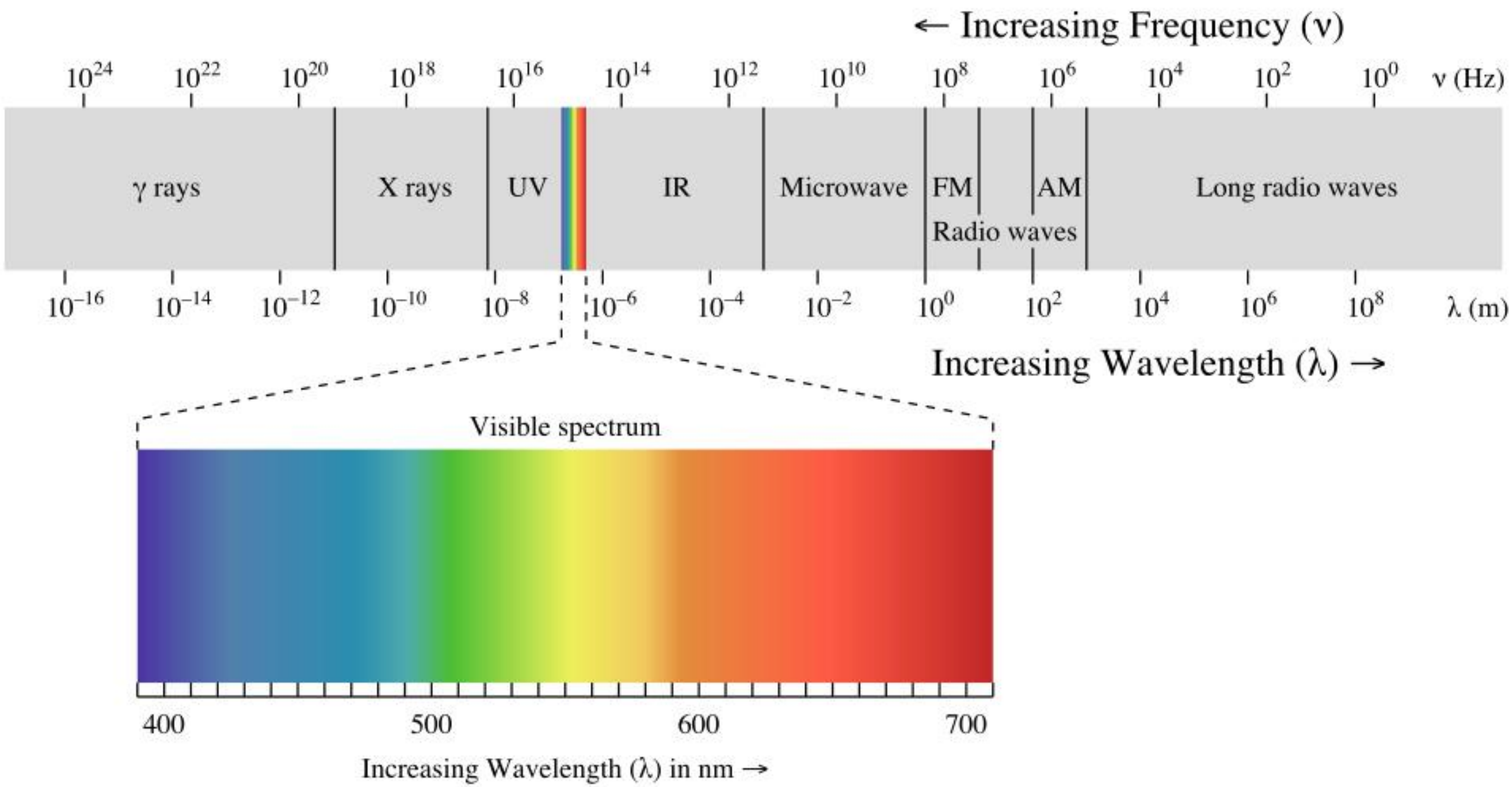
Electromagnetic Waves

Visible light, radio waves, cell communications, UV tanning, HDTV – these are all examples of EM waves



← Increasing Frequency (ν)
10^24
10^22
10^20
10^18
10^16
10^14
10^12
10^10
10^8
10^6
10^4
10^2
10^0
ν (Hz)
γ rays
X rays
UV
IR
Microwave
FM
AM
Long radio waves
Radio waves
10^−16
10^−14
10^−12
10^−10
10^−8
10^−6
10^−4
10^−2
10^0
10^2
10^4
10^6
10^8
λ (m)
Increasing Wavelength (λ) →
Visible spectrum
400
500
600
700
Increasing Wavelength (λ) in nm →

CLASS	FREQUENCY	WAVELENGTH	ENERGY
Y	300 EHz	1 pm	1.24 MeV
HX	30 EHz	10 pm	124 keV
SX	3 EHz	100 pm	12.4 keV
	300 PHz	1 nm	1.24 keV
EUV	30 PHz	10 nm	124 eV
NUV	3 PHz	100 nm	12.4 eV
NIR	300 THz	1 μm	1.24 eV
MIR	30 THz	10 μm	124 meV
FIR	3 THz	100 μm	12.4 meV
EHF	300 GHz	1 mm	1.24 meV
SHF	30 GHz	1 cm	124 μeV
UHF	3 GHz	1 dm	12.4 μeV
VHF	300 MHz	1 m	1.24 μeV
HF	30 MHz	10 m	124 neV
MF	3 MHz	100 m	12.4 neV
LF	300 kHz	1 km	1.24 neV
VLF	30 kHz	10 km	124 peV
VF/ULF	3 kHz	100 km	12.4 peV
SLF	300 Hz	1 Mm	1.24 peV
ELF	30 Hz	10 Mm	124 feV
	3 Hz	100 Mm	12.4 feV

Maxwell's Equations of Electromagnetism

No magnetic Monopoles

Changing B field induces E field and Changing E field induces B field

1) $\nabla \bullet \vec{E} = \rho / \varepsilon_0$ (ρ = charge/volume or Coulomb/m^3) $\quad \oint_\Sigma \vec{E} \bullet d\vec{A} = Q_{enc} / \varepsilon_0$

2) $\nabla \bullet \vec{B} = 0 \quad \oint_\Sigma \vec{B} \bullet d\vec{A} = 0$

(no magnetic monopoles found yet - if they exist then $\nabla \bullet \vec{B} \propto \rho_m$ (Mag monopole density)

3) $\nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad \oint_C \vec{E} \bullet d\vec{l} = -\frac{d}{dt}\int_\Sigma \vec{B} \bullet d\vec{A}$

4) $\nabla x \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J} \quad \oint_C \vec{B} \bullet d\vec{l} = \varepsilon_0 \mu_0 \frac{d}{dt}\int_\Sigma \vec{E} \bullet d\vec{A} + \mu_0 I_{enc}$

($\vec{J}$ = current density vector (amp/m^2) $\; I = \int_\Sigma \vec{J} \bullet d\vec{A}$)

Define Displacement Current as:

$$\vec{J}_D \equiv \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

4) $\nabla x \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J} = \mu_0 (\vec{J}_D + \vec{J}) \quad \oint_C \vec{B} \bullet d\vec{l} = \mu_0 I_D + \mu_0 I_{enc}$

Note that $\vec{J}$ is real current density while $\vec{J}_D$ is a ficticious current density

Note that Stokes Theorem for ANY well behaved vector field F: $\quad \oint_C \vec{F} \bullet d\vec{l} = \int_\Sigma \nabla x \vec{F} \bullet d\vec{A}$

Maxwell's Equations of Electromagnetism

Allow existence of magnetic monopoles

1) $\nabla \bullet \vec{E} = \rho / \varepsilon_0$ (ρ = charge/volume or Coulomb/m^3) $\quad \oint_\Sigma \vec{E} \bullet d\vec{A} = Q_{enc} / \varepsilon_0$

2) $\nabla \bullet \vec{B} = \mu_0 \rho_m$ (IF magnetic monopoles) $\quad \oint_\Sigma \vec{B} \bullet d\vec{A} = \mu_0 Q_{m-enc}$

3) $\nabla x \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{J}_m$ (IF magnetic monopoles) $\quad \oint_C \vec{E} \bullet d\vec{l} = -\frac{d}{dt} \int_\Sigma \vec{B} \bullet d\vec{A} - I_{m-enc}$

4) $\nabla x \vec{B} = \varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}$ $\quad \oint_C \vec{B} \bullet d\vec{l} = \varepsilon_0 \mu_0 \frac{d}{dt} \int_\Sigma \vec{E} \bullet d\vec{A} + \mu_0 I_{enc}$

($\vec{J}$ = current density (flux) vector (amp/m^2) $I = \int_\Sigma \vec{J} \bullet d\vec{A}$) - can also be a mag monopole current

Force Law $\vec{F} = q_e(\vec{E} + \vec{v}\, x\, \vec{B}) + q_m(\vec{B} - \frac{\vec{v}}{c^2}\, x\, \vec{E})$ (right/left hand rule for electric/magnetic charges)

Note that Divergence (Gauss' Law) for ANY well behaved vector field $\vec{\mathrm{F}}$:

$$\oint_\Sigma \vec{F} \bullet d\vec{A} = \int_V \nabla \bullet \vec{F}\, dV \rightarrow \text{continuity eq for flux field } \vec{\mathrm{F}} \rightarrow \nabla \bullet \vec{F} + \partial \rho / \partial t = 0$$

Units – not just MKS

What about charge? – Farad's – Henry's

$$\mu_0 \equiv 4\pi \times 10^{-7}\ \text{H/m}$$

$$H = Henry$$

What is a Henry (unit of inductance)?

Recall:

V=Ldi/dt

L=inductance and units are in Henry's

V has units of J/Q (Joule/ Coulomb)

di/dt has units of $Coul/\text{s}^2$

$$H \equiv L \equiv \frac{\text{J/Coul}}{\text{Coul/s}^2} = \frac{\text{J-s}^2}{Coul^2} = \frac{\text{kg-m}^2}{Coul^2} = \text{Henry}$$

$$\epsilon_0 \cong 8.854 \times 10^{-12}\ \text{F/m}\ \ (\text{Farad/m})$$

What is a Farad (unit of capacitance C)?

$$F \equiv C \equiv \frac{Q}{V} = \frac{\text{Coul}}{\text{J/Coul}} = \frac{Coul^2}{\text{J}} = \frac{Coul^2\text{-s}^2}{\text{kg-m}^2} = F\text{arad}$$

$$\mu_0 \equiv 4\pi \times 10^{-7}\ \text{kg-m-Coul}^{-2} (H/m)$$

$$\epsilon_0 \cong 8.854 \times 10^{-12}\ \text{Coul}^2\text{s}^2\text{kg}^{-1}\text{m}^{-3} (F/m)$$

$$Farad * Henry = s^2$$

EM Wave Equation in General – Allow Mag Mono

$\nabla x(\nabla x\vec{F}) = \nabla(\nabla \bullet \vec{F}) - \nabla^2(\vec{F})$ for any well behaved vector field $\vec{F}(\vec{x},t)$

Well behave means no singularities and differentiable in space and time

There is no Physics here - just mathematics

Now let $\vec{F}(\vec{x},t)$ be the $\vec{E}$ or $\vec{B}$ field

E field

$$\nabla x(\nabla x\vec{E}) = \nabla(\nabla \bullet \vec{E}) - \nabla^2(\vec{E}) = \nabla(\rho_e / \varepsilon_0) - \nabla^2(\vec{E})$$

$$= \nabla x(-\frac{\partial \vec{B}}{\partial t} - \vec{J}_m) = -\frac{\partial(\nabla x\vec{B})}{\partial t} - \nabla x\vec{J}_m = -\frac{\partial(\varepsilon_0\mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0\vec{J})}{\partial t} - \nabla x\vec{J}_m = -\varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial \vec{J}}{\partial t} - \nabla x\vec{J}_m$$

$$\rightarrow \nabla^2(\vec{E}) - \varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial \vec{J}}{\partial t} + \nabla x\vec{J}_m + \nabla(\rho_e) / \varepsilon_0$$

$$\rightarrow \text{(in vacuum and no mag monopoles)} \;\; \nabla^2(\vec{E}) - \varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

