) over a closed loop
- Charge across the capacitor $Q = CV$ $I = dQ/dt = C\, dV/dt$
- But the same $I = -V/R$ (minus as V across cap is minus across R if we go in a loop)
- $CdV/dt = - V/R$ or $C\, dV/dt + V/R = 0$ or $dV/dt + V/RC = 0$
- Solution is $V(t) = V_0\, e^{-t/RC}$
- Same solution as before
- The time required to fall from the initial voltage V_0 to V_0/e is time $\tau = RC$

Complex impedances

- Consider the following series circuit
- If we put an input Voltage V_{in} across the system
- We get a differential eq as before but with V_{in}
- $V_{in} + IR + Q/C = 0$ $\int E \bullet dl = 0$ around the closed loop
- $V_{in} + R\, dQ/dt + Q/C = V_{in} + IR + \int I\, dt/C$ – we can write the solution as a complex solution $I = I_0\ e^{i\omega t}$
- $V_{in} + IR + \int I\, dt/C$
- We can make this more
- General letting $V_{in} = V_0\ e^{i\omega t}$
- This allows a driven osc term – freq ω
- $V_0\ e^{i\omega t} + R\, I_0\ e^{i\omega t} + I_0\ e^{i\omega t}\ /(i\omega C)$
- $V_0 + R\, I_0 + I_0/(i\omega C)$ - thus we can interpret this as a series of impedances (resistance) Z (general impedance) where $Z_R = R$ is the normal impedance of a resistor and $Z_c = 1/(i\omega C) = -i/(\omega C)$ is the impedance of a capacitor
- Note the impedance of a capacitor is complex and proportional to $1/\omega C$ - the minus i will indicate a 90 degree phase shift



Q26.9

A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the maximum charge stored on the capacitor?

A. the emf $\mathcal{E}$ of the battery

B. the capacitance C of the capacitor

C. the resistance R of the resistor

D. both $\mathcal{E}$ and C

E. all three of $\mathcal{E}$, C, and R

A26.9

A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the maximum charge stored on the capacitor?

A. the emf ε of the battery

B. the capacitance C of the capacitor

C. the resistance R of the resistor

D. both ε and C

E. all three of ε, C, and R

Q26.10

A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the rate at which the capacitor charges?

A. the emf ε of the battery

B. the capacitance C of the capacitor

C. the resistance R of the resistor

D. both C and R

E. all three of ε, C, and R

A26.10

A battery, a capacitor, and a resistor are connected in series. Which of the following affect(s) the rate at which the capacitor charges?

A. the emf $\mathcal{E}$ of the battery

B. the capacitance *C* of the capacitor

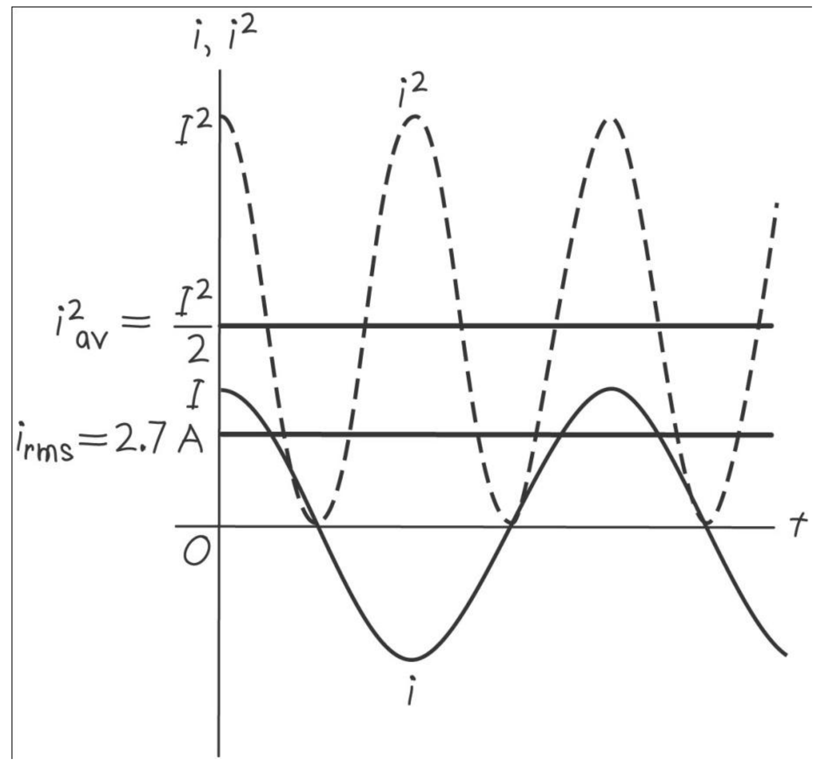
C. the resistance *R* of the resistor

D. both *C* and *R*

E. all three of $\mathcal{E}$, *C*, and *R*

Current in the device I'm using right now

- A desktop PC draws current from a plug to the wall, but what are the details?



