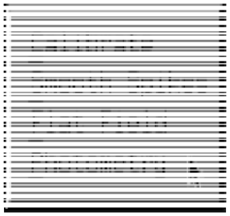
Chapter 6: Photometry



Aperture Photometry

In aperture photometry, concentric circular apertures are used to compute the sky subtracted flux of a star. The inner circle is made large enough to cover almost all of the flux from the star and the outer one is large enough to obtain a good sky value but not too large. We assume the image to be analyzed is already flat fielded, though for some applications, this is not critical.

In general, we want to sum up the contributions of all the pixels where significant light from the star occurs. Since there are other sources of signal, such as CCD dark current, atmospheric emission, etc., we must subtract these so that the result we get is only due to star. We call this corrected value the **sky subtracted value**.

Heuristically we let:

$g(\lambda)$ = pixel value in A/D units from all sources, star, background, dark current, etc.

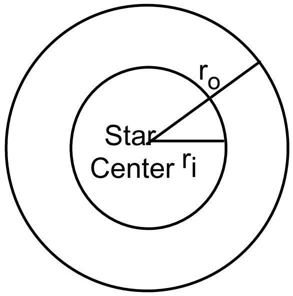
$g_b(\lambda)$ = pixel value in A/D units if **NO** star is present. This is the background value and is assumed to have the same integration time.

Here we have written $g(\lambda)$ and $g_b(\lambda)$ to denote that in general these values are wavelength dependent and will vary as the filter is changed.

$$f(\lambda) = \Sigma[g(\lambda) - g_b(\lambda)]$$

The **sky subtracted signal f(λ)** (star only) is then
where the sum is over all pixels where starlight is present. We can get $g_b(\lambda)$ by either taking a separate exposure with no star present, but of equal time or as is more common, by using pixels near to where the star is located to calculate the background level.

To analyze the problem in detail, we introduce the following notation:



r_i = inner aperture radius
r_o = outer aperture radius
IA = inner aperture
OA= outer aperture

We assume: r_i and r_o are measured from the centroided star position.
N_{IA} = # pixels in inner aperture
N_{OA} = # pixels in outer aperture
G(j,k) = image array - already flat fielded
R = A/D counts per e^-
N(j,k) = G(j,k)/R = pixel value in e^-

The sky subtracted stellar flux n in electrons is:

$$n = \tfrac{1}{R}\,\Sigma(g(\lambda) - g_b(\lambda)) = \tfrac{1}{R}\,f(\lambda)$$

Where the sum is over all pixels where starlight is present. In terms of the image arrays, we have:

$$n = \sum_{IA} N(j,k) - \frac{N_{IA}}{N_{OA}} \sum_{OA} N(j,k)$$

Here: $\frac{1}{R} g_b(\lambda) = \frac{1}{NOA} \sum_{OA} N(j,k)$=Avg background level per pixel in e^-

Error Analysis

We can compute the error in the sky subtracted flux f as follows (assuming uncorrelated errors)

$$\delta n = \langle \delta n^2 \rangle^{\frac{1}{2}} = [\sum_{IA}(\delta N)^2 + (\frac{N_{IA}}{N_{OA}})^2 \sum_{OA}(\delta N)^2]^{\frac{1}{2}}$$

Here δN is the uncertainty in a given pixel measurement.

$$\delta N = (N_R^2 + (\sqrt{N}\,)^2)^{\frac{1}{2}} = (N_R^2 + N)^{\frac{1}{2}}$$

$$\delta n = [\sum_{IA}(N_R^2 + N(j,k)) \quad + (\frac{N_{IA}}{N_{OA}})^2 \sum_{OA}(N_R^2 + N(j,k))]^{\frac{1}{2}}$$

Where:

N	= total number of electrons produced in a pixel
N_R	= CCD readout noise in e^-
m	= magnitude of star determined
n	= sky subtracted stellar flux in e^-
$m-m_o$	= -2.5 log(n/n_o)
m_o, n_o	- known object in background
	n_o, m_o can also refer to sky background per pixel
m	- magnitude associated with f(λ)
m	= -2.5 log(n) + 2.5 log (n_o) + m_o
m	= -2.5 log(n) + c : c=constant
m	= -2.5 log(e) ln(n) + c
δm	= 2.5 |(δn/n)| log (e) ~ 1.086 (δn/n)

Note, we have assumed n, N(j,k), N_R are given in number of e^- since we are using $(N)^{1/2}$ statistics.

Chapter 6: Photometry

Standard Fluxes and Stars

In order to be able to compare the magnitudes and intensities of stars, we need a standard of measurement so that different measurements using various telescopes, CCDs etc. will yield the same results. For this, we need a standard measure of flux (i.e. photons/cm^2-s-μ or ergs/cm^2-s-μ for example). In addition, we would like a standard set of stars to calibrate our instruments on. In the section entitled "Finding the Absolute Flux of a Star" we will look in detail at the question of measurements of flux. For now, it is sufficient to assume the detector (CCD) and electronics are linear. Thus the relationship between the intensity of a star we measure in A/D units as f(λ) and the actual flux of the star $\mathcal{F}(\lambda)$ in photons/cm^2-s-μ for example is just

$$f(\lambda) = c(\lambda)\, F(\lambda)$$

where c(λ) is a "constant" that depends on the specifics of out telescope, filter, CCD, A/D etc. In general, this "constant" depends on the wavelength being measured for a variety of reasons (filter response, CCD quantum efficiency, etc.).