B field

$$\nabla x(\nabla x\vec{B}) = \nabla(\nabla \bullet \vec{B}) - \nabla^2(\vec{B}) = \nabla(\mu_0\rho_m) - \nabla^2(\vec{B})$$

$$= \nabla x(\varepsilon_0\mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0\vec{J}) = \varepsilon_0\mu_0 \frac{\partial(\nabla x\vec{E})}{\partial t} + \mu_0\nabla x\vec{J} = -\varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} - \varepsilon_0\mu_0 \frac{\partial \overrightarrow{J_m}}{\partial t} + \mu_0\nabla x\vec{J}$$

$$\rightarrow \nabla^2(\vec{B}) - \varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = \varepsilon_0\mu_0 \frac{\partial \overrightarrow{J_m}}{\partial t} - \mu_0\nabla x\vec{J} + \mu_0\nabla(\rho_m)$$

$$\rightarrow \text{(in vacuum and no mag monopoles)} \;\; \nabla^2(\vec{B}) - \varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

Maxwell's Equation in Vacuum

No charges – No mag monopoles

1) $\nabla \bullet \vec{E} = 0$ $\qquad$ $\oint_{\Sigma} \vec{E} \bullet d\vec{A} = 0$

2) $\nabla \bullet \vec{B} = 0$ $\qquad$ $\oint_{\Sigma} \vec{B} \bullet d\vec{A} = 0$

3) $\nabla x \vec{E} = -\dfrac{\partial \vec{B}}{\partial t}$ $\qquad$ $\oint_{C} \vec{E} \bullet d\vec{l} = -\dfrac{d}{dt}\int_{\Sigma} \vec{B} \bullet d\vec{A}$

4) $\nabla x \vec{B} = \varepsilon_0 \mu_0 \dfrac{\partial \vec{E}}{\partial t}$ $\qquad$ $\oint_{C} \vec{B} \bullet d\vec{l} = \varepsilon_0 \mu_0 \dfrac{d}{dt}\int_{\Sigma} \vec{E} \bullet d\vec{A}$

EM Wave Equation in Materials

No Magnetic Monopoles

E field

$$\nabla x(\nabla x\vec{E}) = \nabla(\nabla \bullet \vec{E}) - \nabla^2(\vec{E}) = \nabla(\rho_e / \varepsilon_0) - \nabla^2(\vec{E})$$

$$= \nabla x(-\frac{\partial \vec{B}}{\partial t}) = -\frac{\partial(\nabla x\vec{B})}{\partial t} = -\frac{\partial(\varepsilon_0\mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J})}{\partial t} = -\varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} - \mu_0 \frac{\partial \vec{J}}{\partial t}$$

$$\rightarrow \nabla^2(\vec{E}) - \varepsilon_0\mu_0 \frac{\partial^2 \vec{E}}{\partial t^2} = \mu_0 \frac{\partial \vec{J}}{\partial t} + \nabla(\rho_e / \varepsilon_0)$$

B field

$$\nabla x(\nabla x\vec{B}) = \nabla(\nabla \bullet \vec{B}) - \nabla^2(\vec{B}) = \nabla(0) - \nabla^2(\vec{B})$$

$$= \nabla x(\varepsilon_0\mu_0 \frac{\partial \vec{E}}{\partial t} + \mu_0 \vec{J}) = \varepsilon_0\mu_0 \frac{\partial(\nabla x\vec{E})}{\partial t} + \mu_0 \nabla x\vec{J} = -\varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} + \mu_0 \nabla x\vec{J}$$

$$\rightarrow \nabla^2(\vec{B}) - \varepsilon_0\mu_0 \frac{\partial^2 \vec{B}}{\partial t^2} = -\mu_0 \nabla x\vec{J}$$

Wave Equations in Materials

No free charges or currents

Homogeneous Wave Equation

$$\nabla^2 \vec{E} - \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} = 0 \qquad \nabla^2 \vec{B} - \mu\varepsilon \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

$$\mu = \mu_m \mu_0$$

$$\mu_0 = \text{free space permeability}$$

$$\mu_m = \kappa_m = \text{magnetic material relative permeability}$$

$$\varepsilon = \varepsilon_r \varepsilon_0$$

$$\varepsilon_0 = \text{free space permittivity}$$

$$\varepsilon_r = \kappa_e = \text{dielectric material relative permittivity}$$

$$\text{Wave Speed } v = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0\varepsilon_0}\sqrt{\mu_m\varepsilon_r}} = \frac{c}{\sqrt{\mu_m\varepsilon_r}} = \frac{c}{n}$$

$$n = \sqrt{\mu_m\varepsilon_r} = \text{"effective index of refraction"}$$

Solutions for Wave Equation

Any well behaved function is a solution of the homogeneous wave equation i it has the following form:
$f(\vec{k}\bullet\vec{x}\pm\omega t)$ where $\vec{k}$ is called the "wave vector" and ω is the angular frequency $\omega = 2\pi f$ & $k = 2\pi/\lambda$
The homogeneous wave function is linear so a sum of solutions is a solution.
We can thus form basis sets of solutions

Any well behaved function can be written as a Fourier series
We can use sin/co functions as a basis set by taking a discrete or continuous
Fourier transform of $f(\vec{k}\bullet\vec{x}\pm\omega t)$
We can write a general solution set consisting of the sum of the following:
$\vec{E}(\vec{x},t)=\vec{E}_0\sin(\vec{k}\bullet\vec{x}\pm\omega t)$
$\vec{B}(\vec{x},t)=\vec{B}_0\sin(\vec{k}\bullet\vec{x}\pm\omega t)$
Use $\sin(\vec{k}\bullet\vec{x}-\omega t)$ for simplicity
$\vec{E}_0, \vec{B}_0$ are amplitude vectors

No free charges solution:
1) $\nabla\bullet\vec{E}=0\rightarrow\vec{k}\bullet\vec{E}=\vec{k}\bullet\vec{E}_0=0\rightarrow\vec{k}\perp\vec{E}_0$
2) $\nabla\bullet\vec{B}=0\rightarrow\vec{k}\bullet\vec{B}=\vec{k}\bullet\vec{B}_0=0\rightarrow\vec{k}\perp\vec{B}_0$

$\nabla^2(\vec{E})-\varepsilon_0\mu_0\dfrac{\partial^2\vec{E}}{\partial t^2}=0\rightarrow k^2=\varepsilon_0\mu_0\omega^2=\omega^2/c^2$ (this is called a dispersion relation where ω=ck)
$\nabla^2(\vec{B})-\varepsilon_0\mu_0\dfrac{\partial^2\vec{B}}{\partial t^2}=0\rightarrow same$
3) $\nabla x\vec{E}=-\dfrac{\partial\vec{B}}{\partial t}\rightarrow\vec{k}\,x\,\vec{E}_0=\omega\vec{B}_0\rightarrow\vec{B}_0=\dfrac{k}{\omega}(\hat{k}\,x\,\vec{E}_0)\rightarrow\vec{B}_0\perp\vec{E}_0$ & $E_0=cB_0\rightarrow 1Tesla=3x10^8\,Volt/m$
4) $\nabla x\vec{B}=\varepsilon_0\mu_0\dfrac{\partial\vec{E}}{\partial t}\rightarrow\vec{k}\,x\,\vec{B}_0=\varepsilon_0\mu_0\omega\vec{E}_0\rightarrow\vec{E}_0=\varepsilon_0\mu_0\dfrac{\omega}{k}(\hat{k}\,x\,\vec{B}_0)=\dfrac{\omega}{c^2k}(\hat{k}\,x\,\vec{B}_0)\rightarrow\vec{E}_0\perp\vec{B}_0$
$\rightarrow\vec{k},\vec{E}_0,\vec{B}_0$ are all perpendicular to each other with $\vec{k}\propto\vec{E}_0\,x\,\vec{B}_0$, E and B always in phase

Poynting Vector

It Points in the Direction of Energy Flow

Assume Free Space (J=0 - no free currents)

Some math:

$$\nabla \bullet (\vec{E}\, x\, \vec{B}) = \vec{B} \bullet (\nabla\, x\, \vec{E}) - \vec{E} \bullet (\nabla\, x\, \vec{B}) = \vec{B} \bullet (-\frac{\partial \vec{B}}{\partial t}) - \vec{E} \bullet (\varepsilon_0 \mu_0 \frac{\partial \vec{E}}{\partial t}) = -\mu_0 \frac{\partial}{\partial t}[\frac{B^2}{2\mu_0} + \varepsilon_0 \frac{E^2}{2}]$$

Add some Physics:

ρ_M =Magnetic field energy density (J/m^3)= $\dfrac{B^2}{2\mu_0}$

ρ_E =Electric field energy density (J/m^3)= $\varepsilon_0 \dfrac{E^2}{2}$

Define the Poynting vector $\vec{S}(\vec{x},t) = \dfrac{1}{\mu_0}(\vec{E}(\vec{x},t)\, x\, \vec{B}(\vec{x},t))$ (units are watts/m^2)

$\vec{S}(\vec{x},t) \propto \hat{k} \rightarrow \hat{k}$ points along wave propagation $\rightarrow \vec{S}$ is along direction of energy flow

$$\left|\vec{S}(\vec{x},t)\right| = \frac{1}{\mu_0}\left|(\vec{E}(\vec{x},t)\, x\, \vec{B}(\vec{x},t))\right| = \frac{1}{\mu_0}\left|\vec{E}(\vec{x},t)\right|\left|\vec{B}(\vec{x},t)\right| = \frac{1}{c\mu_0}\mathrm{E}_0^{\,2}\sin^2(\vec{k} \bullet \vec{x} \pm \omega t)$$

Continuity equation (use divergence theorem):

$$\oint_\Sigma \vec{S} \bullet d\vec{A} = \int_V \nabla \bullet \vec{S}\, dV = -\mu_0 \frac{\partial}{\partial t}\int_V (\rho_M + \rho_E)\, dV \rightarrow \nabla \bullet \vec{S} + \partial\rho / \partial t = 0$$

$$\rho = \rho_M + \rho_E$$

Time Averaged Poynting Vector

$$\left|\vec{S}(\vec{x},t)\right| = \frac{1}{\mu_0}\left|(\vec{E}(\vec{x},t)\, x\, \vec{B}(\vec{x},t))\right| = \frac{1}{\mu_0}\left|\vec{E}(\vec{x},t)\right|\left|\vec{B}(\vec{x},t)\right| = \frac{1}{c\mu_0}\vec{E}_0{}^2\sin^2(\vec{k}\bullet\vec{x}\pm\omega t)$$

$\left|\vec{S}(\vec{x},t)\right|$ oscillates at twice the frequency of the EM wave

For example for visible light at λ=0.55 microns (peak of human vision)

$\rightarrow f = 545$ THz $\rightarrow$ Poynting vector oscillates at 1090 THz

This is too fast for virtually normal detectors

Time average of Poyting vector is useful in practical measurements

$$\left\langle\left|\vec{S}(\vec{x},t)\right|\right\rangle = \left\langle\frac{1}{c\mu_0}\vec{E}_0{}^2\sin^2(\vec{k}\bullet\vec{x}\pm\omega t)\right\rangle = \frac{1}{c\mu_0}\left|\vec{E}_0{}^2\right|\left\langle\sin^2(\vec{k}\bullet\vec{x}\pm\omega t)\right\rangle = \frac{1}{2c\mu_0}\left|\vec{E}_0{}^2\right|$$

$$\left\langle\sin^2(\vec{k}\bullet\vec{x}\pm\omega t)\right\rangle = 1/2$$

$$\rightarrow\left|\vec{E}_0{}^2\right| = 2c\mu_0\left\langle\left|\vec{S}(\vec{x},t)\right|\right\rangle \rightarrow \left|\vec{E}_0\right| = \sqrt{2c\mu_0\left\langle\left|\vec{S}(\vec{x},t)\right|\right\rangle} = 27.5\sqrt{\left\langle\left|\vec{S}(\vec{x},t)\right|\right\rangle}$$

Example : Solar insolation (flux from sun)

1) At surface of Earth $\left\langle\left|\vec{S}(\vec{x},t)\right|\right\rangle \sim 1000 w/m^2 \rightarrow \left|\vec{E}_0\right| \sim 868\ volts/m$

2) At top of atmosphere $\left\langle\left|\vec{S}(\vec{x},t)\right|\right\rangle \sim 1350 w/m^2 \rightarrow \left|\vec{E}_0\right| \sim 1010\ volts/m$

Wave Equations

$$\nabla^2 \mathbf{E} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla^2 \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2}$$