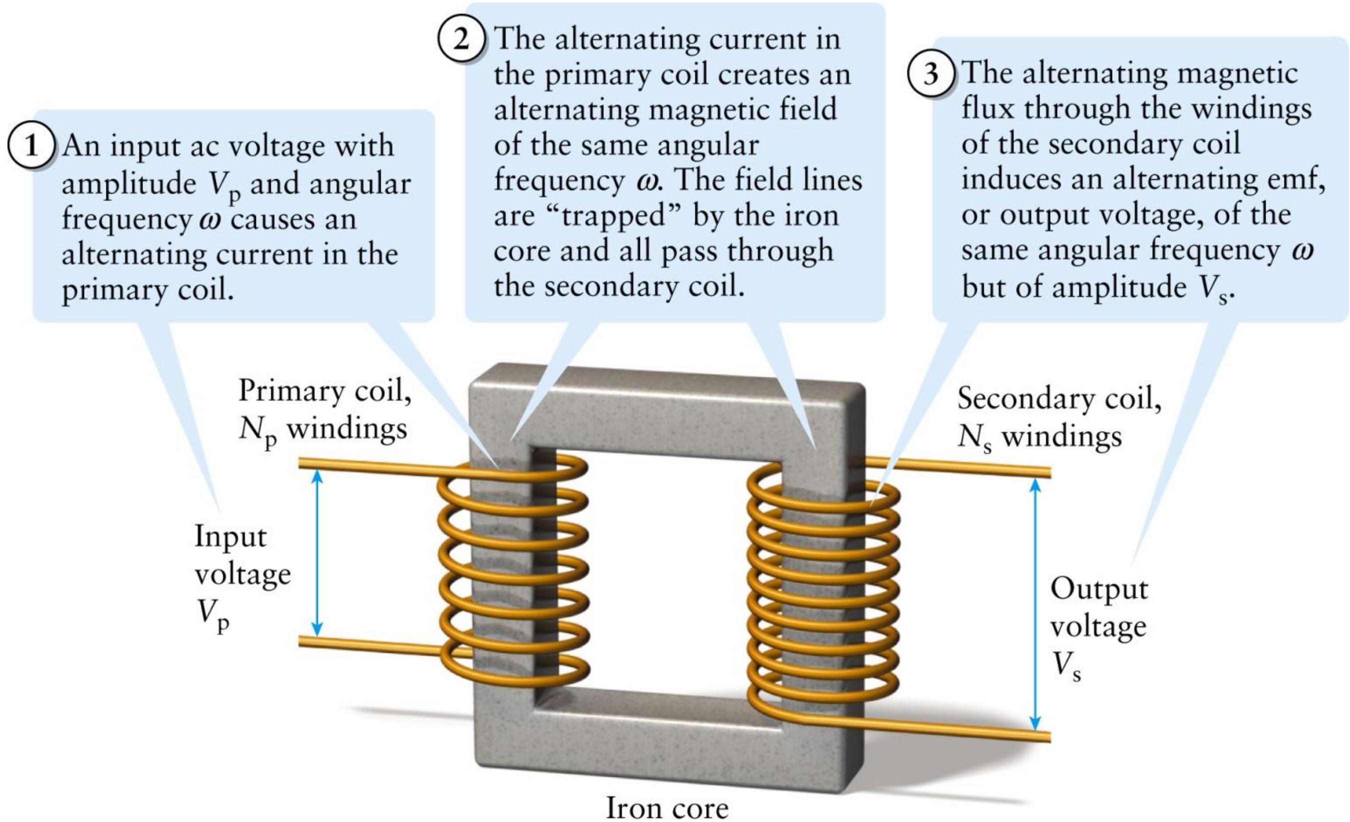
Power produced by or transferred into a circuit element

$$P = iV$$

Current through the circuit element

Voltage (absolute value) across the circuit element

Source of emf: Power flows out of the source and into the moving charges.
Resistor: Power flows out of the charges and into the resistor.



The amplitude V_s of the output voltage equals (N_s/N_p) times the amplitude V_p of the input voltage.

- If $N_s > N_p$, then $V_s > V_p$. This is a step-up transformer.
- If $N_s < N_p$, then $V_s < V_p$. This is a step-down transformer.

Amplitudes of the input (p) and output (s) voltages

$$\frac{V_s}{V_p} = \frac{V_{s,rms}}{V_{p,rms}} = \frac{N_s}{N_p}$$

Number of windings in the primary coil (p) and secondary coil (s)

Rms values of the input (p) and output (s) voltages

$$L = \frac{N\Phi_B}{i}$$

Inductance of a coil

Number of windings in the coil

Magnetic flux through each winding of the coil due to the field produced by the coil itself