The magnitude scale is defined so that the difference in magnitudes is related to the log (base 10) of the ratio of fluxes as

$$m_1 - m_2 = -2.5 \log(\mathcal{F}_1(\lambda_1)/\mathcal{F}_2(\lambda_2))$$

where m1, m2 and $\mathcal{F}_1(\lambda_1)$, $\mathcal{F}_2(\lambda_2)$ refer to the magnitudes and fluxes of two stars. We have to be careful here though to specify the wavelength accepted by our instrument.

If we assume both measurements are done at a fixed wavelength λ , then one can write this in terms of the measured intensity $f_1(\lambda)$, $f_2(\lambda)$ as:
Since c(λ) is the same in both cases. Here the assumption of fixed

$$m_1 - m_2 = -2.5 \log\left(\frac{f_1(\lambda)/c(\lambda)}{f_2(\lambda)c(\lambda)}\right) = -2.5 \log\left(\frac{f_1(\lambda)}{f_2(\lambda)}\right)$$

wavelength was critical.

Here we have implicitly assumed that $f(\lambda)$ is the **sky subtracted signal** in the language of the previous section so that the background, sky, dark current, etc., has been subtracted.

So far we can only get magnitude differences. What we need are stars of known flux and magnitude at given wavelengths. These are standard stars. If m_o is the known magnitude of a standard star and $f_o(\lambda)$ is the measured intensity in A/D units, then the magnitude m_1 of another star whose intensity $f_1(\lambda)$ is measured at the same wavelength is:

$$m_1 = m_o - 2.5 \log[f_1(\lambda)/f_o(\lambda)]$$

In this way, we calibrate the measured magnitudes.

Dealing with the atmosphere

Our goal is to calculate the apparent magnitude of a star as it would appear above the earth's atmosphere and to take into account the band pass and efficiencies of the whole system (filters, telescope, detector, atmosphere) so that we can compare our results to those measured by others or so they can compare their results to ours. We will use the following parameters:

$f(\lambda)$ = intensity measured (in general it will depend of wavelength)
$f^*(\lambda)$ = intensity that would be measured outside the earth's atmosphere (i.e. corrected for atmospheric absorption)
$m(\lambda)$ = magnitude measured (in general it will depend on wavelength)
$m^*(\lambda)$ = magnitude that would be measured outside of the earth's atmosphere. This is what we are trying to solve for. (i.e. corrected for atmospheric absorption)
$\alpha(\lambda,\theta)$ = opacity of atmosphere. Depends on wavelength and zenith angle θ of object. In general $\alpha(\lambda,\theta)$ is complicated as it varies with time and depends on moisture and dust in the air, altitude of site, etc. In practice, it must be measured at least once per night and often times for each object.
Formally $\alpha(\lambda,\theta) = \ln(f^*(\lambda)/f(\lambda))$
$\alpha_o(\lambda)$ = opacity at zenith (looking straight up) $\theta=0$ here.

We define **$K(\lambda) = 2.5 \log(e)\alpha_o(\lambda)$** = **extinction coefficient** $\sim 1.086\alpha_o(\lambda)$.

We can model the earth's atmosphere as a horizontally stratified slab so that we can relate $\alpha(\lambda,\theta)$ and $\alpha_o(\lambda)$ as follows:

$$\alpha(\lambda,\theta) = \alpha_o(\lambda)X(\theta) \cong \frac{\alpha_o(\lambda)}{\cos(\theta)} = \alpha_o(\lambda)\sec(\theta)$$

Where $X(\theta)$ is called the "air mass" and for angles $\theta <= 60$ degrees is well approximated as $X\theta) = \sec(\theta)$

The "air mass" is the ratio of the atmosphere column density at the observation zenith angle θ to the column density at $\theta=0$ (often referred to sea level for $\theta=0$). The term is loosely used in the literature unfortunately.

