# Measuring the Separation and Position Angle of Two Double Stars: STF 1169 AB and STF 42 AB 


#### Abstract

Researchers have been observing multiple star systems for centuries in the hopes of understanding them more and calculating their orbits. Our paper aimed to add to the historical data of two binary stars, STF 1169 AB and STF 42 AB, by measuring their separation and position angle. The values measured for STF 1169 AB were 20.94 arcsec and $15.46^{\circ}$, while the values for STF 42 AB were 6.26 arcsec and $20.41^{\circ}$.


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## I. INTRODUCTION

Although the idea of a double star was first proposed in the 2nd century AD, these stellar objects were not observed until much later with the invention of the telescope in 1608 [9][7]. The first binary star system - now known as Mizar - was discovered in 1650 by Giovanni Battista Riccioli, with many more observed after. These discoveries were eventually collected by Christian Mayer in 1779 who then compiled them into the first double star catalogue [9].

Our modern understanding of binary stars didn't begin to take shape, though, until the publication of Sir William Herschel's discoveries in 1802 [1]. Since then Astronomers have continued to observe double star systems in order to understand them better and discover if they are in fact true binaries. One way this is accomplished is through measuring the separation between the A and B stars of the system and the position angle of the B star relative to the A star; these values function as the $\rho$ and $\theta$ attributed to polar coordinates and, after measuring these values over several years, can be used to create an estimated orbit of the B star around the A star. Continued observations allow astronomers to further refine this estimation or declassify the star as a binary if the A and B stars appear to be drifting apart [10]. The goal of this project was to create new data for two double stars, STF 1169 AB and STF 42 AB , in order to help future astronomers make better estimated orbits and to better our understanding of these two systems.
STF 1169 (WDS $08165+7930$ ) is a 3 star system located in the Camelopardalis constellation and is classified as a physical double. It was first observed by Friedrich Georg Wilhelm von Struve in 1932; the third star (C) in the system was not discovered until 1962, though [13]. This paper focuses on the analysis of the A and B stars only. The orbit of this binary was first estimated by Kisselev, et al. in 2009 but, due to the stars large estimated orbital period of 200 centuries, STF 1169 is consid-


FIG. 1: The estimated orbital plot of STF 1169 AB calculated by Kisselev, et al. The historical data of STF 1169 AB is represented by the multicolored dots along the orbit; two colors were used, green and pink, and represent data taken through micrometric and photographic processes respectively.
ered a Short-Arc Binary. Short-Arc Binaries are labeled as such since their data has been collected for such a relatively short period of time that the data only spans a small arc of the stars orbit; this makes it especially difficult for astronomers to estimate the orbit and for this orbit to be taken as a good approximation. Continuing to measure the separation and position angle is the only way to increase the accuracy of the estimated orbits of Short-Arc Binaries [4][13]. The current estimated orbit is shown in Figure 1. Our measurements of STF 1169 AB are meant to aid in the process of making this plot more accurate.

STF 42 (WDS $00360+2959$ ) is also a 3 star, physical system and, as with STF 1169, this paper only discusses
the A and B stars. This binary is located in the Andromeda constellation and is part of the Arcturus moving group; it is also classified as a sub-dwarf triple system [6][14]. The first estimated orbit for STF 42 AB was made as a single-lined orbit in 1988 by Latham, et al. and has a period of approximately 1,900 years [12]. Since then, two more orbits have been calculated for STF 42 AB , one in 2009 and the other more recently in 2019. Both of these orbits are considered as potentially accurate and it is still up for debate on which one is correct; these orbits can be seen in Figure 2. Although the period is not as large as that of STF 1169 AB , STF 42 AB is still considered a Short-Arc Binary since data for it has been collected over a relatively short amount of time compared to its orbital period; the arc created by the current data is much smaller than the estimated orbit. Therefore, we approached our measurements of STF 42 AB in the same way as STf 1169 AB : with the intent of creating a larger data set for the binary star in order for future astronomers to create more accurate orbits.