Current in the coil

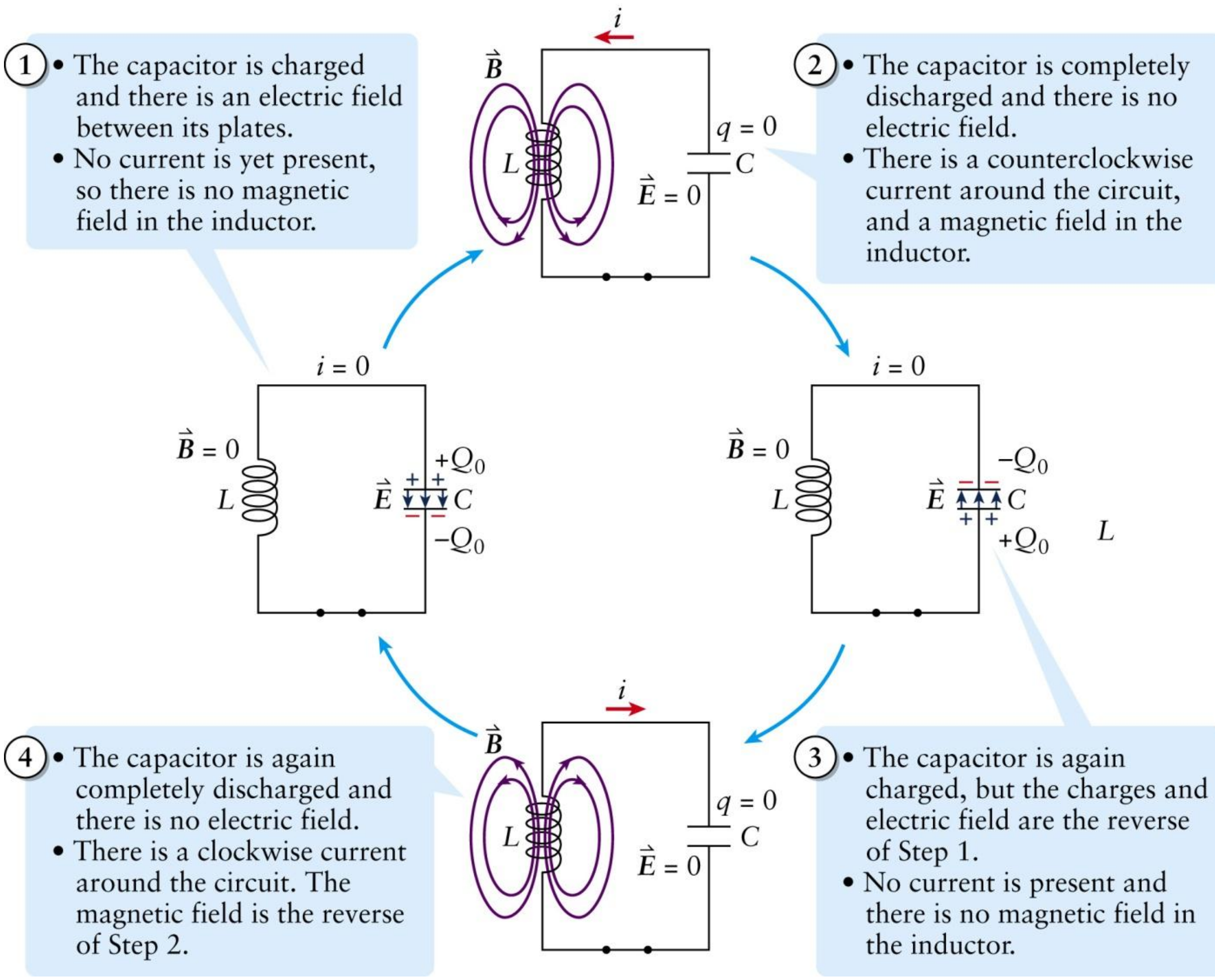
Voltage across an inductor

Inductance of the inductor

$$V = -L\frac{\Delta i}{\Delta t}$$

Rate of change of the current i in the inductor

The negative sign means that the voltage opposes any change in the current.



1
• The capacitor is charged and there is an electric field between its plates.
• No current is yet present, so there is no magnetic field in the inductor.
2
• The capacitor is completely discharged and there is no electric field.
• There is a counterclockwise current around the circuit, and a magnetic field in the inductor.
3
• The capacitor is again charged, but the charges and electric field are the reverse of Step 1.
• No current is present and there is no magnetic field in the inductor.
4
• The capacitor is again completely discharged and there is no electric field.
• There is a clockwise current around the circuit. The magnetic field is the reverse of Step 2.
i
$\vec{B}$
L
q = 0
C
$\vec{E}$ = 0
i = 0
$\vec{B}$ = 0
$+Q_0$
$-Q_0$
$\vec{E}$