This is similar to a scalar wave equation where c is the speed of the wave

$$\nabla^2 f = \frac{1}{{c_0}^2} \frac{\partial^2 f}{\partial t^2}$$

$$c_0 = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$$

Solutions to the Wave Equations

The following is a solution for ANY well behaved function f
K is called the wave vector – here it is a unit vector that points in the direction of wave propagation

$$v = f\lambda$$

$$\mathbf{E} = \mathbf{E}_0 f\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right)$$

$$\nabla \cdot \mathbf{E} = \hat{\mathbf{k}} \cdot \mathbf{E}_0 f'\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right) = 0 \qquad \mathbf{E} \cdot \hat{\mathbf{k}} = 0$$

$$\nabla \times \mathbf{E} = \hat{\mathbf{k}} \times \mathbf{E}_0 f'\left(\hat{\mathbf{k}} \cdot \mathbf{x} - c_0 t\right) = -\frac{\partial \mathrm{B}}{\partial t} \qquad \mathbf{B} = \frac{1}{c_0}\hat{\mathbf{k}} \times \mathbf{E}$$

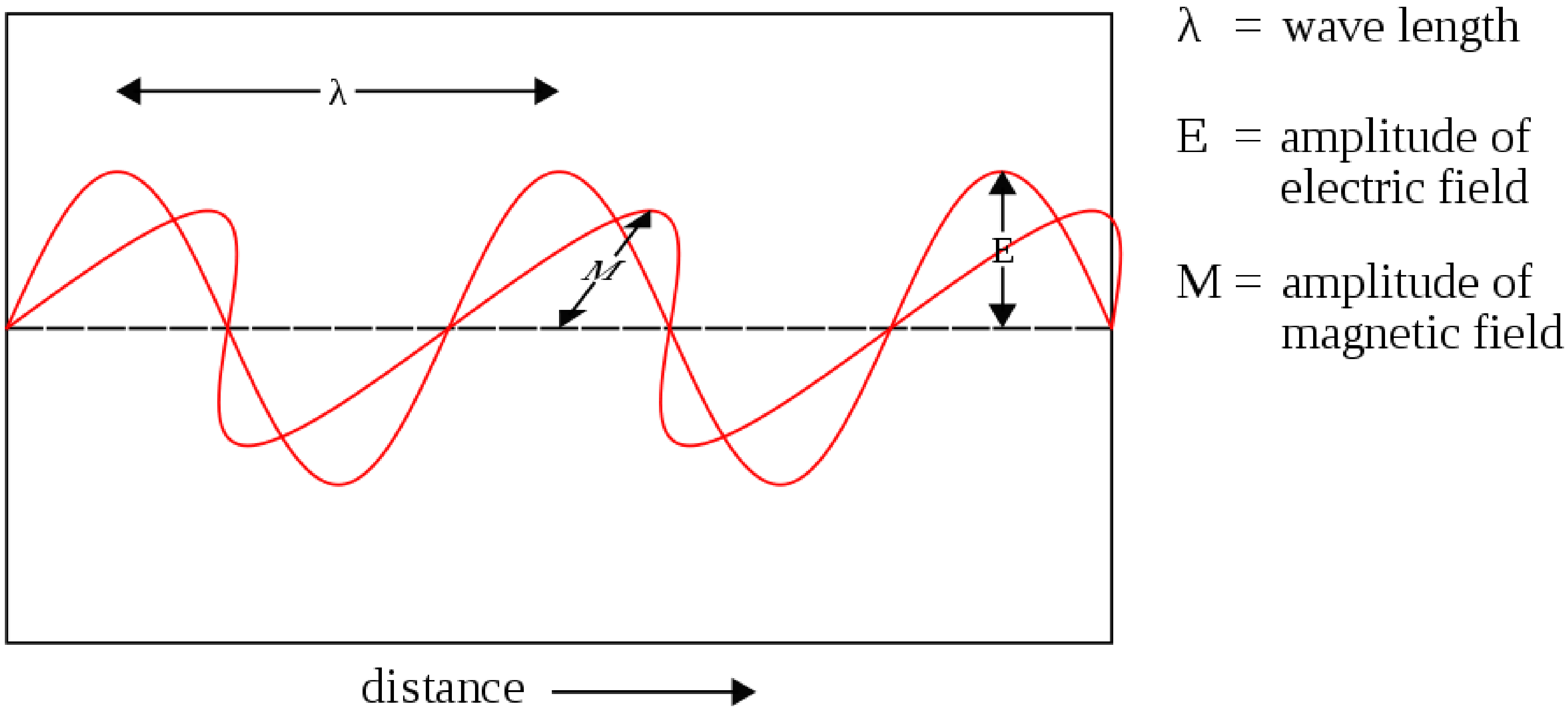
E, B and K form a mutual orthogonal system

E, B (M) and wavelength

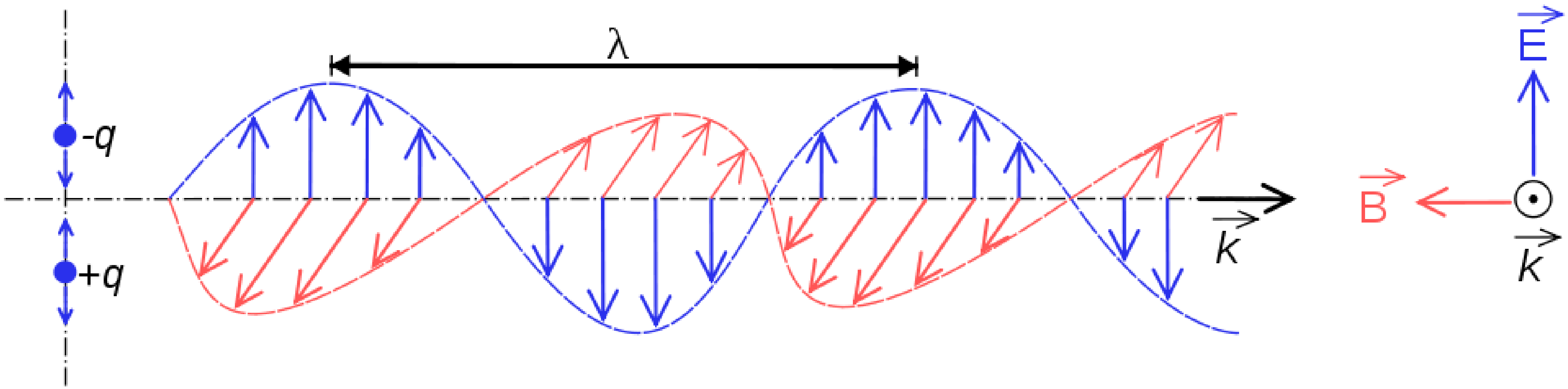
$$K = 2\pi/\lambda$$

$$v = f\lambda$$

Light wave



E, B and K – Right handed coordinate system
E and B are "in phase"



Properties of electromagnetic waves

- Maxwell's equations imply that in an electromagnetic wave, both the electric and magnetic fields are perpendicular to the direction of propagation of the wave, and to each other.
- In an electromagnetic wave, there is a definite ratio between the magnitudes of the electric and magnetic fields: $E = cB$.
- Unlike mechanical waves, electromagnetic waves require no medium. In fact, they travel in vacuum with a definite and unchanging speed:
- Inserting the numerical values of these constants, we obtain
  $c = 3.00 \times 108$ m/s.

Speed of electromagnetic waves in vacuum

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$$

Electric constant (ϵ_0)

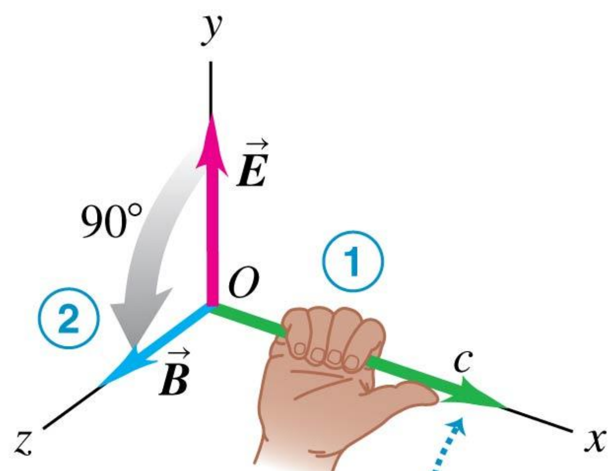
Magnetic constant (μ_0)

Properties of electromagnetic waves

- The direction of propagation of an electromagnetic wave is the direction of the vector product of the electric and magnetic fields.

Right-hand rule for an electromagnetic wave:

1. Point the thumb of your right hand in the wave's direction of propagation.
2. Imagine rotating the $\vec{E}$-field vector 90° in the sense your fingers curl. That is the direction of the $\vec{B}$ field.



Direction of propagation = direction of $\vec{E} \times \vec{B}$.

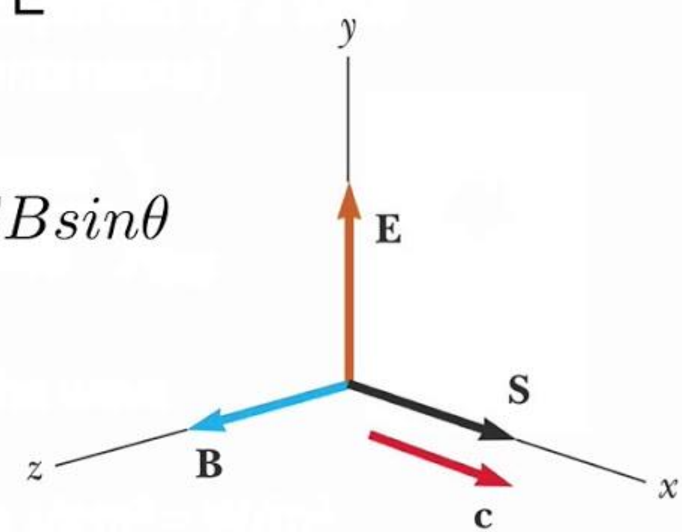
The Poyting Vector
Units are Flux (w/m^2)

The Poynting Vector

- The Poynting vector depends on E and B

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}, \text{ so } \quad S = \frac{1}{\mu_0} EB\sin\theta$$

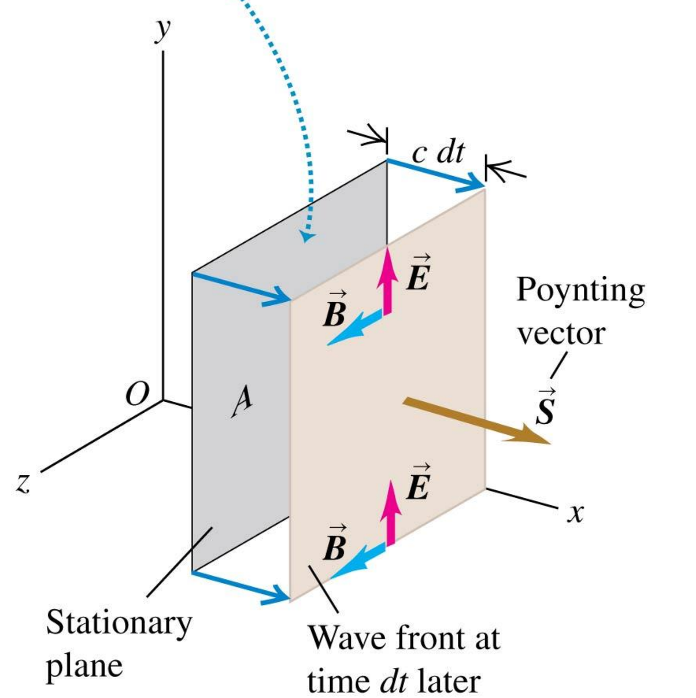
- Its direction is the direction of propagation
- This is time dependent
  - Its magnitude varies in time
  - Its magnitude reaches a maximum at the same instant as **E** and **B**



$$\theta = 90°, \text{ so } \quad S = \frac{1}{\mu_0} EB$$

Energy in electromagnetic waves

- Electromagnetic waves such as those we have described are traveling waves that transport energy from one region to another.
- The British physicist John Poynting introduced the **Poynting vector** $\vec{S}$
- The magnitude of the Poynting vector is the power per unit area in the wave, and it points in the direction of propagation.



Poynting vector in vacuum

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

Electric field ($\vec{E}$); Magnetic field ($\vec{B}$); Magnetic constant (μ_0)

Energy in electromagnetic waves

- The magnitude of the average value of is called the **intensity**. The SI unit of intensity is 1 W/m^2.
- These rooftop solar panels are tilted to be face-on to the sun so that the panels can absorb the maximum amount of wave energy.

