Homework 3: Phys 141

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Problem 3.6

The optical path difference is $d = 2l(n-1)$. When a fringe changes from bright to dark, the optical path difference changes by $\frac{\lambda}{2}$. The number of fringes, N, is given by

$$N = \frac{2\Delta d}{\lambda} = \frac{4l(n-1)}{\lambda} = \frac{4 * 10 * 10^{-2} * (0.0003)}{590 * 10^{-9}} = 203$$

Problem 3.9

The transverse coherence width is given by

$$l_t = \frac{1.22\lambda}{\theta_s}$$

Which, for $\lambda = 500nm$ and $\theta = 0.5 * \frac{pi}{180}$ is

$$l_t = 0.084nm$$

Problem 3.10

Again, we start from

$$l_t = \frac{1.22\lambda}{\theta_s}$$

Now, $\theta_s \approx \frac{s}{r} = 10^{-3}$ and $\lambda = 590nm$, so

$$l_t = 0.72nm$$

Problem 4.1

$$T_{max} = \frac{T^2}{(1-R)^2} = \frac{0.05^2}{(1-0.9)^2} = 0.25$$

$$T_{min} = \frac{T^2}{(1+R)^2} = \frac{0.05^2}{(1+0.9)^2} = 7x10^{-4}$$

$$\mathfrak{F} = \pi\frac{\sqrt{R}}{1-R} = \pi\frac{0.9}{0.1} = 29.8$$

$$F = \frac{4R}{(1-R)^2} = \frac{4(0.9)}{0.1^2} = 360$$

Problem 4.5
For a single layer,

$$M = \begin{bmatrix} cos(kl) & -\frac{i}{n}sin(kl) \\ -insin(kl) & cos(kl) \end{bmatrix} \equiv \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

now,

$$t = \frac{2n_0}{An_0Bn_Tn_0 + C + Dn_T} = \frac{2}{A+B+C+D}$$

since $n_0 = n_T = 1$.

$$t = \frac{2}{2\cos(kl) - i\sin(kl)(n + \frac{1}{n})}$$

$$T = tt^* = \frac{2}{2\cos(kl) - i\sin(kl)(n + \frac{1}{n})}\frac{2}{2\cos(kl) + i\sin(kl)(n + \frac{1}{n})} = \frac{4}{4\cos^2(kl) + \sin^2(kl)(n + \frac{1}{n})^2}$$

now, $kl = \frac{2\pi nd}{\lambda}$. If $\lambda = \frac{2\pi nd}{N}$, then $T = 1$, which is the maxima.

Problem 4.7

$$R = \frac{(n_T - n_1^2)^2}{(n_T + n_1^2)^2} = 0.008 = 0.8\%$$

Problem 4.9
We start with the boundary conditions that

$$1) E_0 + E_0' = E_1 + E_1'$$

$$2) n_0(E_0 - E_0') = n_1(E_1 - E_1')$$

$$3) E_1e^{ikl} + E_1'e^{-ikl} = E_T$$

$$4) E_1n_1e^{ikl} - n_1E_1'e^{-ikl} = n_TE_T$$

Equations 3 and 4 can be combined to give the equations

$$5) E_1 = \frac{n_1 + n_T}{2n_1}E_Te^{-ikl}$$

$$6) E_1' = \frac{n_1 - n_T}{2n_1}E_Te^{ikl}$$

Equations 5 and 6 can be substituted into equation 1 to give

$$7) 1 + \frac{E_0'}{E_0} = \frac{E_T}{E_0}\left(\cos(kl) - i\frac{n_T}{n_1}\sin(kl)\right)$$

Equations 5 and 6 can also be substituted into equations 2 to give

$$8) n_0(1 - \frac{E_0'}{E_0}) = \frac{E_T}{E_0}\Big(n_T \cos(kl) - i n_1 \sin(kl)\Big)$$

Lastly, equations 7 and 8 can be combined to give

$$\begin{bmatrix} 1 \\ n_0 \end{bmatrix} + \frac{E_0'}{E_0}\begin{bmatrix} 1 \\ -n_0 \end{bmatrix} = \begin{bmatrix} \cos(kl) & -\frac{i}{n_1}\sin(kl) \\ -n_1 i \sin(kl) & \cos(kl) \end{bmatrix}\begin{bmatrix} 1 \\ n_T \end{bmatrix}\frac{E_T}{E_0}$$

$$\Rightarrow \begin{bmatrix} 1 \\ n_0 \end{bmatrix} + r\begin{bmatrix} 1 \\ -n_0 \end{bmatrix} = M\begin{bmatrix} 1 \\ n_T \end{bmatrix} t$$