$m^*(\lambda)$ and $m(\lambda)$

The relationship between the two magnitudes $m(\lambda)$ and $m^*(\lambda)$ and the corresponding fluxes $f(\lambda)$ and $f^*(\lambda)$ is as follows:

$$m^*(\lambda) - m(\lambda) = -2.5 \log [f^*(\lambda)/f(\lambda)]$$

Since $\log [f^*(\lambda)/f(\lambda)] = \log (e) \ln[f^*(\lambda)/f(\lambda)] = \log (e)\alpha(\lambda,\theta)$

Therefore

$$m^*(\lambda) \cong m(\lambda) - 2.5 \log (e)\alpha(\lambda,\theta)$$
$$m^*(\lambda) \cong m(\lambda) - 2.5 \log (e)\alpha_o(\lambda)x(\theta)$$
$$m^*(\lambda) \cong m(\lambda) - K(\lambda)x(\theta)$$
$$m^*(\lambda) \cong \mathbf{m(\lambda) - K(\lambda)sec(\theta)} \qquad \boldsymbol{\theta <= 60 \text{ degrees}}$$

Hence once we measure $m(\lambda)$ we can get $m^*(\lambda)$ if we know or can calculate $K(\lambda)$. The problem now becomes one of finding (measuring) $K(\lambda)$.

Note that we have really only determined the difference $m^*(\lambda)-m(\lambda)$ and unless we use a calibration (known) star to set the "reference level", then $m(\lambda)$ and hence $m^*(\lambda)$ will be uncalibrated.

In Table I, we give the "air mass" and refraction of an object versus zenith angle θ. The "air mass" includes effects due to the earth's curvature and is slightly different from $\sec(\theta)$ for angles greater than 60 degrees. The refraction angle assumes observations at sea level. Objects are always lower than they appear.
If we define:

Z_o = Zenith angle (angle from zenith (vertical)) of star if no atmosphere were present
Z = Actual zenith angle of star

then at sea level:
$R = Z_o - Z$ in arc seconds
$R = 58.3 \tan Z - 0.067 \tan^3 Z$

A plot of m(λ) vs x(θ) measured over time as a star rises or sets should be

Table 1: Sea Level Air Mass and Refraction versus Zenith Angle

θ (Deg)	Air Mass x	R (arc sec)
0	1	0
10	1.015	10
20	1.064	21
30	1.154	34
40	1.304	49
50	1.553	70
60	1.995	101
70	2.904	159

a straight line if the atmosphere is stable over this time. See Figure 1 as an example. Since $m(\lambda) = m^*(\lambda) + K(\lambda)\cos(\theta)$, the slope of the line would be K(λ) and the zero intercept would be m*(λ), which is the extra atmospheric (above the atmosphere - no extinction), magnitude we are trying to measure. In figure 1, K(λ)=0.2019 mag/airmass and m*(λ)=13.02 mag.

Note that, in theory, if me measure m(λ) for the same star at two air masses, we can then determine m*(λ) and K(λ). Conversely, if we know m*(λ) (from standard stars) we can determine K(λ). Note that we can measure K(λ), but as stated before, we really only measure magnitude differences (i.e. m*(λ)- m(λ)) unless we calibrate our magnitude scale using a standard star. Where the line drawn is the best fit to the data. The line is of the form y=ax+b.

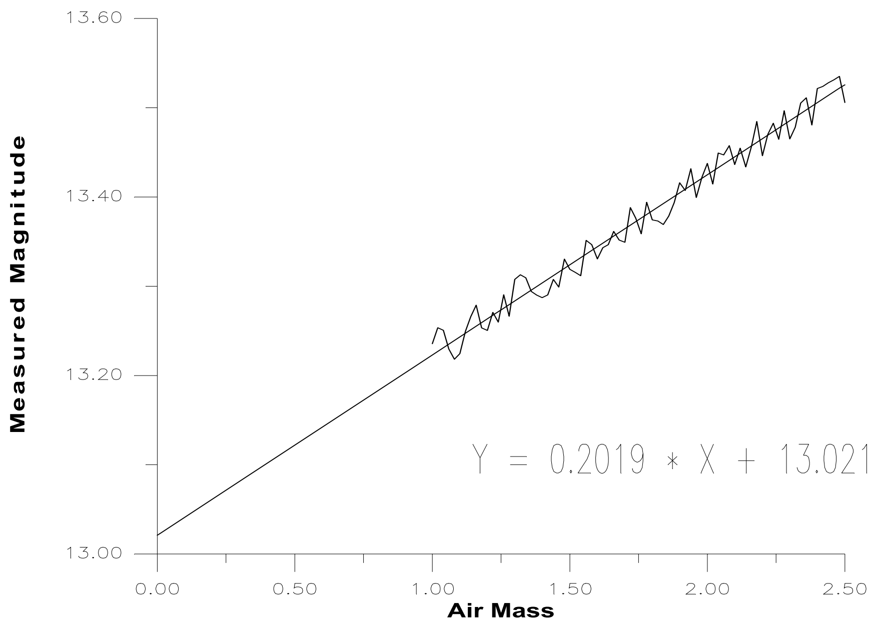


Figure 1: Measured Magnitude versus Air Mass for a Single Star.

The best fit line is Y=0.2019X + 13.021, where Y is the measured magnitude and X is air mass. Therefore, the extinction coefficient is 0.2019 mag / airmass and the true (zero atmosphere) magnitude is 13.021.