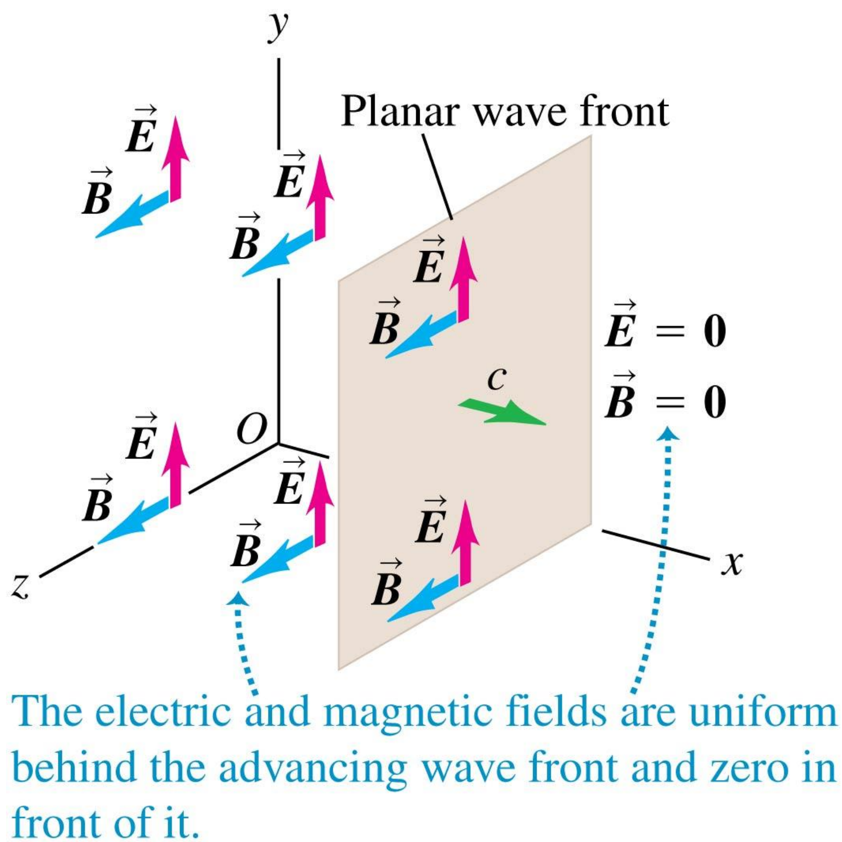
Intensity of a sinusoidal electromagnetic wave in vacuum

$$I = S_{av} = \frac{E_{max}B_{max}}{2\mu_0} = \frac{E_{max}{}^2}{2\mu_0 c} = \frac{1}{2}\sqrt{\frac{\epsilon_0}{\mu_0}}E_{max}{}^2 = \frac{1}{2}\epsilon_0 c E_{max}{}^2$$

Electric-field amplitude; Magnetic-field amplitude; Electric constant; Magnitude of average Poynting vector; Magnetic constant; Speed of light in vacuum

A simple plane electromagnetic wave

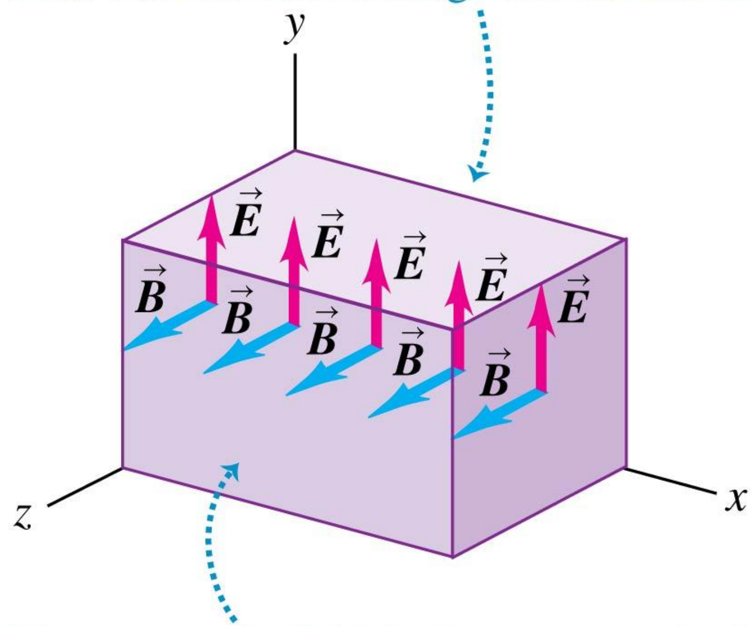
- To begin our study of electromagnetic waves, imagine that all space is divided into two regions by a plane perpendicular to the *x*-axis.
- At every point to the left of this plane there are uniform electric field magnetic fields as shown.
- The boundary plane, which we call the **wave front**, moves in the +*x*-direction with a constant speed *c*.



Gauss's laws and the simple plane wave

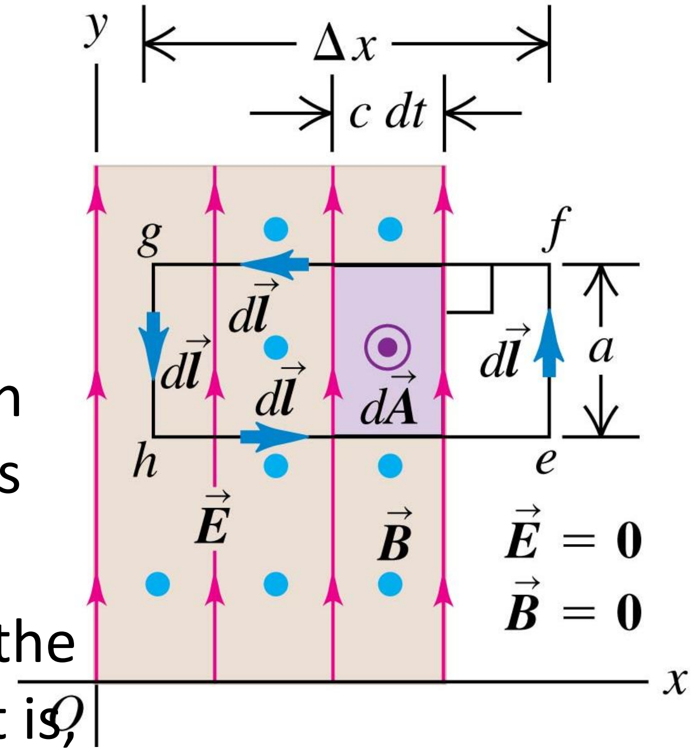
- Shown is a Gaussian surface, a rectangular box, through which the simple plane wave is traveling.
- The box encloses no electric charge.
- In order to satisfy Maxwell's first and second equations, the electric and magnetic fields must be perpendicular to the direction of propagation; that is, the wave must be **transverse**.

The electric field is the same on the top and bottom sides of the Gaussian surface, so the total electric flux through the surface is zero.



The magnetic field is the same on the left and right sides of the Gaussian surface, so the total magnetic flux through the surface is zero.

Faraday’s law and the simple plane wave



- The simple plane wave must satisfy Faraday’s law in a vacuum.
- In a time *dt*, the magnetic flux through the rectangle in the *xy*-plane increases by an amount $d\Phi_B$.
- This increase equals the flux through the shaded rectangle with area *ac dt*; that is, $d\Phi_B = Bac\,dt$.
- Thus $d\Phi_B/dt = Bac$.
- This and Faraday’s law imply:

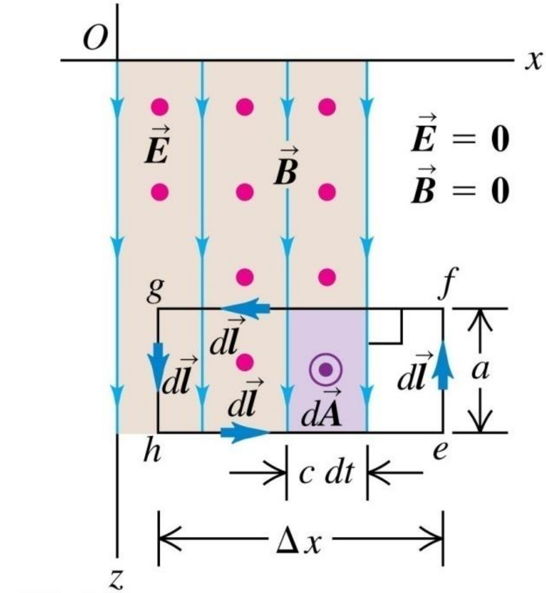
Electromagnetic wave in vacuum:

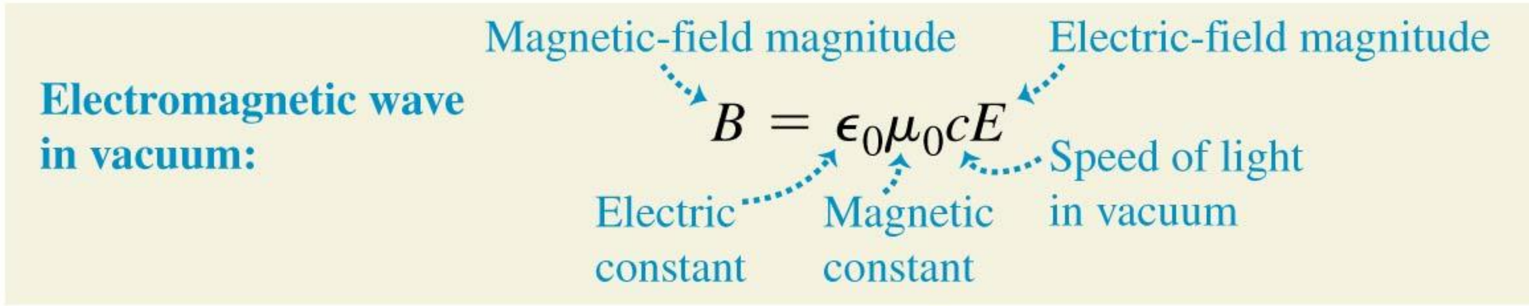
$$E = cB$$

Electric-field magnitude (E); Magnetic-field magnitude (B); Speed of light in vacuum (c)

Ampere's law and the simple plane wave

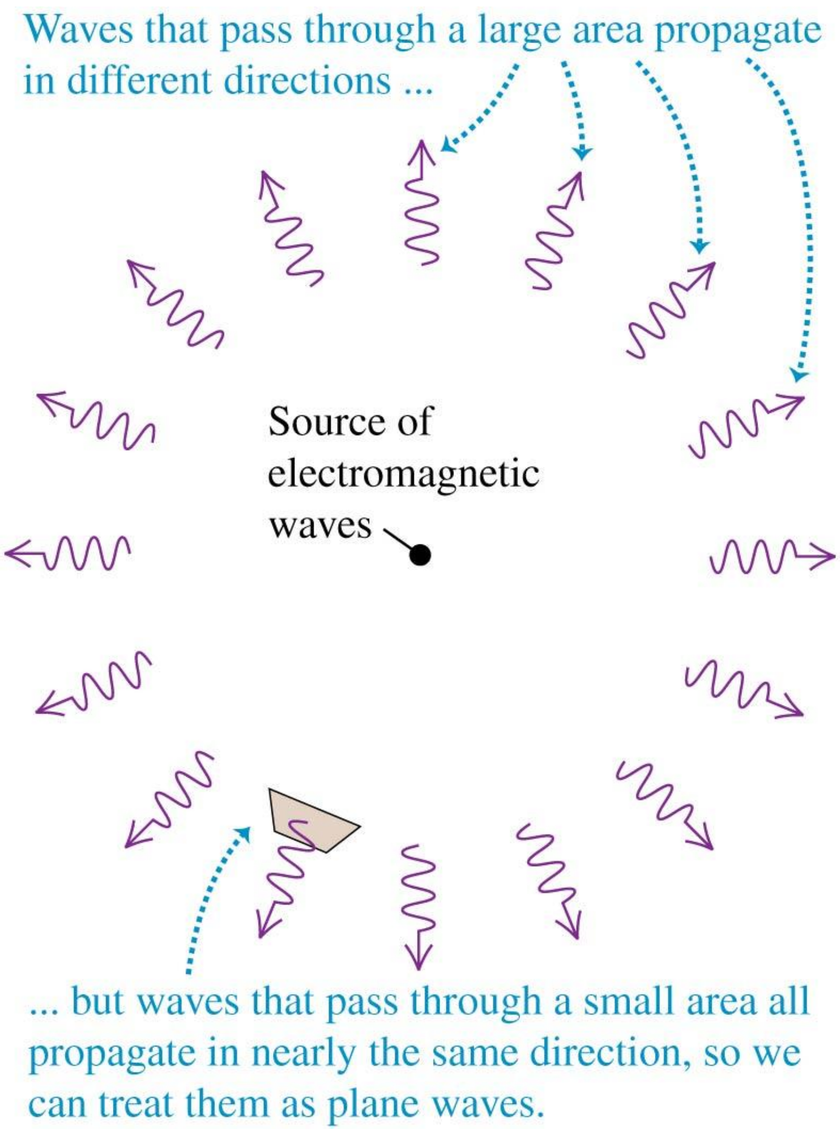
- The simple plane wave must satisfy Ampere's law in a vacuum.
- In a time *dt*, the electric flux through the rectangle in the *xz*-plane increases by an amount $d\Phi_E$.
- This increase equals the flux through the shaded rectangle with area *ac dt*; that is, $d\Phi_E = Eac\,dt$.
- Thus $d\Phi_E/dt = Eac$. This implies E=Bc:





Sinusoidal electromagnetic waves

- Electromagnetic waves produced by an oscillating point charge are an example of sinusoidal waves that are not plane waves.
- But if we restrict our observations to a relatively small region of space at a sufficiently great distance from the source, even these waves are well approximated by plane waves.