Energy in an LC circuit

Magnetic energy U_B in the inductor

Electric energy U_E in the capacitor

$$E = \frac{1}{2}Li^2 + \frac{q^2}{2C}$$

Charge on the capacitor

Inductance of the inductor

Current in the inductor

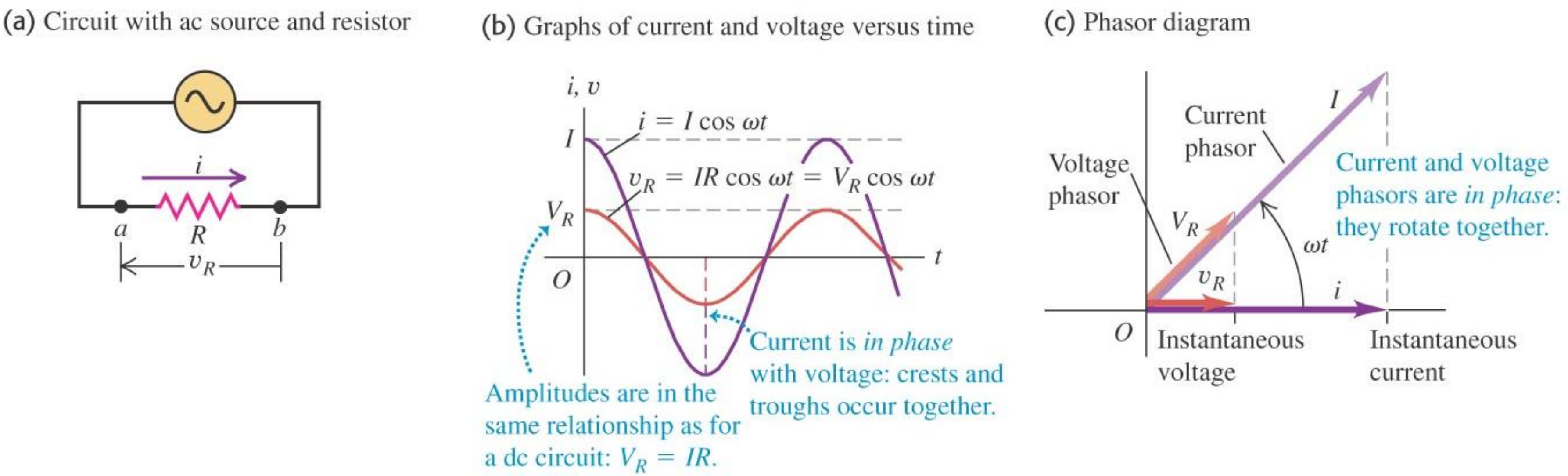
Capacitance of the capacitor

Table 21-1 Comparing a block on an ideal spring to an *LC* circuit

Quantity for a block on an ideal spring	Corresponding quantity for an *LC* circuit
spring displacement, x	capacitor charge, q
block velocity, $v_x = \Delta x / \Delta t$	current, $i = \Delta q / \Delta t$
spring constant of the spring, k	reciprocal of the capacitance, $1/C$
mass of the block, m	inductance, L
oscillation amplitude, A	maximum capacitor charge, Q_0
potential energy of the spring, $U_{\text{spring}} = \frac{1}{2}kx^2$	electric energy in the capacitor, $U_E = \frac{q^2}{2C}$
kinetic energy of the block, $K = \frac{1}{2}mv_x^2$	magnetic energy in the inductor, $U_B = \frac{1}{2}Li^2$

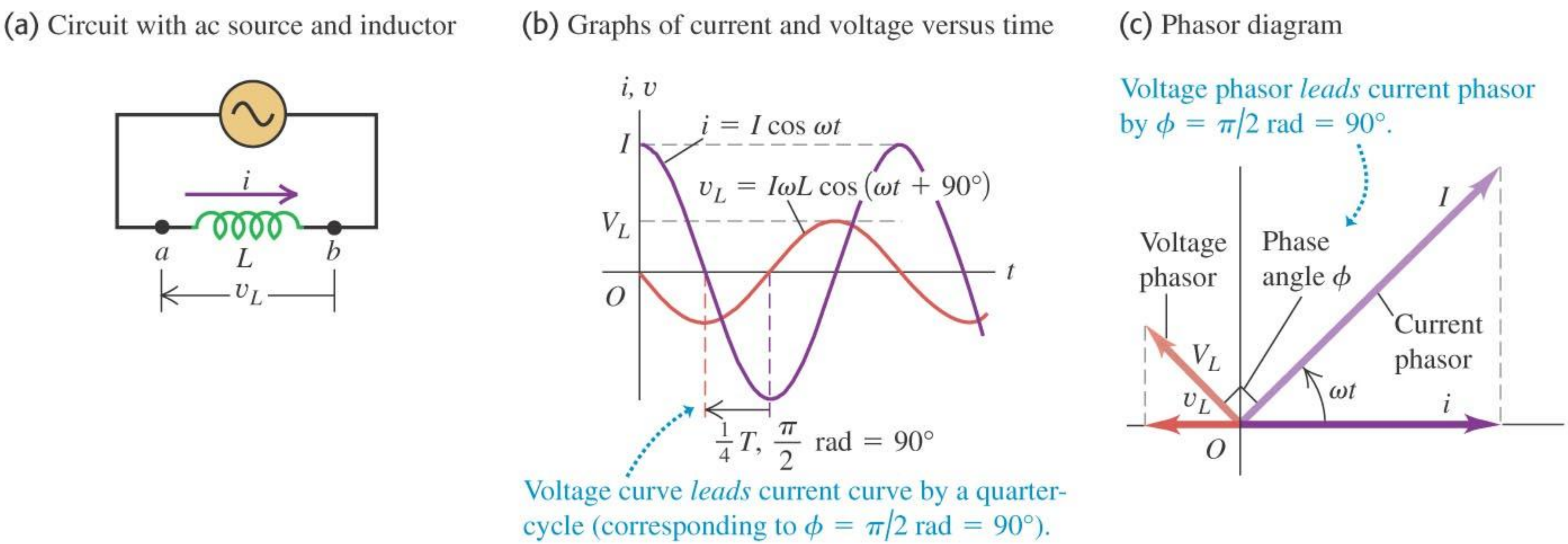
Resistors in an AC circuit

- Ohm's Law applied in oscillatory fashion.