In Table II, we list the extinction coefficient and transmission verus wavelength using a "standard" sea level atmosphere assuming the zenith angle is zero (θ=0). By definition, in this case the air mass $x(\theta=0) = 1$. The extinction coefficient unit of measure is "magnitudes". **Figure 2** gives a plot of $K(\lambda)$ versus λ from 0.3 microns (Ultraviolet) to 2 microns (near infrared).

Atmospheric Extinction. Air Mass=1 (Sea Level)

Figure 2

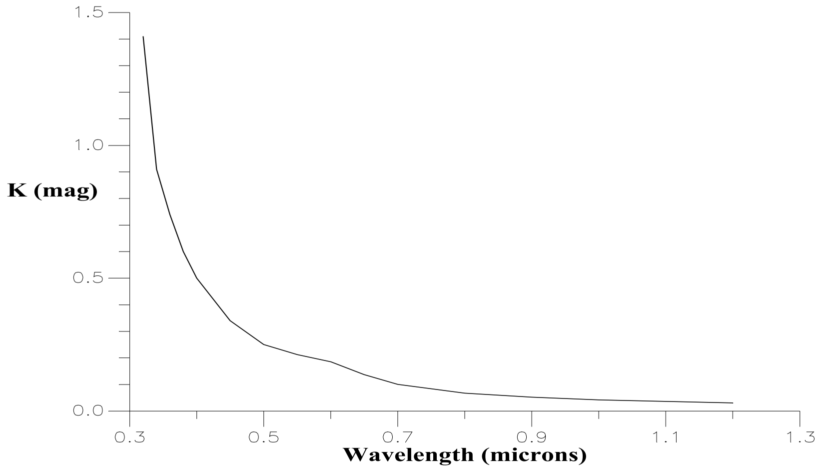


Table II

λ(microns)	K(λ) (mag)	Transmission (%) ($100e^{-\alpha(\lambda,0)}$)
0.3	4.89	1.1
0.32	1.41	27.3
0.34	0.91	43
0.36	0.74	51
0.38	0.6	58
0.4	0.5	63
0.45	0.34	73
0.5	0.25	79
0.55	0.212	82
0.6	0.185	84
0.65	0.137	88
0.7	0.1	91.1
0.8	0.067	93.9
0.9	0.052	95.3
1	0.042	96.2
1.2	0.03	97.2
1.4	0.023	97.9
1.6	0.018	98.3
1.8	0.016	98.5
2	0.014	98.7

Finding $m^*(\lambda)$ and $K(\lambda)$

If we measure the magnitude of a star for two different air masses, we can solve for $m^*(\lambda)$ and $K(\lambda)$ as follows:

let $m_1(\lambda)$, $X_1(\theta_1)$ be measured at angle θ_1

let $m_2(\lambda)$, $X_2(\theta_2)$ be measured at angle θ_2

as before:

$$m_1(\lambda) = m^*(\lambda) + K(\lambda)X_1(\theta_1)$$
$$m_2(\lambda) = m^*(\lambda) + K(\lambda)X_2(\theta_2)$$

Then

$$m^*(\lambda) = \frac{m_1(\lambda)X_2(\theta_2) - m_2(\lambda)X_1(\theta_1)}{X_2(\theta_2) - X_1(\theta_1)}, \quad K(\lambda) = \frac{m_2(\lambda) - m_1(\lambda)}{X_2(\theta_2) - X_1(\theta_1)}$$

The primary disadvantage to this method is that it assumes the atmosphere is stable over the time it takes to star to go from θ_1 to θ_2. Usually it is desirable to have at least a 30 degree difference between θ_1 and θ_2 to give reasonable accuracy for $m^*(\lambda)$ and $K(\lambda)$. In theory, the measured $K(\lambda)$ could now be used for other stars to find $m^*(\lambda)$ as long as the atmosphere is stable.

Another way of determining $K(\lambda)$ is to measure two or more known stars of the same spectral class at significantly different air masses using the same filter(s). Since in this case, we know $m^*(\lambda)$ for each star, we have:

$$m_1(\lambda) = m_1^*(\lambda) + K(\lambda)X_1(\theta_1)$$
$$m_2(\lambda) = m_2^*(\lambda) + K(\lambda)X_2(\theta_2)$$

Since we specified that the same filter is used for each observation:
By using stars of the same spectral class, we minimize any mismatch

$$K(\lambda) = \frac{m_1(\lambda) - m_2(\lambda) - (m_1^*(\lambda) - m_2^*(\lambda))}{X_1(\theta_1) - X_2(\theta_2)}$$

problems our filters may have. Also by writing $K(\lambda)$ as involving only the differences in magnitudes $m_1(\lambda)$-$m_2(\lambda)$ eliminates the need to calibrate the measured magnitudes $m_1(\lambda)$, $m_2(\lambda)$.