Fields of a sinusoidal wave

- We can describe electromagnetic waves by means of wave functions:

Sinusoidal electromagnetic plane wave, propagating in +x-direction:

$$\vec{E}(x,t) = \hat{j}E_{max}\cos(kx - \omega t)$$
$$\vec{B}(x,t) = \hat{k}B_{max}\cos(kx - \omega t)$$

Electric field; Electric-field magnitude; Wave number; Angular frequency; Magnetic field; Magnetic-field magnitude

- The wave travels to the right with speed $c = \omega/k$.
- The amplitudes must be related by:

Sinusoidal electromagnetic wave in vacuum:

$$E_{max} = cB_{max}$$

Electric-field amplitude; Magnetic-field amplitude; Speed of light in vacuum

Electromagnetic waves in matter

- Electromagnetic waves can travel in certain types of matter, such as air, water, or glass.
- When electromagnetic waves travel in nonconducting materials—that is, dielectrics—the speed *v* of the waves depends on the dielectric constant of the material.

$$v = \frac{1}{\sqrt{\epsilon\mu}} = \frac{1}{\sqrt{KK_m}}\frac{1}{\sqrt{\epsilon_0\mu_0}} = \frac{c}{\sqrt{KK_m}}$$

Speed of electromagnetic waves in a dielectric (v); Permittivity (ϵ); Permeability (μ); Dielectric constant (K); Relative permeability (K_m); Electric constant (ϵ_0); Magnetic constant (μ_0); Speed of light in vacuum (c)

- The ratio of the speed *c* in vacuum to the speed *v* in a material is known in optics as the **index of refraction** *n* of the material.

$$\frac{c}{v} = n = \sqrt{KK_m} \cong \sqrt{K}$$

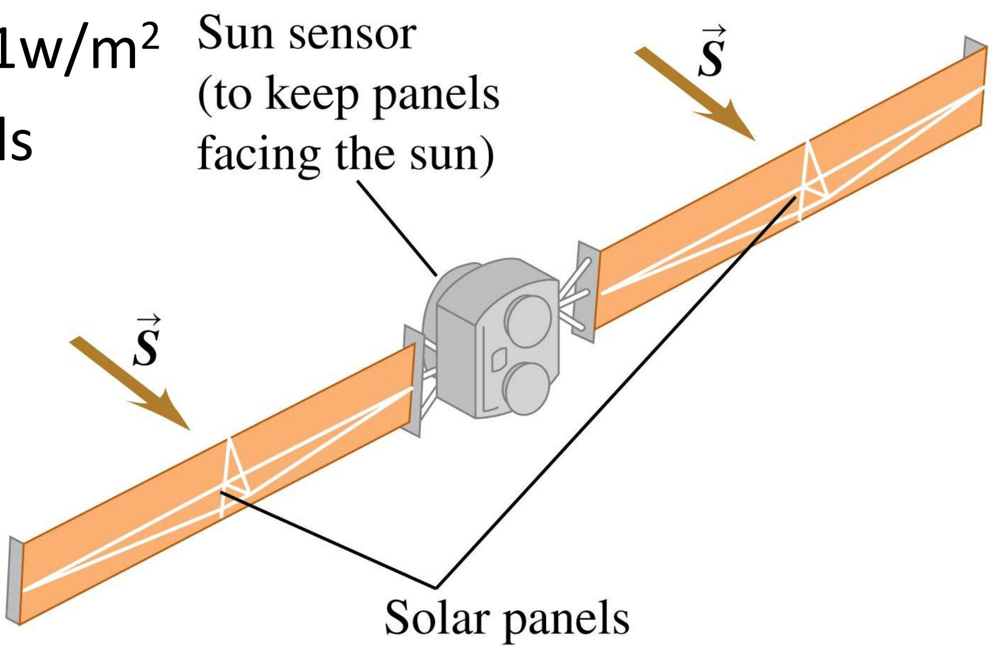
Electromagnetic radiation pressure

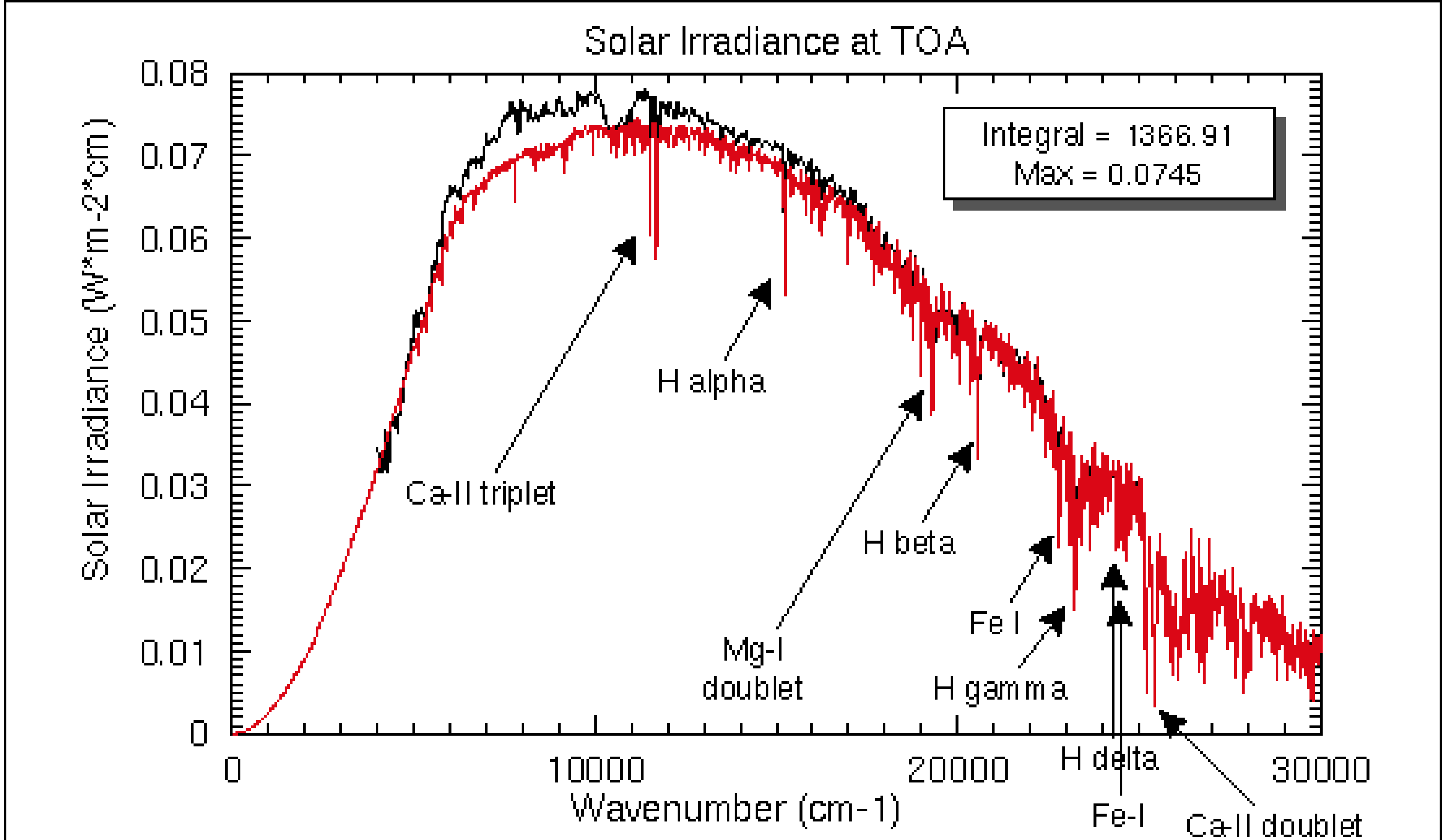
- Electromagnetic waves carry momentum and can therefore exert **radiation pressure** on a surface:

$$p_{\text{rad}} = \frac{S_{\text{av}}}{c} = \frac{I}{c} \quad \text{(radiation pressure, wave totally absorbed)}$$

$$p_{\text{rad}} = \frac{2S_{\text{av}}}{c} = \frac{2I}{c} \quad \text{(radiation pressure, wave totally reflected)}$$

- At surface of Earth - S_{AV} ~1000 w/m^2
- Above atmosphere - S_{AV} ~1361w/m^2
- For example, if the solar panels on an earth-orbiting satellite are perpendicular to the sunlight, and the radiation is completely absorbed, the average radiation pressure is 4.7×10^{-6} N/m^2.





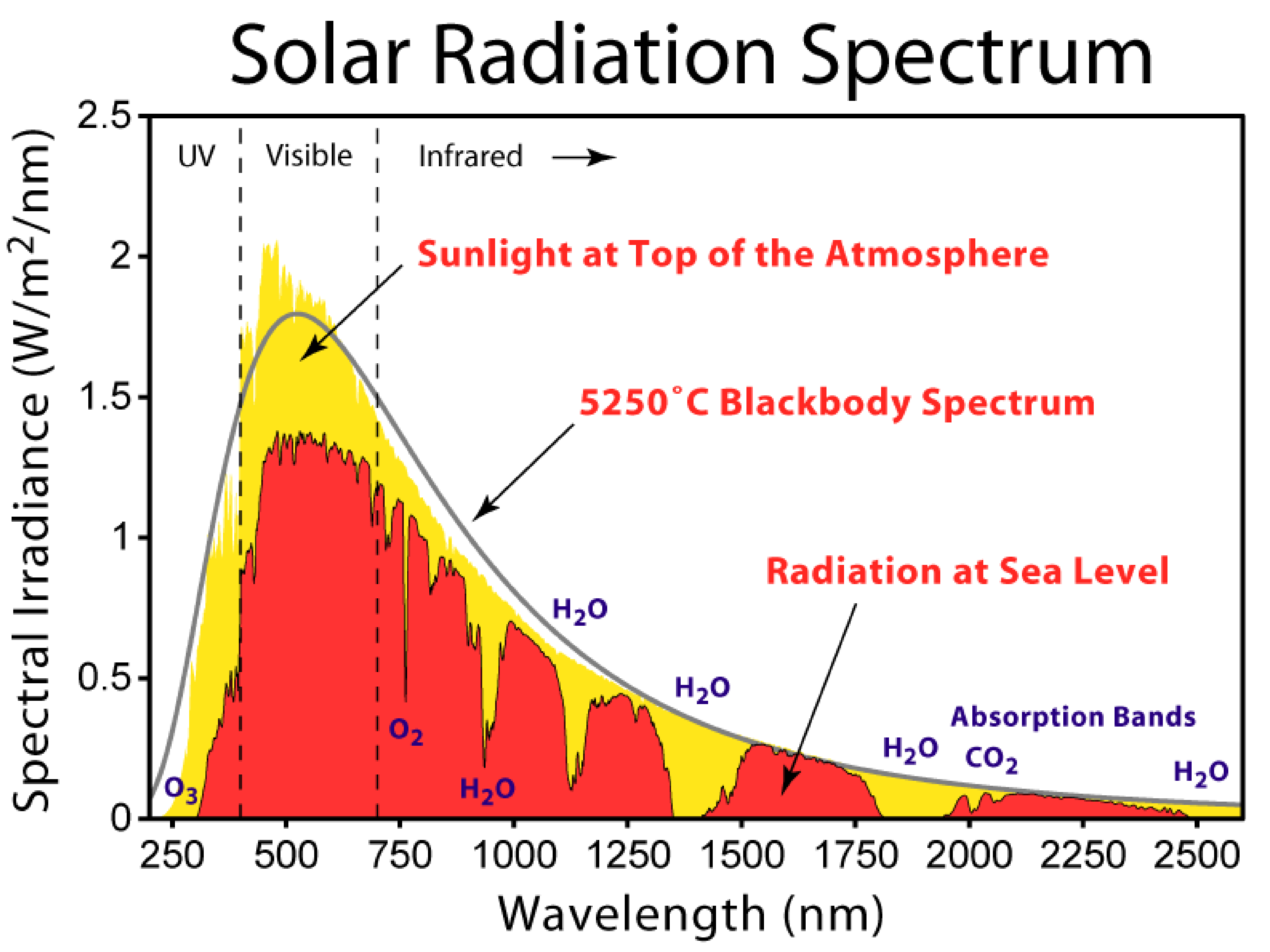
Ca-II triplet: 11545, 11707, 11767
H alpha: 15237
Mg-I doublet: 19292, 19332
H beta: 20571
Fe-I: 22812
H gamma: 23039
H delta: 24380
Fe-I: 24723
Ca-II doublet: 25202, 25426

Balmer Series, n = 2,3,4,5...

