Effects of Matter

Matter is polarized by both electric and magnetic fields. The dipoles (both electric and magnetic) create their own additional E and B fields which add (or subtract) to externally applied field to form a total field that is modified from the applied field.

Maxwell's Equations of Electromagnetism

No different in matter than in vacuum BUT **E** and **B** are the TOTAL **E,B**

ρ is the total charge density and $\vec{J}$ is the total current density - these include internal effects of matter

1) $\nabla \bullet \vec{E} = \rho / \varepsilon_0$ (ρ = charge/volume or Coulomb/m^3) $\quad \oint_\Sigma \vec{E} \bullet d\vec{A} = Q_{enc} / \varepsilon_0$

2) $\nabla \bullet \vec{B} = 0 \quad \oint_\Sigma \vec{B} \bullet d\vec{A} = 0$

(no magnetic monopoles found yet - if they exist then $\nabla \bullet \vec{B} \propto \rho_m$ (Mag monopole density)

3) $\nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \oint_C \vec{E} \bullet d\vec{l} = -\frac{d}{dt} \int_\Sigma \vec{B} \bullet d\vec{A}$

4) $\nabla x \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J} \quad \oint_C \vec{B} \bullet d\vec{l} = \varepsilon_0 \mu_0 \frac{d}{dt} \int_\Sigma \vec{E} \bullet d\vec{A} + \mu_0 I_{enc}$

($\vec{J}$ = current density vector (amp/m^2) $I = \int_\Sigma \vec{J} \bullet d\vec{A}$)

Define Displacement Current as:

$$\vec{J}_D \equiv \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

4) $\nabla x \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J} = \mu_0 (\vec{J}_D + \vec{J}) \quad \oint_C \vec{B} \bullet d\vec{l} = \mu_0 I_D + \mu_0 I_{enc}$

Note that $\vec{J}$ is real current density while $\vec{J}_D$ is a ficticious current density

Note that Stokes Theorem for ANY well behaved vector field $\vec{F}$: $\quad \oint_C \vec{F} \bullet d\vec{l} = \int_\Sigma \nabla x \vec{F} \bullet d\vec{A}$

EM Wave Equation in Materials

→Note the matters terms $\frac{\partial \vec{J}}{\partial t} + \nabla(\rho_e / \varepsilon_0)$ and $\nabla x \vec{J}$

E field

$$\nabla x(\nabla x \vec{E}) = \nabla(\nabla \bullet \vec{E}) - \nabla^2(\vec{E}) = \nabla(\rho_e / \varepsilon_0) - \nabla^2(\vec{E})$$

$$= \nabla x(-\frac{\partial \vec{B}}{\partial t}) = -\frac{\partial(\nabla x \vec{B})}{\partial t} = -\frac{\partial(\varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J})}{\partial t} = -\varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial \vec{J}}{\partial t}$$

$$\rightarrow \nabla^2(\vec{E}) - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial \vec{J}}{\partial t} + \nabla(\rho_e / \varepsilon_0)$$

B field

$$\nabla x(\nabla x \vec{B}) = \nabla(\nabla \bullet \vec{B}) - \nabla^2(\vec{B}) = \nabla(0) - \nabla^2(\vec{B})$$

$$= \nabla x(\varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}) = \varepsilon_0 \mu_0 \frac{\partial(\nabla x \vec{E})}{\partial t} + \mu_0 \nabla x \vec{J} = -\varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} + \mu_0 \nabla x \vec{J}$$

$$\rightarrow \nabla^2(\vec{B}) - \varepsilon_0 \mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = -\mu_0 \nabla x \vec{J}$$

Polarized matter

No free charges in this case

We will allow free flow later

- E,B dipoles are formed by applied EM wave
- EM wave has time varying (oscillating) E,B
- Oscillating E,B → induced oscillating dipoles in matter
- Oscillating dipoles → accelerated charges
  - Can think if time varying dipole moments
- Oscillating charges → radiation
- Radiation is EM wave → adds/ subtracts
  - From applied EM wave
  - → new EM wave is sum of the applied and induced wave
- It is this effect (polarization) that is the basis of most optical properties you will deal with
- While BOTH E and B dipoles are allowed – most materials are dominated by the effect of electric dipole moments
  - Ex: “non magnetic doped” glass, plastic, water, air, …

Electric Dipoles in Matter

Consider neutral atoms and molecules that can be polarized

$\overrightarrow{E_a} =$ atomic/molecular E field induced by local (nearby) E field

$\overrightarrow{E_L} =$ local electric field due to all other polarized matter + external field

For class A dielectrics (most of the materials you deal with): $\overrightarrow{E_a} \propto \overrightarrow{E_L}$ BUT opposes it

$\overrightarrow{E_T} = \overrightarrow{E_L} + \overrightarrow{E_a}$ total electric field DUE to BOTH the polarized local matter field + rest of matter + external field

$\left\langle \overrightarrow{E_T} \right\rangle = \left\langle \overrightarrow{E_L} \right\rangle + \left\langle \overrightarrow{E_a} \right\rangle$ what we measure in matter

Note: $\left| \left\langle \overrightarrow{E_L} \right\rangle \right| > \left| \left\langle \overrightarrow{E_T} \right\rangle \right|$ since $\overrightarrow{E_a}$ opposes $\overrightarrow{E_L}$

Consider a uniform spherical charge distribution with density ρ

$$\oint_{\Sigma} \vec{E} \bullet d\vec{A} = 4\pi r^2 E(r) = Q_{enc} / \varepsilon_0 = \frac{4\pi}{3} r^3 \rho \rightarrow E(r) = \frac{r\rho}{3\varepsilon_0}$$

