Homework 2: Phys 141

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Problem 2.14
The critical angle for internal reflection is:

$$\theta_c = \arcsin(\frac{n_2}{n_1})$$

So, for water

$$\theta_c = \arcsin(\frac{1}{1.33}) = 48.8^\circ$$

and for diamond

$$\theta_c = \arcsin(\frac{1}{2.42}) = 24.4^\circ$$

Problem 2.15
The Brewser angle is

$$\theta_B = \arcsin(\frac{n_2}{n_1})$$

For water this is

$$\theta_B = \arctan(\frac{1.33}{1}) = 53.1^\circ$$

and for diamond this is

$$\theta_B = \arctan(\frac{2.42}{1}) = 67.6^\circ$$

Problem 2.16
Water

$$r_s = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{\frac{1}{\sqrt{2}} - \sqrt{1.33^2 - \frac{1}{2}}}{\frac{1}{\sqrt{2}} + \sqrt{1.33^2 - \frac{1}{2}}} = -0.23$$

$$\Rightarrow R_s = |r_s|^2 = 0.053$$

$$r_p = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{-1.33^2\frac{1}{\sqrt{2}} + \sqrt{1.33^2 - \frac{1}{2}}}{1.33^2\frac{1}{\sqrt{2}} + \sqrt{1.33^2 - \frac{1}{2}}} = -0.05$$

$$\Rightarrow R_p = |r_p|^2 = 0.0027$$

Diamond

$$r_s = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{\frac{1}{\sqrt{2}} - \sqrt{2.42^2 - \frac{1}{2}}}{\frac{1}{\sqrt{2}} + \sqrt{2.42^2 - \frac{1}{2}}} = -0.53$$

$$\Rightarrow R_s = |r_s|^2 = 0.28$$

$$r_p = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} = \frac{-2.42^2\frac{1}{\sqrt{2}} + \sqrt{2.42^2 - \frac{1}{2}}}{2.42^2\frac{1}{\sqrt{2}} + \sqrt{2.42^2 - \frac{1}{2}}} = -0.28$$

$$\Rightarrow R_p = |r_p|^2 = 0.08$$

Problem 2.18

To produce circularly polarized light, the Mooney rhombus must induce a phase difference between TE and TM of $\frac{\pi}{2}$ in total, or $\frac{\pi}{4}$ for each instance of reflection. So then, $\Delta = \frac{\pi}{4}$.

$$\tan(\frac{\delta}{2}) = \frac{\cos\theta\sqrt{\sin^2\theta - n^2}}{\sin^2\theta}$$

if $\Delta = \frac{\pi}{4}$ and $n = \frac{1}{1.65}$
then, we can find that $\theta = 60°$

Problem 2.19

a)

The amplitude of an evanescent wave is

$$E'' e^{-\alpha|y|}$$

So then,

$$|y| = \frac{1}{\alpha}$$

Now,

$$\alpha = k''\sqrt{\frac{\sin^2\theta}{n^2} - 1}$$

and

$$k'' = \frac{2\pi}{\lambda_0} = \frac{2\pi}{n\lambda}$$

Now, since $\theta = 45°$, $n = \frac{1}{1.5}$ and $\lambda = 500nm$

So,

$$\alpha = \frac{2\pi}{1.5500}\sqrt{\frac{1.5^2}{2} - 1} = 2.96x10^{-3}nm^{-1}$$

b)
for $|y| = 1$mm,

$$E_2 = E'' e^{-\alpha|y|} = E'' e^{-2960}$$

$$I_2 = |E_2|^2 = E''^2 e^{-5920}$$

So,

$$\frac{I_2}{I_1} = \frac{|E_2|^2}{|E_1|^2} = e^{-5920}$$

Problem 2.20
If incident angle into the fiber is α, the angle of refraction is

$$n_0 \sin(\alpha) = n_1 \sin(\theta_r)$$

By geometry,

$$n_1 \sin(\theta_r) = n_1 \cos(\theta_c) = n_1\sqrt{1 - \sin^2\theta_c}$$

Now,

$$\sin(\theta_c) = \frac{n_2}{n_1}$$

So,

$$\sin(\alpha) = n_1\sqrt{1 - \frac{n_2^2}{n_1^2}} = \sqrt{n_1^2 - n_2^2}$$

$$\Rightarrow \alpha = \arcsin(\sqrt{n_1^2 - n_2^2})$$

Problem 2.21

$$\tan(\frac{\Delta}{2}) = \tan(\frac{1}{2}(\delta_p - \delta_s)) = \tan(\beta - \alpha) = \frac{\tan(\beta) - \tan(\alpha)}{1 + \tan(\beta)\tan(\alpha)}$$

Now,

$$\tan(\beta) = \frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}$$

$$\tan(\alpha) = \frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}$$

So then,

$$\tan(\frac{\Delta}{2}) = \frac{\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta} - \frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}}{1 + \frac{\sqrt{\sin^2\theta - n^2}}{\cos\theta}\frac{\sqrt{\sin^2\theta - n^2}}{n^2\cos\theta}} = \frac{\cos\theta\sqrt{\sin^2\theta - n^2}}{\sin^2\theta}$$

Problem 2.23

The light is unpolarized, so $I(TE) = I(TM) = \frac{1}{2}I_0$

According to 2.27

$$P = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{I(TM) - I(TE'')_t}{I(TM) + I(TE'')_t}$$

Now,

$$r_{S1} = \frac{\cos\theta - \sqrt{n^2 - \sin^2\theta}}{\cos\theta + \sqrt{n^2 - \sin^2\theta}}$$

Since $\sin\theta = \frac{n_2}{\sqrt{n_1^2 + n_2^2}}$ and $\cos\theta = \frac{1}{1 + \frac{n_2^2}{n_1^2}}$, we find that

$$r_{S1} = \frac{1 - \frac{n_2^2}{n_1^2}}{1 + \frac{n_2^2}{n_1^2}}$$

$$\Rightarrow R_{S1} = \left|\frac{1 - \frac{n_2^2}{n_1^2}}{1 + \frac{n_2^2}{n_1^2}}\right|^2 = \left|\frac{1 - 1.5^2}{1 + 1.5^2}\right|^2 = 0.148$$

Now, since the light is still incident on the second boundary at Brewster's angle,

$$R_{S1} = R_{S2}$$

Now, $I(TE'')_t = I(TE')_t(1 - R_{S1}) = I(TE)_t(1 - R_{S1})(1 - R_{S2}) = I(TE)_t(1 - R_{S1})^2$

So,

$$P = \frac{1 - (1 - R_{S1})^2}{1 + (1 - R_{S1})^2} = \frac{1 - (1 - 0.148)^2}{1 + (1 - 0.148)^2} = 0.159$$