27427*(1 -- 4/n^2)

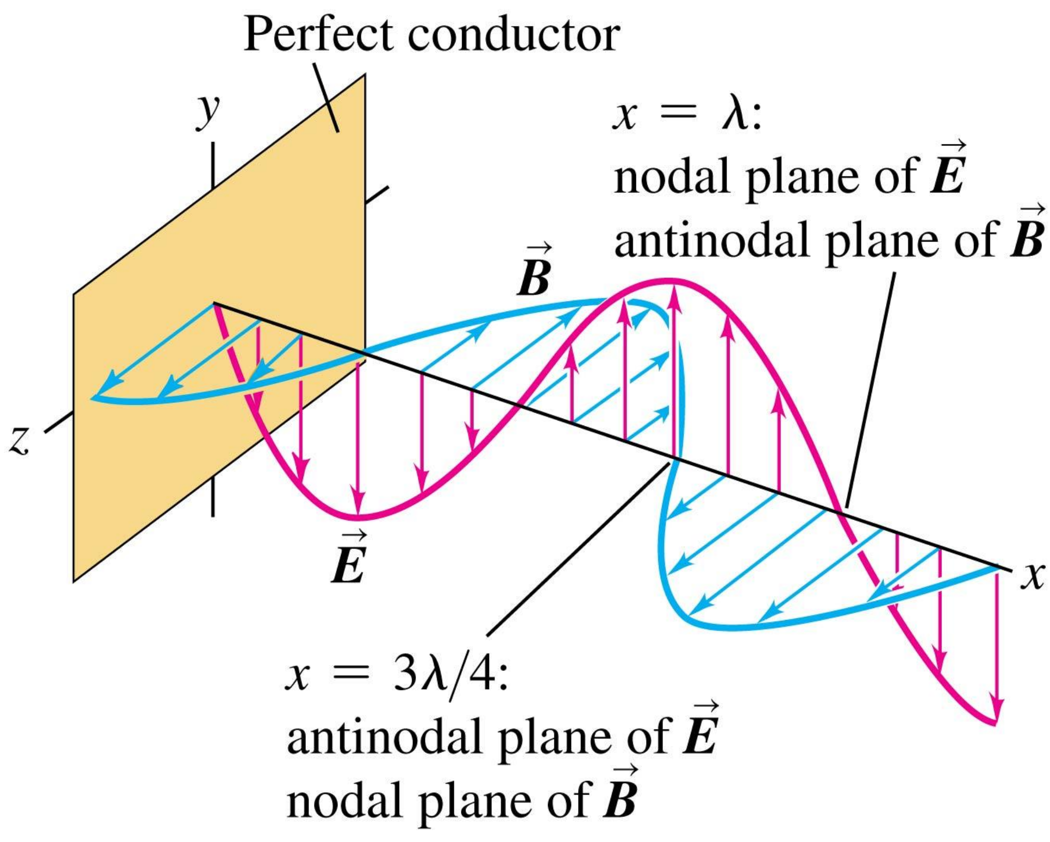
= 27430 * (5/9, 3/4, 21/25, 8/9)

= 15237,20570,23039,24380


Solar Radiation Spectrum
Spectral Irradiance (W/m2/nm)
2.5
2
1.5
1
0.5
0
UV
Visible
Infrared
Sunlight at Top of the Atmosphere
5250˚C Blackbody Spectrum
Radiation at Sea Level
H2O
H2O
Absorption Bands
O2
H2O
H2O
CO2
H2O
O3
250
500
750
1000
1250
1500
1750
2000
2250
2500
Wavelength (nm)

Standing electromagnetic waves

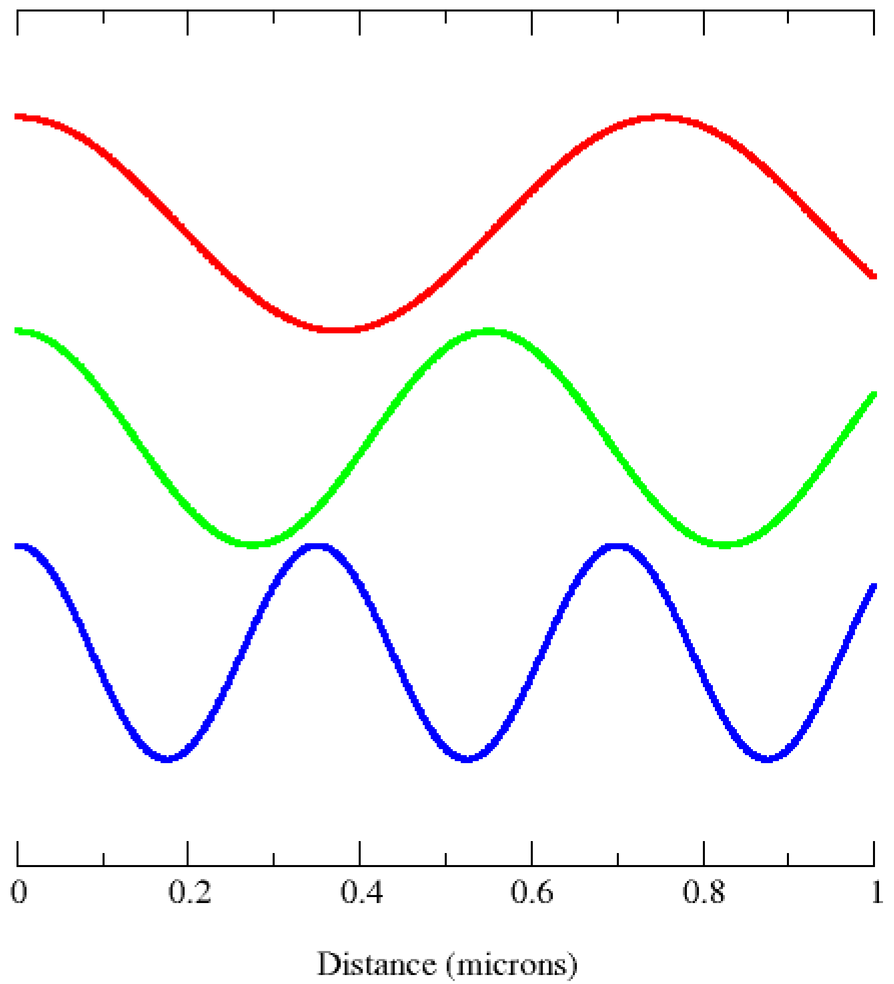
- Electromagnetic waves can be reflected by a conductor or dielectric, which can lead to *standing waves*.
- As time elapses, the pattern does not move along the *x*-axis; instead, at every point the electric and magnetic field vectors simply oscillate.



Standing waves in a cavity

- A typical microwave oven sets up a standing electromagnetic wave with λ = 12.2 cm, a wavelength that is strongly absorbed by the water in food.
- Because the wave has nodes spaced $\lambda/2$ = 6.1 cm apart, the food must be rotated while cooking.
- Otherwise, the portion that lies at a node—where the
electric-field amplitude is zero—will remain cold.

Visible light – Red-Green-Blue



Visible light

- *Visible light* is the segment of the electromagnetic spectrum that we can see.
- Visible light extends from the violet end (400 nm) to the red end (700 nm).

TABLE 32.1 Wavelengths of Visible Light

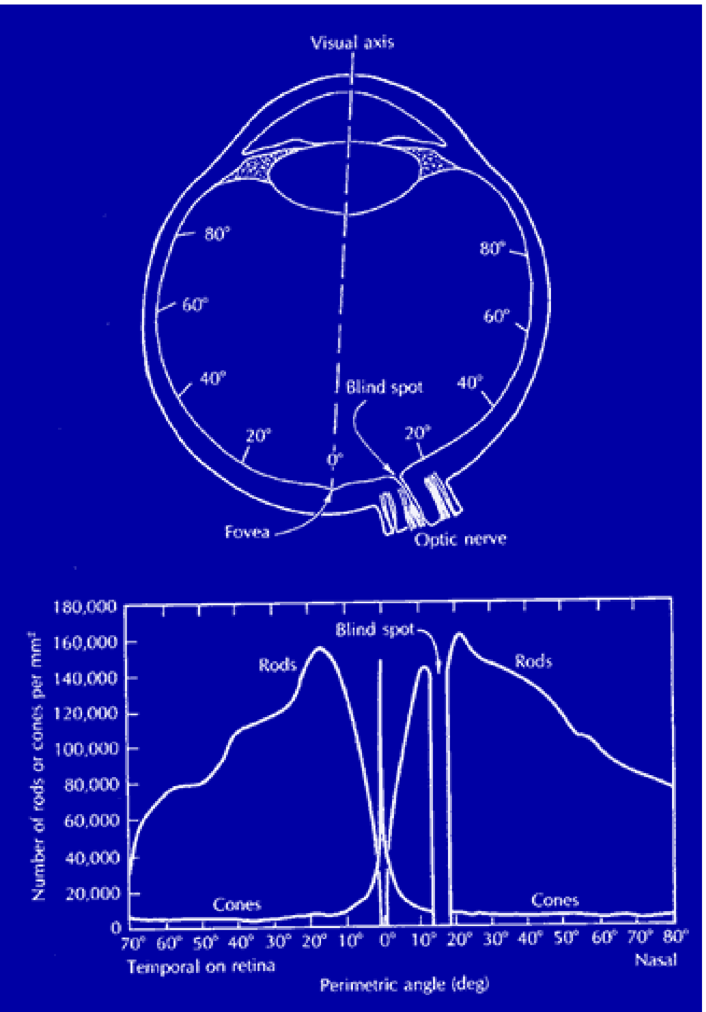
380–450 nm	Violet
450–495 nm	Blue
495–570 nm	Green
570–590 nm	Yellow
590–620 nm	Orange
620–750 nm	Red