Chapter 6: Photometry

Finding the absolute flux of a star

Absolute Flux

To find the absolute flux of a star, we need to know the response of all of the elements of our system including telescope, filters, detector, sky background and atmospheric opacity. We define these responsivities quantitatively as follows:

$f(\lambda)$ = measured star intensity in A/D units
$F^*(\lambda)$ = actual star flux above atmosphere in photons/cm^2-sec-μ
$F(\lambda)$ = star flux at telescope aperture = $\mathcal{F}^*(\lambda)e^{-\alpha(\lambda,\theta)}$
$\varepsilon(\lambda)$ = optical efficiency - including telescope, filter, glass, etc. (fraction of phtons entering telescope aperture that make it to the detector)
$QE(\lambda)$ = quantum efficiency of CCD in e$^-$/photon
A = effective aperture area of telescope in cm^2
$F_B(\lambda)$ = emitted sky background photons/cm^2-steradian-μ
R = CCD response = A/D counts per e$^-$
R_o = A/D no signal value (offset)
$\alpha(\lambda,\theta)$ = atmospheric opacity. Depends on λ and zenith angle of observation (θ here) $\alpha(\lambda,\theta)=\ln[\mathcal{F}^*(\lambda)/\mathcal{F}(\lambda)]$
i_{DC} = CCD dark current in e$^-$/s
τ_{DC} = integration time in seconds.
$\Omega(\lambda)$ = solid angle per CCD pixel in steradians
$\Delta\lambda$ = optical bandpass of system (filter) in μ

Define $A_\varepsilon(\lambda)=A\varepsilon(\lambda)QE(\lambda)\Delta\lambda$

$$f(\lambda)=\int[F(\lambda)A\varepsilon(\lambda)QE(\lambda)+F_B(\lambda)A\varepsilon(\lambda)QE(\lambda)\Omega(\lambda)]R\tau\Delta\lambda+i_{DC}\tau R+R_o$$

$$\sim[(F^*(\lambda)e^{-\alpha(\lambda,\theta)}+F_B(\lambda)\Omega(\lambda))A\varepsilon(\lambda)QE(\lambda)\Delta\lambda+i_{DC}]\tau R+R_o$$

We can rewrite $F^*(\lambda)$ as follows in terms of measured quantities.
Define $f_B(\lambda)=F_B(\lambda)\Omega(\lambda)A\varepsilon(\lambda)\Delta\lambda + i_{DC})R\tau + R_o$
Then:

$$f(\lambda)=F^*(\lambda)e^{-\alpha(\lambda,\theta)}A_\varepsilon(\lambda)\tau R+f_B(\lambda)$$

So:

Notice that $f_B(\lambda)$ is precisely the value that the same pixel would have if

$$F^*(\lambda) = \frac{e^{a(\lambda,\theta)}[f(\lambda) - f_B(\lambda)]}{A_\varepsilon(\lambda)\tau}$$

there were no star present (i.e. if we were only measuring the background and dark current. We can easily get $f_B(\lambda)$ by making another measurement of the same integration time of a blank field (same zenith angle approximately) or by using a nearby pixel value which should be equivalent (assuming flat fielding was done first).

Notice that $A_\varepsilon(\lambda)$ is only a function of system parameters and does not depend on the atmosphere. In theory, we need only determine $A_\varepsilon(\lambda)$ once for each filter used and it should be consistent thereafter. This assumes that the CCD is stable from one observation to the next.

Since a star will usually deposit photons in more that one pixel we should sum over all pixels that have significant star light. We then write $F^*(\lambda)$ as:

$$F^*(\lambda) = \frac{e^{a(\lambda,\theta)}}{A_\varepsilon(\lambda)R\tau} \Sigma[f(\lambda) - f_B(\lambda)]$$

In the section on aperture photometry, we calculated the total number of electrons **n** produced in the CCD associated with the star as:

So we can rewrite $F^*(\lambda)$ as:

$$n = \frac{1}{R} \Sigma[f(\lambda) - f_B(\lambda)]$$

$$F^*(\lambda) = \frac{e^{a(\lambda,\theta)}}{A_\varepsilon(\lambda)\tau} n$$

Chapter 6: Photometry

Color Bands Historically, photometry has been done in a number of different color bands. Of particular interest to CCD based photometry are the UBVRI color bands. The rough transmission bands for each are given below in Table III in microns. U=Ultraviolet, B=Blue, V=Visible, R=Red, I=Infrared.

Table III

Band	λ Center	$\Delta\lambda$
U	0.36	0.07
B	0.44	0.1
V	0.55	0.09
R	0.65	0.1
I	0.8	0.15

Figure 3 shows the approximate transmission bands. Transmission normalization is arbitrary.