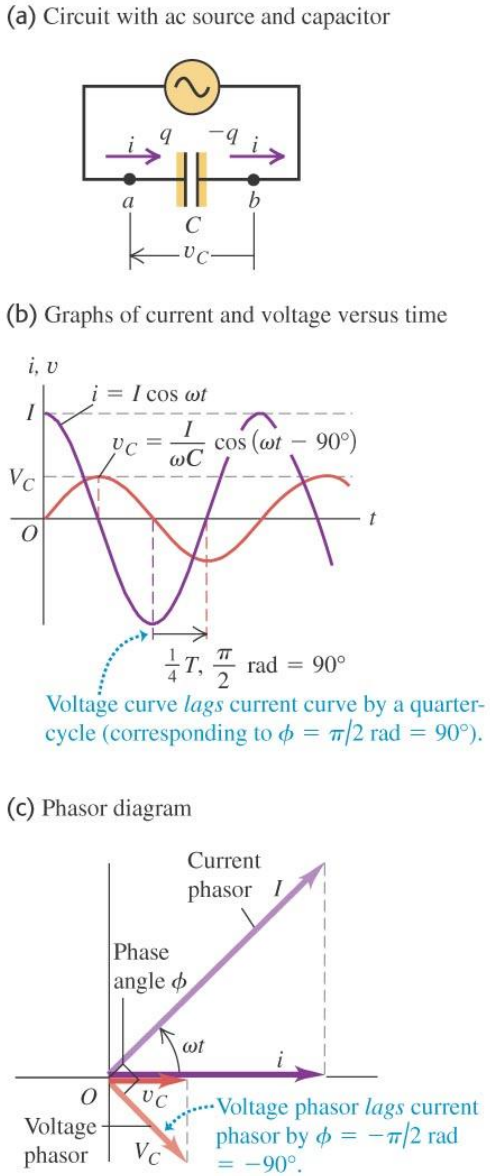
Inductors in an AC circuit

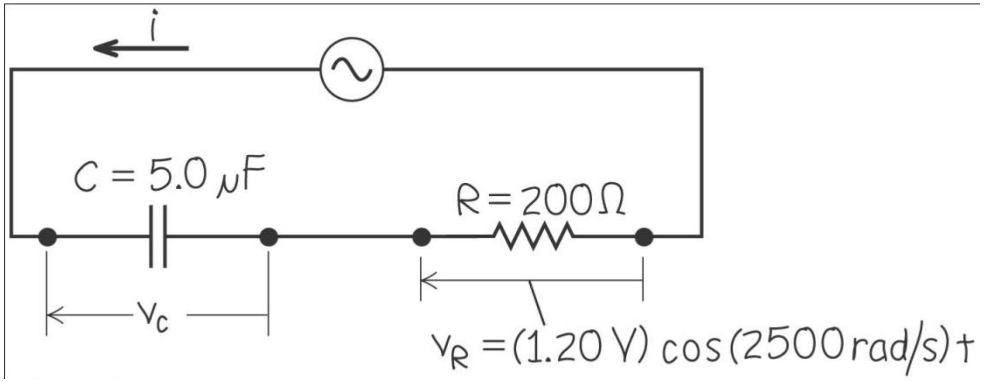
- Replace the resistor in the previous slide with an inductor.



Capacitance in an AC circuit

- Because this is a series circuit, the current is the same through the capacitor as through the resistor just considered.





Comparing AC circuit elements

- Summary of circuit elements.
- Frequency dependence of reactance

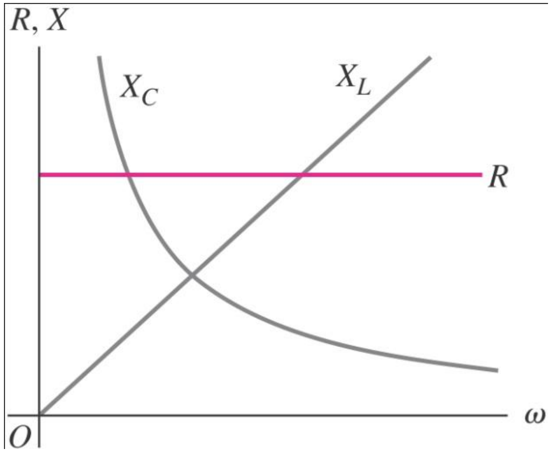


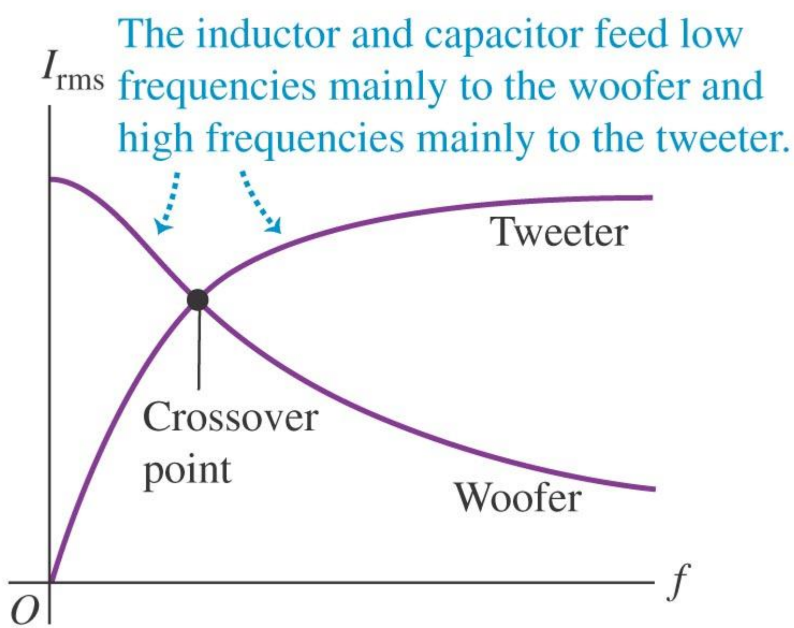
Table 31.1 Circuit Elements with Alternating Current

Circuit Element	Amplitude Relationship	Circuit Quantity	Phase of v
Resistor	$V_R = IR$	R	In phase with i
Inductor	$V_L = IX_L$	$X_L = \omega L$	Leads i by 90°
Capacitor	$V_C = IX_C$	$X_C = 1/\omega C$	Lags i by 90°