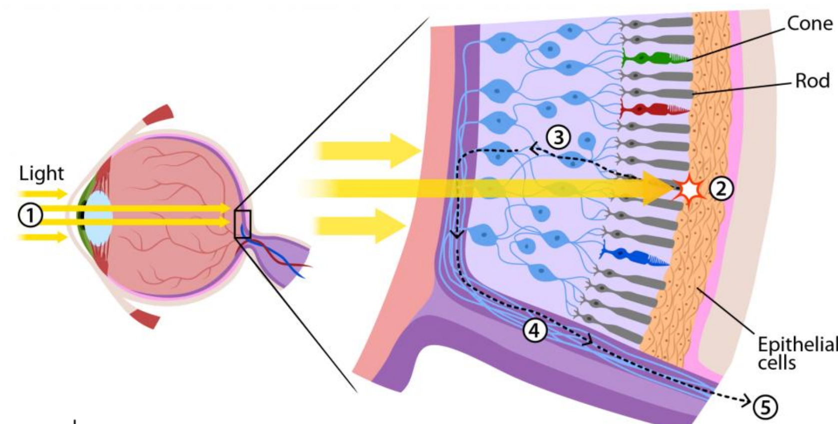
Eye Response to Colors and BW

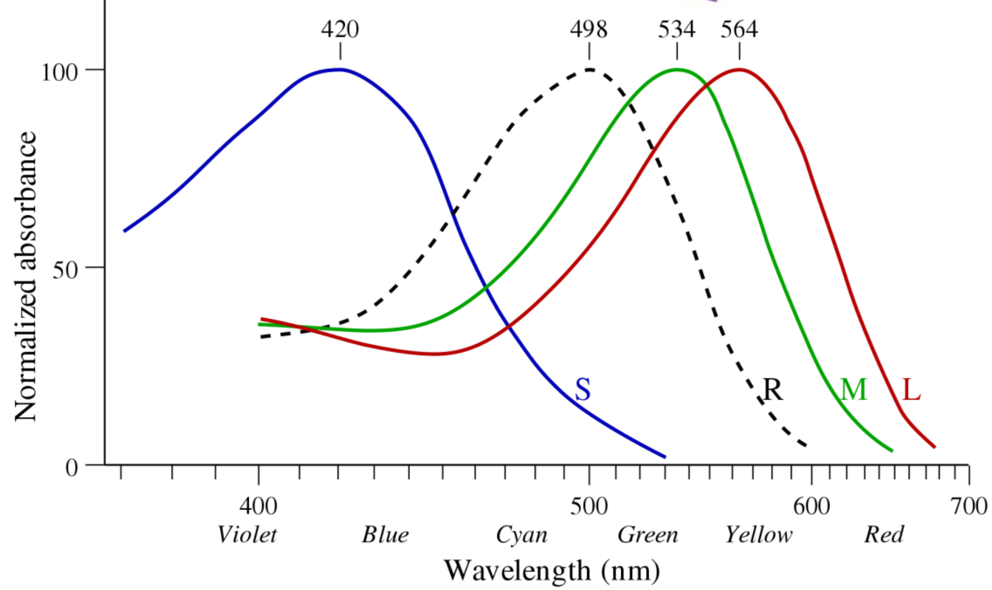
Rods (Scotopic - BW – night vision ~ 120 million) and Cones (Photopic – Color - ~ 6 million)

Modern Electronic Focal Plane Arrays Now Exceed Human Resolution

Central fovea (retina) is primarily populated by cones – NOTE Bind spot



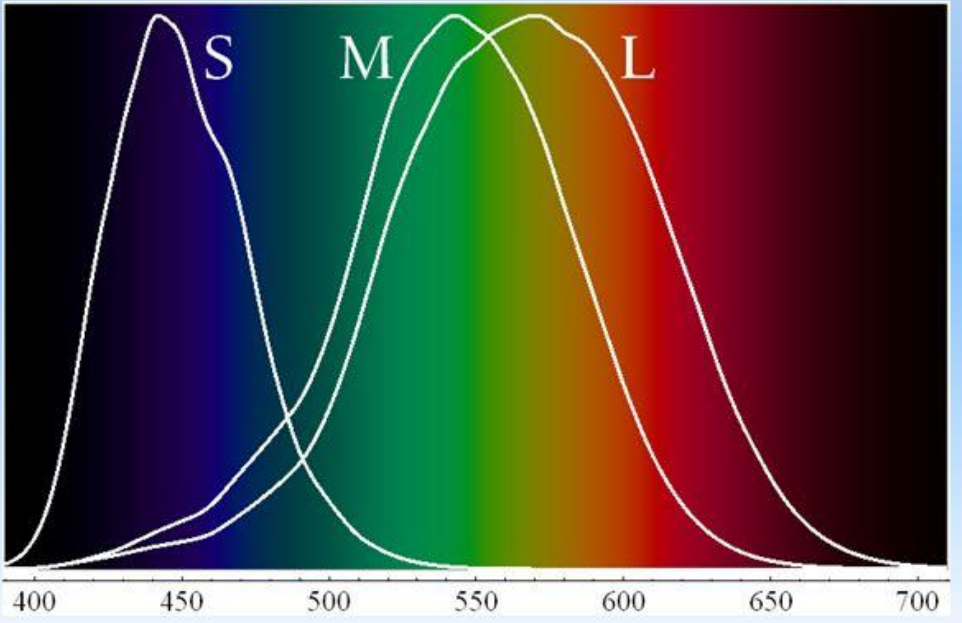




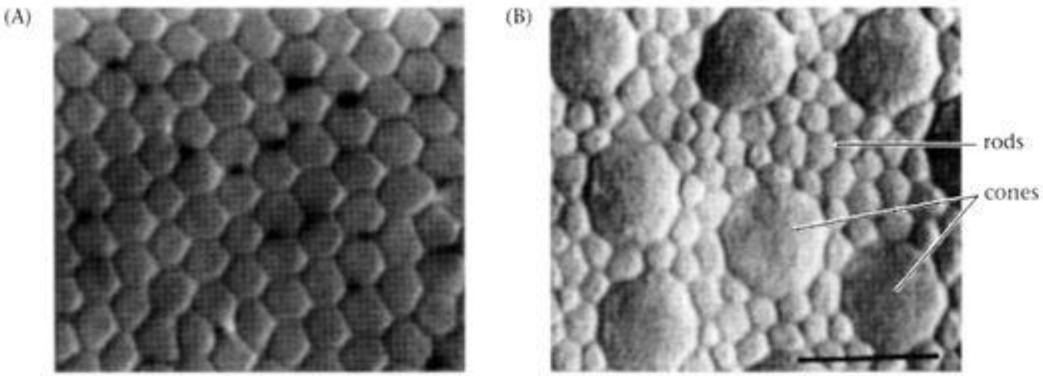
Three Types of Cones for Color Vision

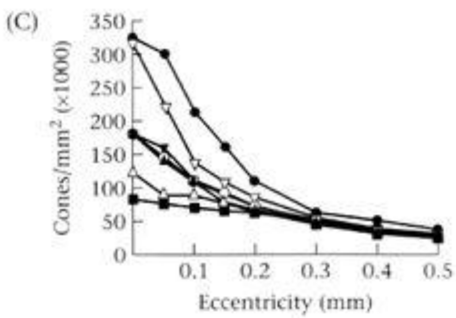
There are three types of cones in the human eye.

Long wavelength cones with a peak detection of greenish-yellow.
Medium wavelength cones with a peak detection of green .
Short wavelength cones that detect principally blue and violet colors.

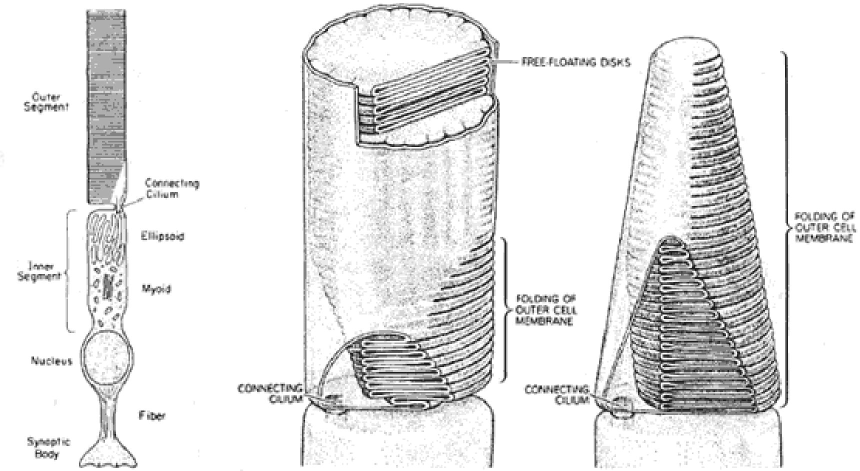


Retina Distribution of Rods and Cones

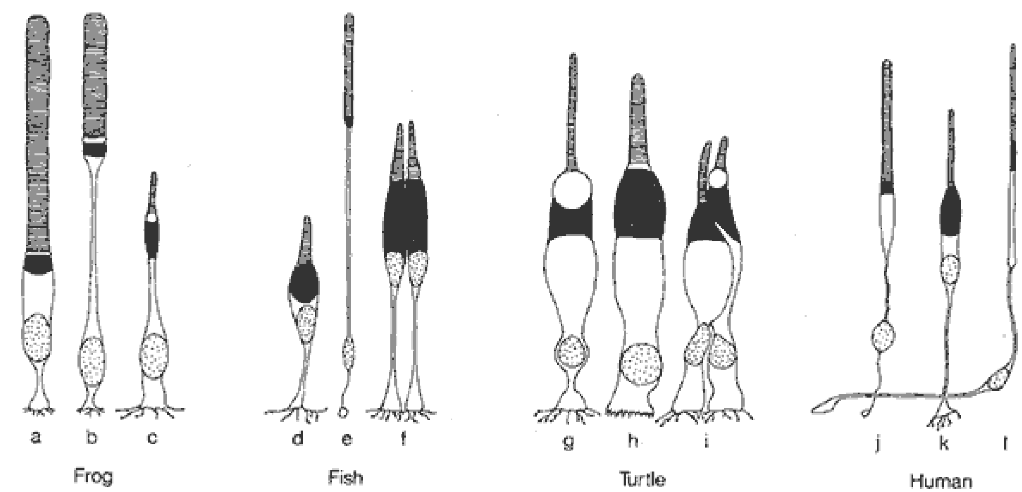




3.4 THE SPATIAL MOSAIC OF THE HUMAN CONES. Cross sections of the human retina at the level of the inner segments showing (A) cones in the fovea, and (B) cones in the periphery. Note the size difference (scale bar = 10 μm), and that, as the separation between cones grows, the rod receptors fill in the spaces. (C) Cone density plotted as a function of distance from the center of the fovea for seven human retinas; cone density decreases with distance from the fovea. Source: Curcio et al., 1990.



At the left is a generalized conception of the important structural features of a vertebrate photoreceptor cell. At the right are shown the differences between the structure of rod (left) and cone (right) outer segments. These diagrams are from Young (1970) and Young (1971).



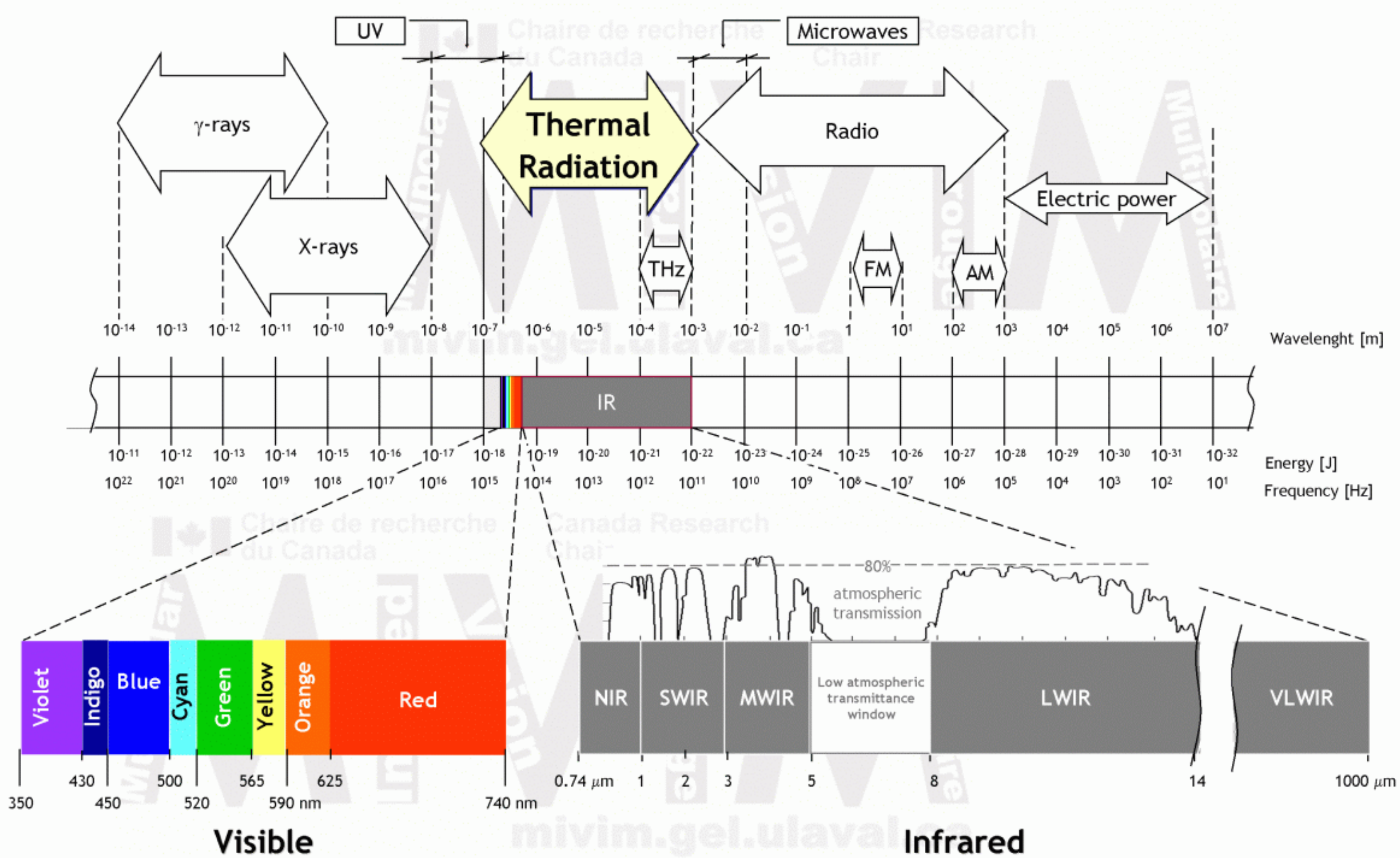
2.4 Drawings of rod and cone cells from a variety of animals. **a**, leopard frog red rod; **b**, leopard frog green rod; **c**, leopard frog cone; **d**, goldfish cone; **e**, goldfish rod; **f**, bluegill twin cone; **g**, snapping turtle cone; **h**, snapping turtle rod; **i**, western painted turtle double cone; **j**, human rod; **k**, human cone; **l**, human foveal cone.