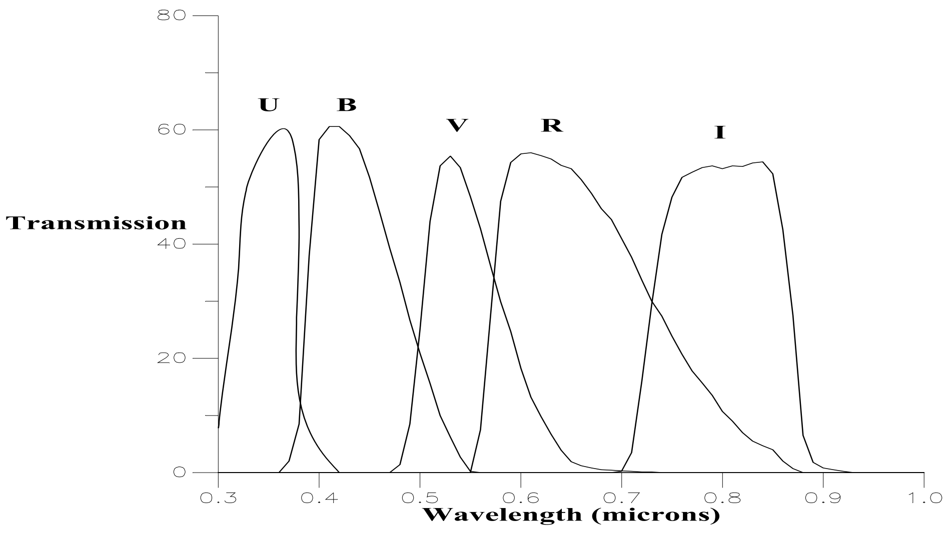


Figure 3: Approximate UBVRI bandpass

Chapter 6: Photometry

By measuring objects in several different bands, the spectral class and temperature can be determined. To aid in this determination a "color index" is often used. A "color index" is simply a difference between magnitudes measured in different bands and hence is just the log of a flux ratio in these bands. For example, the color index B-V is just the magnitude measured in the B band minus that in the V band. We assume for the following that all magnitudes have been corrected for absorption in the earth's atmosphere. Similarly for U-B, R-I, etc. Knowing the spectrum of an object, we can compute the color indices and vice versa. These are summarized in Table IV for stars of varying spectral classification. Note that, by definition, the color indices U-B and B-V are identically zero for an A0 spectral class star. The color indices B-V and U-B versus stellar spectral temperature are plotted in Figures 4 and 5.

Table IV: Spectral Class, Temperature and Color Indicies

Class	T (Kelvin)	B-V	U-B
O5	38,000	-0.32	-1.15
O7	37,000	-0.32	-1.14
O9	31,900	-0.31	-1.12
B0	30,000	-0.3	-1.08
B1	24,200	-0.26	-0.93
B2	22,100	-0.24	-0.86
B3	18,800	-0.2	-0.71
B5	16,400	-0.16	-0.56
B6	15,400	-0.14	-0.49
B7	14,500	-0.12	-0.42
B8	13,400	-0.09	-0.3
B9	12,400	-0.06	-0.19
A0	10,800	0.00	0.00
A1	10,200	0.03	
A2	9,730	0.06	
A3	9,260	0.09	
A5	8,620	0.15	0.11
A7	8,190	0.20	
F0	7,240	0.33	0.06
F2	6,930	0.38	
F5	6,540	0.45	0.00
F6	6,450	0.47	
F7	6,320	0.50	
F8	6,200	0.53	
G0	5,920	0.60	0.11
G2	5,780	0.64	
G5	5,610	0.68	0.20
G8	5,490	0.72	
K0	5,240	0.81	0.47
K2	4,780	0.92	
K3	4,590	0.98	
K5	4,410	1.15	1.03
K7	4,160	1.30	
M0	3,920	1.41	1.26
M1	3,680	1.48	
M2	3,500	1.52	
M3	3,360	1.55	
M4	3,230	1.56	
M5	3,120	1.61	1.19
M8	2,660	2.00	

Graphs From
Table IV :

Figure 4:
B-V Color
Index vs.
Spectral
Temperature
(Kelvin)

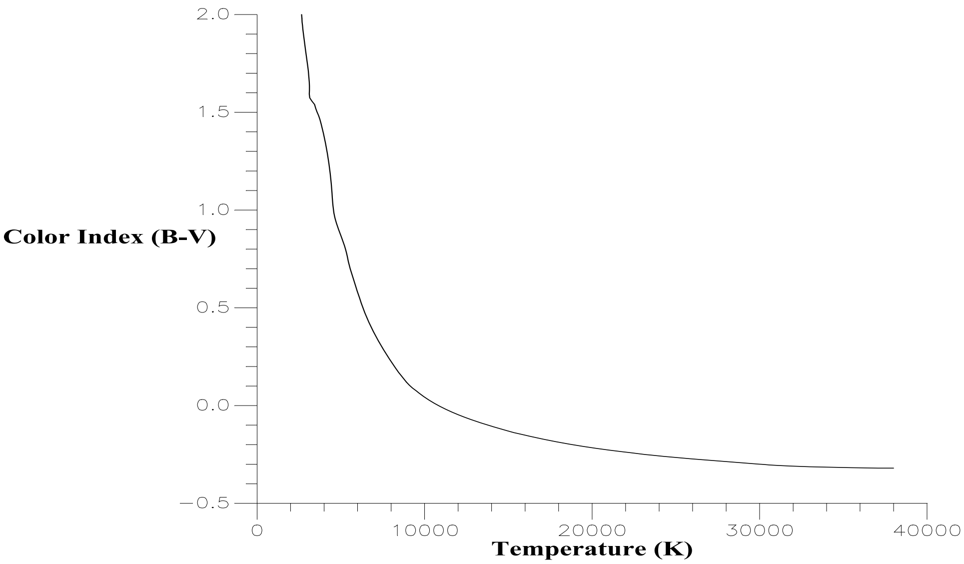


Figure 5:
U-B Color
Index vs.
Spectral
Temperature
(Kelvin)