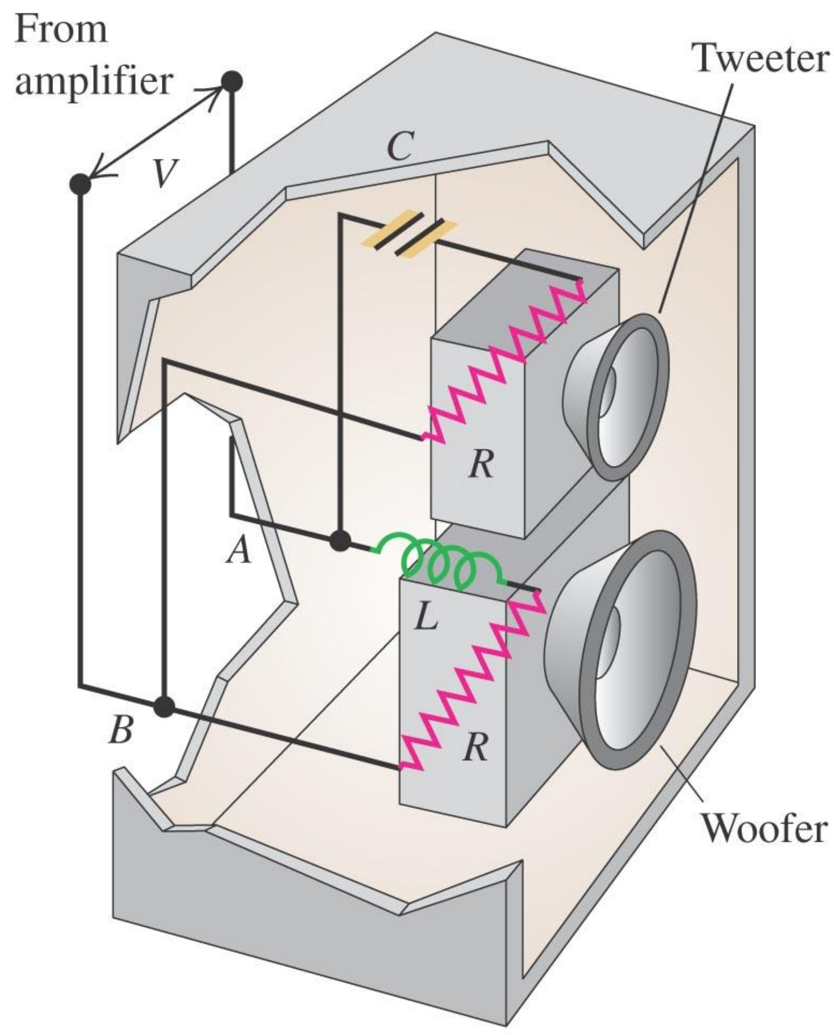
The loudspeaker, a useful application

- The woofer (low tones) and the tweeter (high tones) connect in parallel through a "crossover."

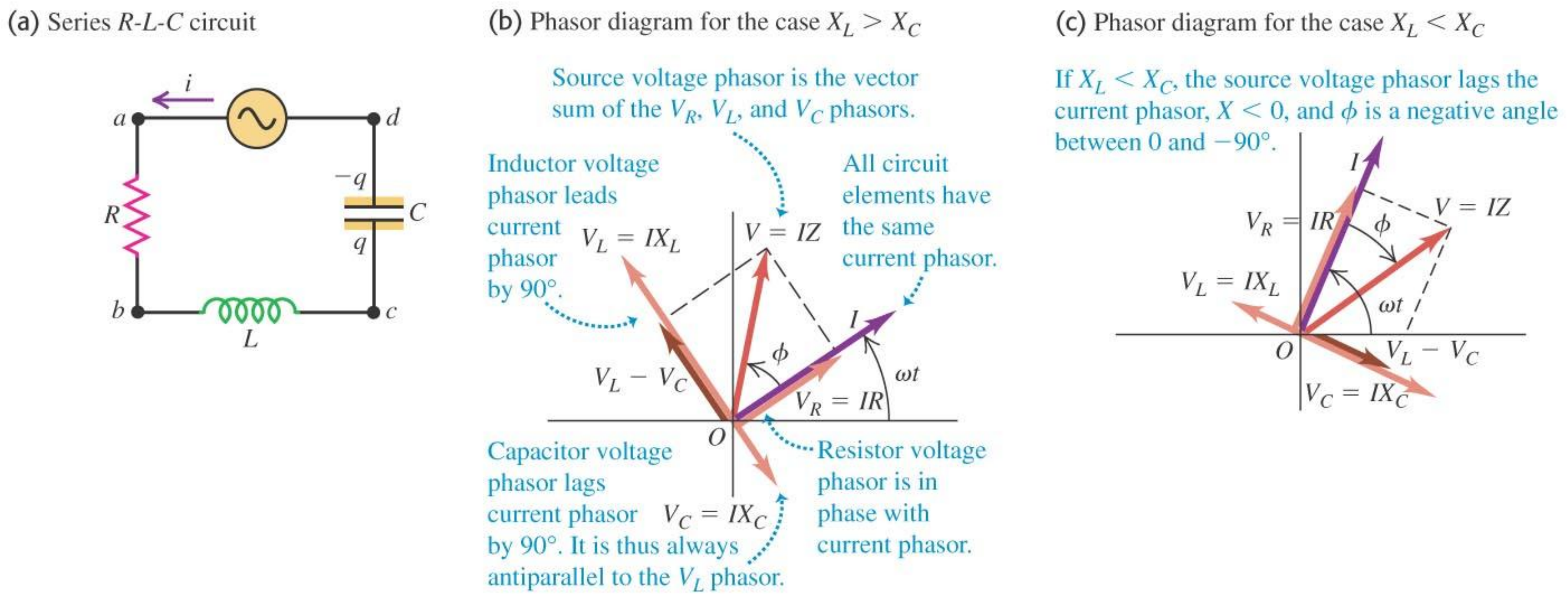
Graphs of rms current as functions of frequency for a given amplifier voltage