Ultraviolet Light and Vision
Our colors looks quite different

- Many insects and birds can see ultraviolet wavelengths that humans cannot.
- As an example, the left-hand photo shows how black-eyed Susans look to us.
- The right-hand photo (in false color), taken with an ultraviolet-sensitive camera, shows how these same flowers appear to the bees that pollinate them.
- Note the prominent central spot that is invisible to humans.

Infrared Vision

Santa Barbara is the IR Capital (Industrial Base) of the US



IR Imaging is very important

Remote sensing for weather, crop health, industrial, medical, astronomy

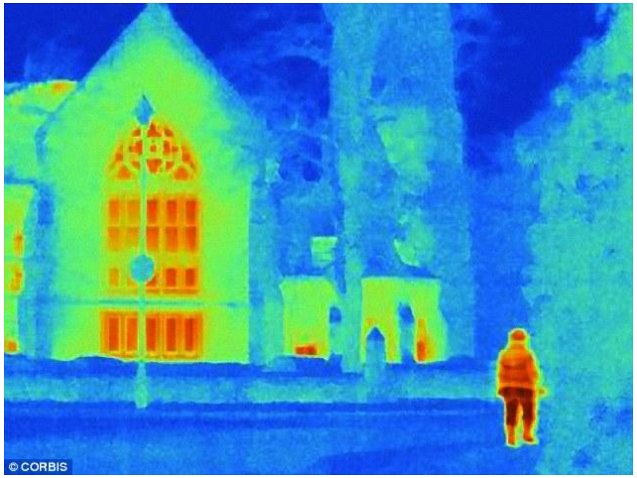
Thermal IR (8-12 microns) is Common Now (Microbolometers)

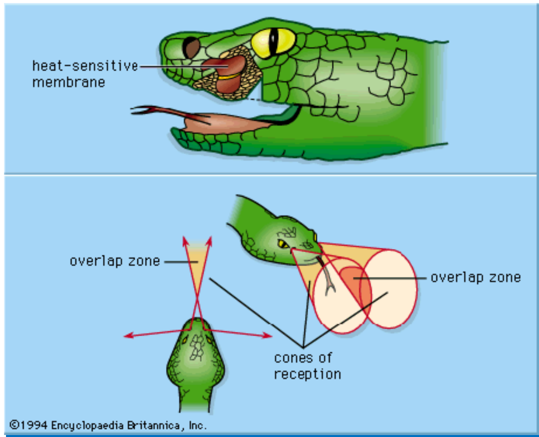
Night Vision Systems in Cars for example

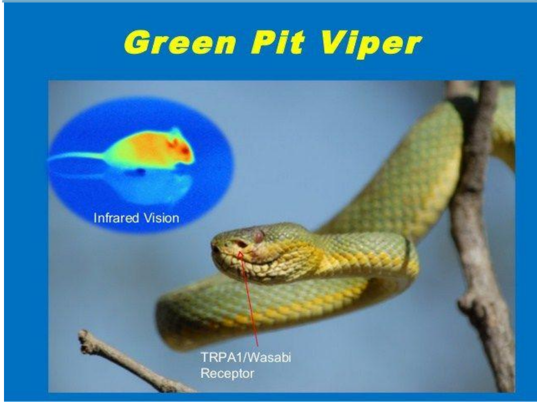
Spectral bands	Range [μm]	Detector materials*	Applications
NIR	0.74 - 1	SiO_2	Telecommunications
SWIR	1 - 3	InGaAs, PbS	Remote sensing
MWIR	3 - 5	InSb, PbSe, PtSi, HgCdTe	High temperature inspection (indoors, scientific research)
LWIR	8 - 14	HgCdTe	Ambient temperature (outdoor, industrial inspection)
VLWIR	14 - 1000	-	Spectrometry, astronomy

*Si: silicon; SiO_2: silica; In: Indium; Ga: gallium; As: arsenic; Pb: lead; S: sulfur; Sb: antimony; Se: selenium; Pt: platinum; Hg: mercury; Cd: cadmium; Te: tellurium.

Thermal IR Imaging (8-12 microns)







Impedance of the Vacuum

Our modern view of the vacuum is that it is a sea of all things at negative energy.

Thus it is NOT NOTHING.

It has an impedance

$$Z_0 \stackrel{\text{def}}{=} \mu_0 c_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} = \frac{1}{\varepsilon_0 c_0}$$

$$\varepsilon_0 \stackrel{\text{def}}{=} \frac{1}{\mu_0 {c_0}^2} \approx 8.854\ 187\ 817\ldots \times 10^{-12}$$

$$Z_0 \approx 376.730\ 313\ 461\ 77\ldots\Omega$$

Electromagnetic Energy Density and Flux and Quanta

$$u_e = \frac{\epsilon_0}{2} E^2$$

$$u_m = \frac{1}{2\mu_0} B^2$$

$$\mathbf{S} = \frac{1}{\mu} \mathbf{E} \times \mathbf{B},$$

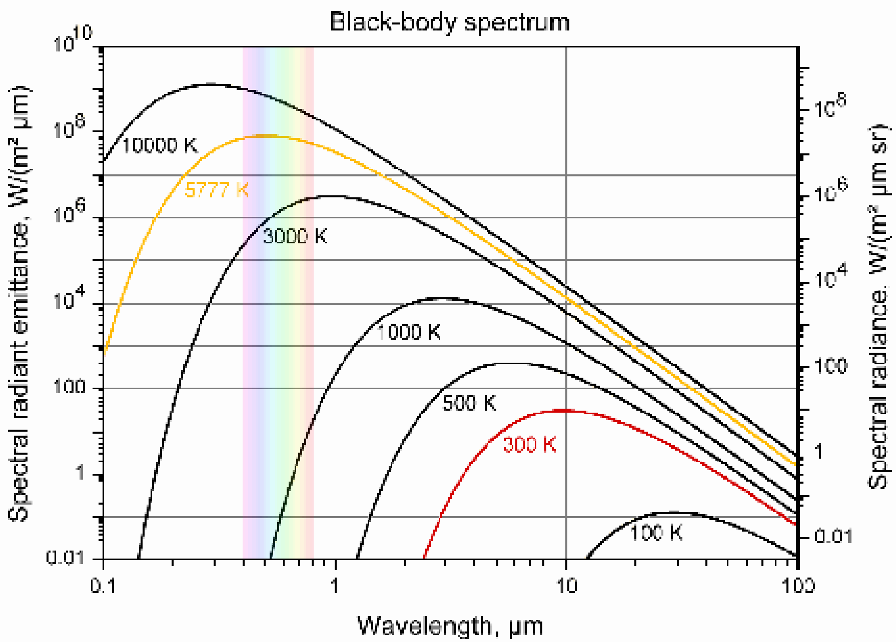
$$E = hf$$

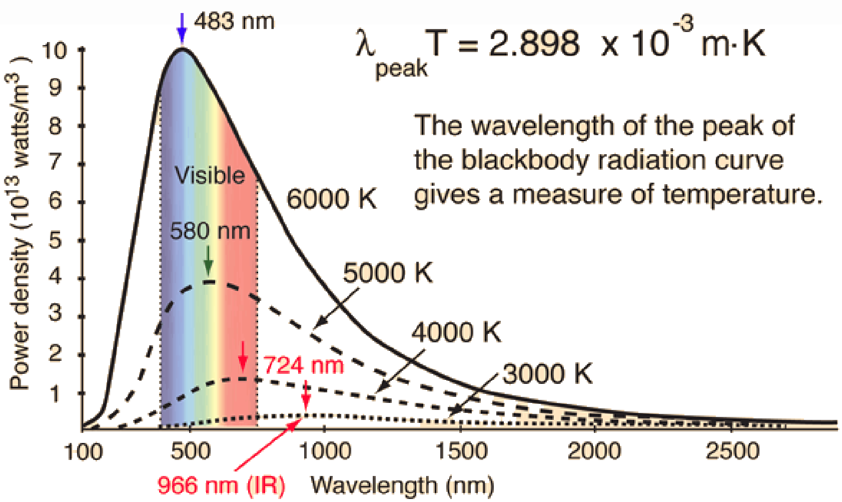
$$h = 6.626\ 068\ 96(33) \times 10^{-34}\ \mathrm{J\ s} = 4.135\ 667\ 33(10) \times 10^{-15}\ \mathrm{eV\ s}.$$

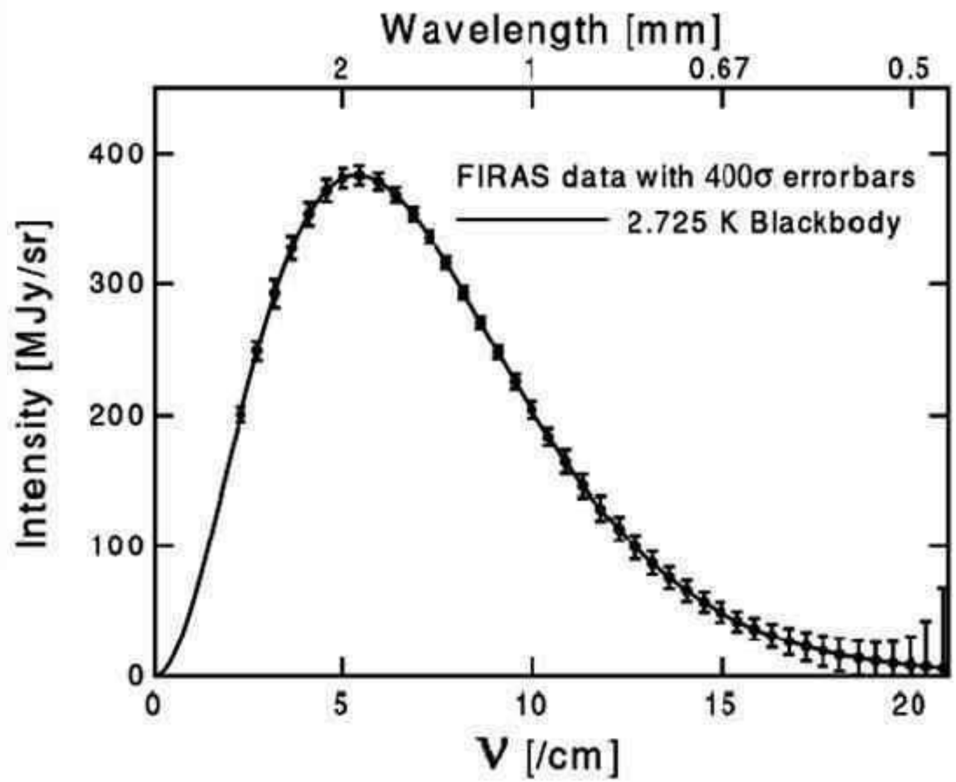
Blackbody Radiation – Planck Function

Blackbody = Body that Absorbs all Radiation

The Universe is a Nearly Perfect Blackbody







Dispersion in materials – polarization depends on frequency