$\vec{E}(r)$ is radial from center of charge distribution

Atomic and Molecular Dipole

Two charges +-q set a distance vector apart of $\vec{l}$

Distance vector from +q to obs point is $\vec{r}\,'$

Distance vector from -q to obs point is $\vec{r}$

Potenital ϕ(r)=kq$(\frac{1}{r'}-\frac{1}{r})$ $k=\frac{1}{4\pi\varepsilon_0}$

$\vec{r}=\vec{r}\,'+\vec{l} \quad \vec{r}\,'=\vec{r}-\vec{l}$

$$r'=\left|\vec{r}\,'\right|=\left[\left(\vec{r}-\vec{l}\right)\bullet\left(\vec{r}-\vec{l}\right)\right]^{1/2}=\left[r^2+l^2-2\vec{r}\bullet\vec{l}\right]^{1/2}=r\left[1+l^2/r^2-2\vec{r}\bullet\vec{l}/r^2\right]^{1/2}$$

Assume distances r,r' are large compared to atomic distance l

$l^2/r^2\sim 0$

$$\rightarrow r'\sim r\left[1-2\vec{r}\bullet\vec{l}/r^2\right]^{1/2}\sim r(1-\vec{r}\bullet\vec{l}/r^2)$$

$$\rightarrow \frac{1}{r'}\sim\frac{1}{r}(1+\vec{r}\bullet\vec{l}/r^2)\rightarrow\frac{1}{r'}-\frac{1}{r}\sim\vec{r}\bullet\vec{l}/r^3)$$

$$\rightarrow\phi(\mathrm{r})=\mathrm{kq}(\frac{1}{r'}-\frac{1}{r})\sim k\frac{q\vec{r}\bullet\vec{l}}{r^3}$$

Define $\vec{\mathrm{p}}=q\vec{l}=$ electric dipole moment

$$\phi(\mathrm{r})\sim k\frac{\vec{\mathrm{p}}\bullet\vec{r}}{r^3}=k\vec{\mathrm{p}}\bullet\frac{\vec{r}}{r^3}=-k\vec{\mathrm{p}}\bullet\nabla(1/r)$$

Recall $\nabla(1/r)=-\frac{\vec{r}}{r^3}$ (spherical gradient)

Electric Field from Atomic Dipole

$\overrightarrow{E_a} =$ atomic/molecular E field induced by local (nearby) E field

$\overrightarrow{E_a} = -\nabla(\phi) = \nabla(k\vec{\mathrm{p}} \bullet \nabla(1/r)) \quad (r >> l)$

Average over volume V

$$\left\langle \overrightarrow{E_a} \right\rangle = \frac{1}{V}\int \overrightarrow{E_a} dV = \frac{k}{V}\int \nabla(\vec{\mathrm{p}} \bullet \nabla(1/r))dV = \frac{k}{V}\oint_{\Sigma} \vec{\mathrm{p}} \bullet \nabla(1/r) d\vec{A}$$

Recall $\int \nabla(f)dV = \oint_{\Sigma} f d\vec{A}$

Let the volume V be a sphere of radius b centered on a microscopic dipole

$\nabla(1/r) = -\frac{\vec{r}}{r^3} = -\frac{\vec{r}}{b^3}$ (at surface of sphere) $= -\frac{\hat{n}}{b^2}$ (where $\hat{n}$ is the radial outward normal vector)

$$\left\langle \overrightarrow{E_a} \right\rangle = \frac{k}{V}\oint_{\Sigma} \vec{\mathrm{p}} \bullet \nabla(1/r) d\vec{A} = \frac{-k}{b^2 V}\oint_{\Sigma} \vec{\mathrm{p}} \bullet \hat{n} d\vec{A} = \frac{-k}{b^2 V}\frac{1}{3}4\pi b^2 \vec{\mathrm{p}} = -\frac{\vec{\mathrm{p}}}{3\varepsilon_0 V}$$

Can show $\oint_{\Sigma} \vec{\mathrm{p}} \bullet \hat{n} d\vec{A} = \oint_{\Sigma} \vec{\mathrm{p}} \bullet \hat{n}\,\hat{n}\, dA = \frac{1}{3}4\pi b^2 \vec{\mathrm{p}}$

$\hat{n} = [\sin(\theta)\cos(\phi), \sin(\theta)\sin(\phi), \cos(\theta)]$, let $\vec{\mathrm{p}} = (0, 0, p)$ $(z\ \ axis\ \ p)$

$dA = b^2 \sin(\theta) d\theta d\phi$

$$\overrightarrow{N} \equiv \oint_{\Sigma} (\vec{\mathrm{p}} \bullet \hat{n})\hat{n}\, dA = \oint_{\Sigma} p\cos(\theta)\hat{n}\, dA = \oint_{\Sigma} p\cos(\theta)\hat{n}\, b^2 \sin(\theta) d\theta d\phi = pb^2 \oint_{\Sigma} \hat{n}\cos(\theta)\sin(\theta)d\theta d\phi$$

$N_x = N_y = 0$

$$N_z = pb^2 \oint_{\Sigma} \cos^2(\theta)\sin(\theta)d\theta d\phi = 2\pi pb^2 \int_0^{\pi} \cos^2(\theta)\sin(\theta)d\theta = -2\pi pb^2 \frac{\cos^3(\theta)}{3} 1_0^{\pi} = \frac{4\pi}{3} pb^2$$

(note error in notes - extra factor of b on Physics 141 - Notes 2.pdf)