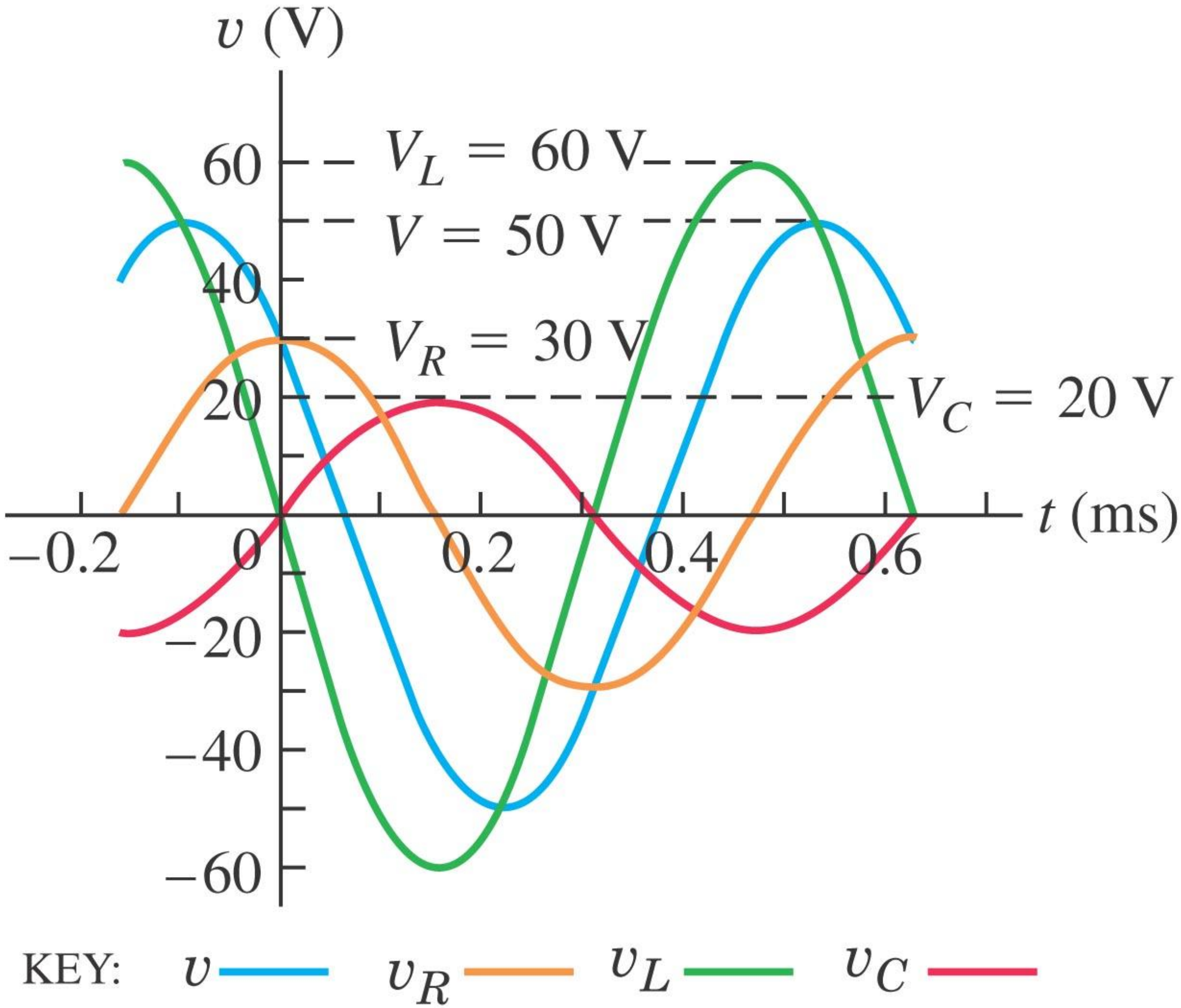
A crossover network in a loudspeaker system