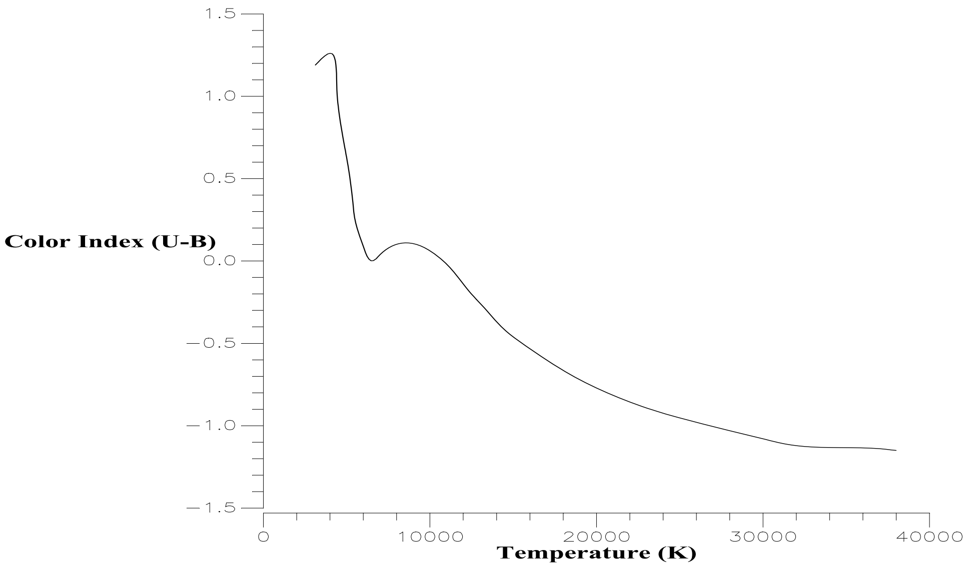


Table V

m_1-m_2	F_1/F_2	% Difference	S/N
0.00	1	0	Infinity
-0.01	1.01	0.9	100
-0.02	1.02	1.9	50
-0.05	1.05	4.7	20
-0.1	1.1	9.6	10
-0.2	1.2	20.2	
-0.5	1.59	58.5	
-1	2.51	251	
-2	6.31	631	

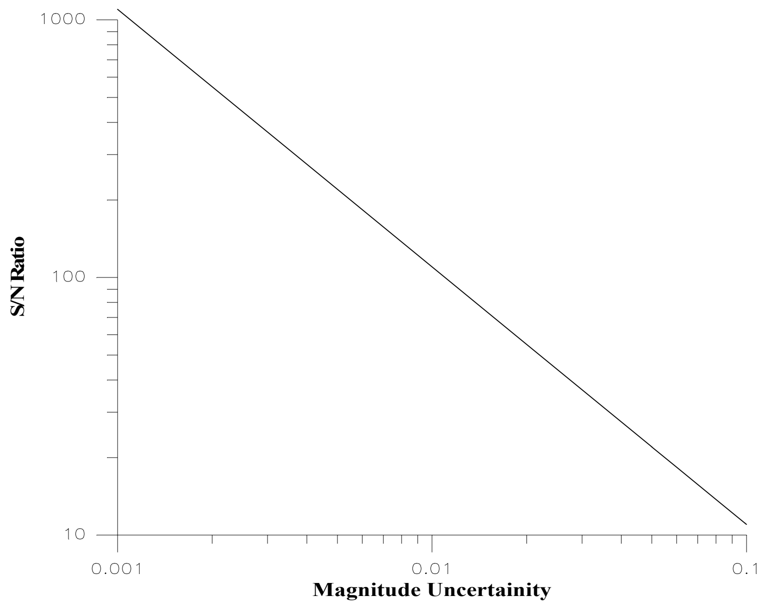
S/N Ratio

$$\frac{d\ln}{dm}F(m) = \frac{dF(m)}{dm}\frac{1}{F(m)} = -0.4\ln(10) = -0.921$$

$$\frac{dF(m)}{F(m)} \cong \frac{1}{F(m)/\Delta F(m)} = \frac{1}{S/N} = -0.921\Delta m$$