The L-R-C circuit

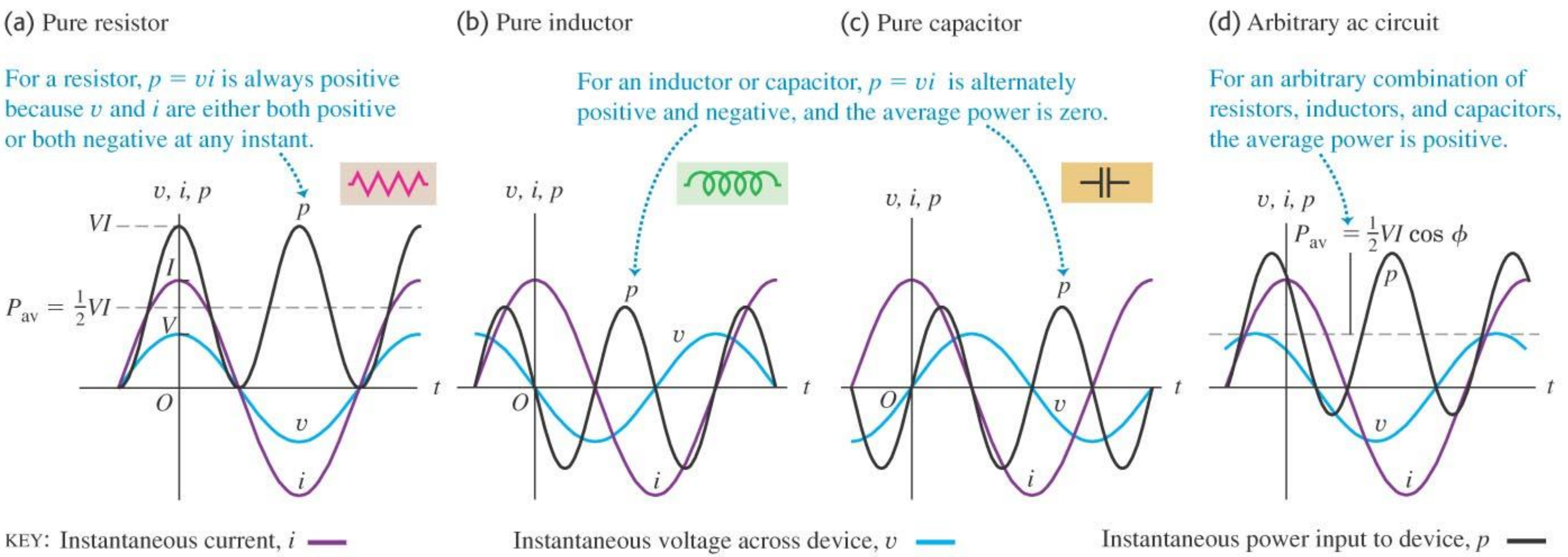


An L-R-C circuit II



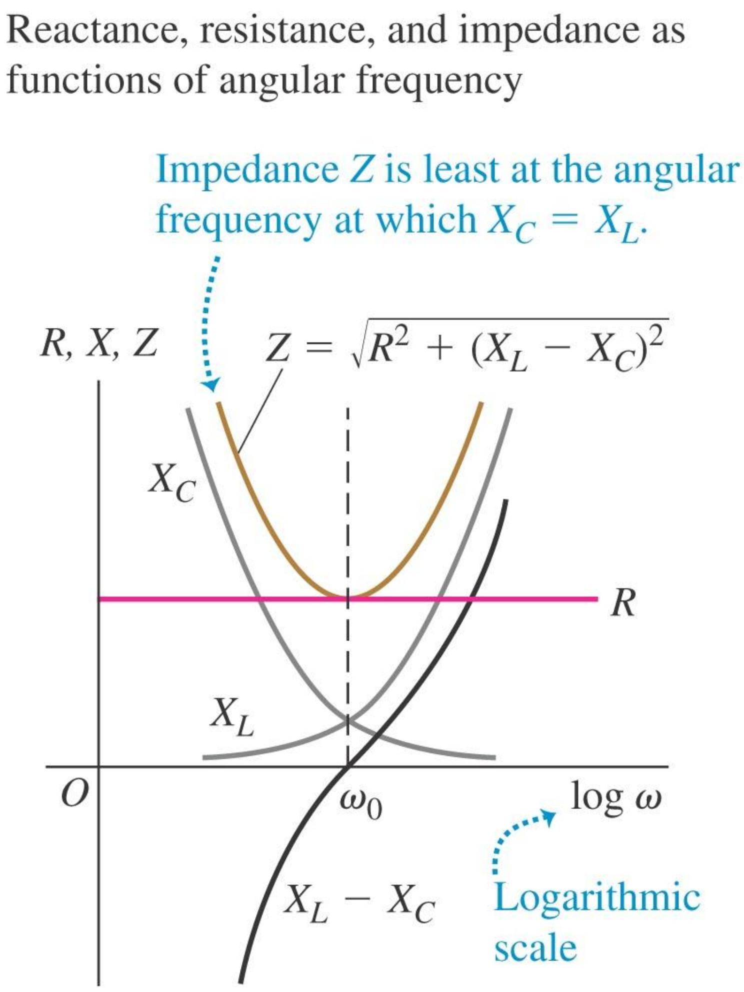
Power in an inductor

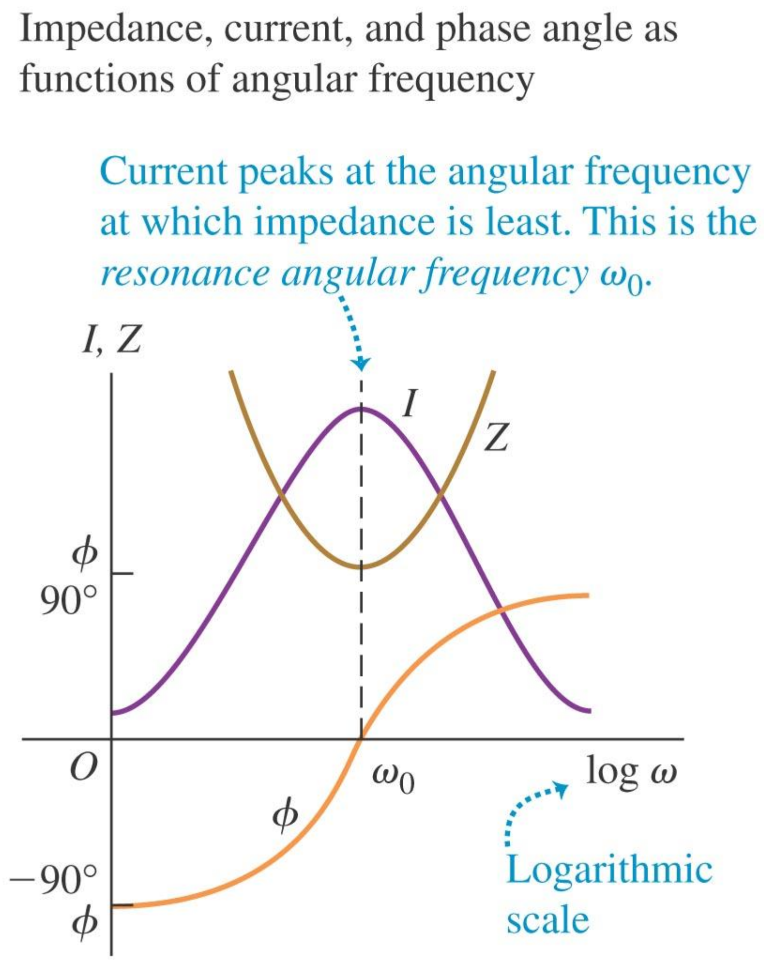
- Consider current, voltage, and power as functions of time.



Circuit behavior at resonance

- Look at the maximum I when the impedance is a minimum.





Tuning a radio – LRC circuit – tuned to station – AM/ FM radio