S/N~ $\frac{1.1}{\Delta m}$ where $\Delta m << 1$

Figure 6



It might seem like the color index differences are small, but remember that these are logarithmic. Since the CCD is highly linear, it is instructive to view the color index as a flux ratio. Since, by definition m_1-m_2 (or m_B-m_V for example) = $-2.5 \log (F_1/F_2)$. $F_1/F_2 = 10^{-(m1-m2)/2.5} = 10^{-0.4(m1-m2)}$

The relation of Signal to Noise Ratio (S/N) required to get a given Magnitude Uncertainty Δm can be computed as follows:

$F(m) = F(0)10^{-0.4m}$
$\ln F(m) = \ln\{F(0)\} - 0.4m \ln(10)$

The column marked "S/N" in Table V is the signal to noise required to measure a given magnitude difference to 1 sigma significance. Generally, a 3 to 5 sigma measurement is needed so you should multiply this column by 3 to 5.

Figure 6 on the next page gives a plot of S/N versus Δm.

Because of the rapidly increasing atmosphere absorption at shorter wavelengths and because of the finite width of the UBVRI filter bands we need some knowledge of the spectrum we are observing to determining the atmospheric extinction correction. This is particularly of concern for U and

Air Mass and Site Altitude

B bands. Since we generally only have the flux in the bands UBVRI (i.e. only 5 numbers) and not a full spectrum, the following approximate corrections are useful.

K(U) = Extinction Coefficient in U band
K(B) = Extinction Coefficient in V band
K(V) = Extinction Coefficient in R band

$$X(P,T,\theta) = \frac{P\,X_o(\theta)}{760(0.962+0.0038\bullet T)}$$

K(U) = 0.65 - 0.01(B-V) magnitude / airmass
K(B) = 0.34 - 0.03 (B-V) magnitude / airmass
K(V) = 0.20 magnitude / airmass
K(R) = 0.14 magnitude / airmass
K(I) = 0.07 magnitude / airmass

$$then\ r = \frac{P}{760(0.962+0.0038\bullet T)}$$

Here "air mass" refers to the equivalent sea level air mass.

Air Mass vs Altitude

Since the effective air mass varies with the altitude of the observing site, it is useful to understand this relationship. In the following we assume an atmospheric profile based on the "US Standard" atmosphere. Ultimately your particular atmosphere will depend on a number of other variables, such as local weather conditions.

The variation with pressure P (in mmHg) and temperature T (in ºC) of the air mass X(θ) is:

Where $X_o(\theta)$ is the airmass at sea level and an angle θ.

Let r = $X(P,T,\theta)/X_o(\theta)$ = ratio of airmass to airmass at sea level

Table VI

height (ft)	T (ºC)	P (mmHg)	r
0		760	1
1,000	13	732.85	0.953
3,000	9	681.22	0.9
5,000	5	632.49	0.848
7,000	1.1	586.58	0.799
9,000	-2.8	543.36	0.752
11,000	-6.8	502.74	0.707
13,000	-10.7	464.6	0.664
15,000	-14.7	428.85	0.623
17,000	-18.6	395.37	0.584
19,000	-22.6	364.06	0.547

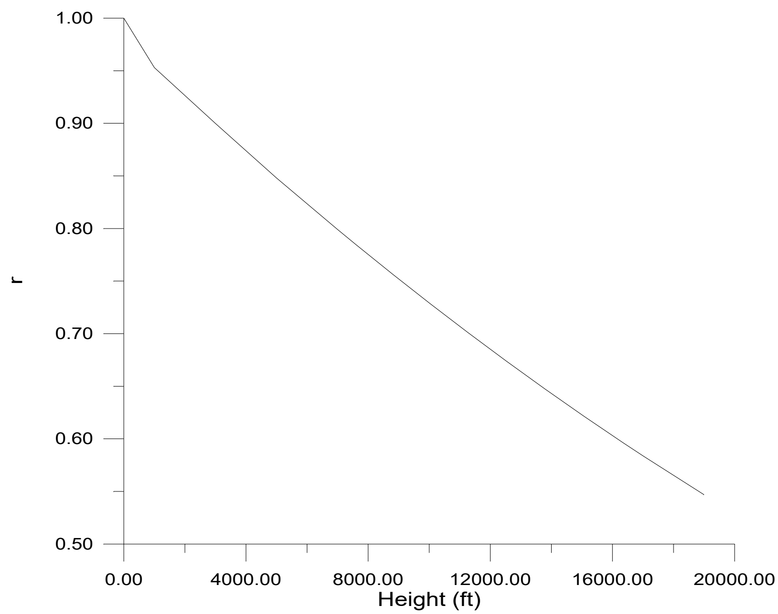


Figure 7: Standard ratio of vertical air mass versus site altitude

(This is also the same relationship to be used for atmospheric refraction vs altitude)

Chapter 6: Photometry

Table VI gives the relation between site altitude (in feet) mean temperature, pressure and r.

Figure 7 plots r versus site altitude.

Height versus temperature and pressure for the "US Standard Atmosphere". r is the ratio of the vertical air mass at a given altitude to that at sea level.