## II. METHODS

In order to measure the separation and position angle, we chose to use the photographic method, which involves taking images of a double star and then measuring the separation and position angle directly from the image. Our images were taken using the Las Cumbres Observatory (LCO), a global network of 25 telescopes; the specific telescopes used were the 0.4 m telescopes located in Spain, Texas, and Hawaii. The LCO's $0.4 m$ telescopes are heavily modified Meade 16 in telescopes and are equipped with an SBIG STL-6303 CCD camera, which can be seen in Figure 3 [3].

When selecting which double stars to observe, two criteria were considered: the magnitudes of the A and B stars and their previously calculated separation. In order for the telescope to be able to take images of our double stars, the A and B stars must have magnitudes between 7 and 10; magnitudes less than 7 would likely be too bright while magnitudes greater than 10 would be too faint. On top of this, the separation must be large enough such that the two stars do not overlap. If one or both stars are too bright and have too small of a separation, the individual stars cannot be resolved, making measurements of their current separation and position angle impossible. Initially, any binary with a separation greater than 3 arcsecs was considered but this was eventually determined to be too small for stars closer to magnitudes of 7 , so this lower limit was changed to 5 or 6 arcsec. To be extra careful, only magnitudes of 8 or greater were consider instead of magnitudes closer to 7 .


FIG. 2: Two potential orbits for STF 42 AB. The top orbit was made in 2009 by Kisselev, et al.; the bottom orbit was calculated in 2019 by Ismailov, et al. In both plots the pink dots represent measurements made photographically, the blue dots were made interferometrically, and the green dots were made micrometrically.

The two stars that met these requirements - and were visible during the time of this project - were STF 1169 AB and STF 42 AB .

STF 42 AB has a primary magnitude of 8.39 , a secondary magnitude of 9.05 , and a most recent separation of 6.037 arcsec. This binary was selected for its relatively low magnitudes and large separation compared to the other stars we had to choose from. STF 1169 AB has a primary magnitude of 8.4 and a secondary magnitude 8.64. It's last measured separation is 20.56 arcsec. We


FIG. 3: The LCO's $0.4 m$ telescope. The component labelled G is the SBIG STL-6303 CCD camera that we used to take our images [3].
initially chose STF 1169 AB as a back-up for if the stars of STF 42 AB were too close and too bright; STF 1169 AB's large separation was the primary factor in this decision, as we knew the A and B stars would not overlap with each other, but we did not choose it as our first subject of observation since we were not sure if this separation would be too big. After receiving the images from the LCO and seeing that both stars were measurable, we decided to make measurements for STF 42 AB and STF 1169 AB.

For each double star, 60 images were taken in total, with 15 images in each of the following filters: Bessel B , Bessel V, SDSS r', and SDSS i'. We chose an exposure time of 1 s and determined the signal to noise ration for both stars in each filter. The equation for the signal to noise ratio is,

$$
\begin{equation*}
\frac{S}{N}=\frac{F A_{\varepsilon} \tau}{\sqrt{N_{R}^{2}+\tau N_{T}}} \tag{1}
\end{equation*}
$$

where $F$ is the point source signal flux on the telescope, $A_{\varepsilon}$ is the effective area, $\tau$ is the integration (exposure) time, $N_{R}$ is the readout noise, and $N_{T}$ is the timedependent noise per unit time. $N_{R}$ is a constant - in this case it is $14 e^{-}$- whereas all other values in the above list can be calculated through various equations, as discussed below.

| Values of Variables |  |
| :--- | :--- |
| Symbol | value |
| $Q_{e}$ | 0.2 |
| $N_{R}$ | $14 e^{-}$ |
| $\Omega$ | $0.571 \operatorname{arcsec}$ |
| $i_{D C}$ | $0.02 \frac{e^{-}}{s}$ |
| $A$ | $1200 \mathrm{~cm}^{2}$ |
| $\varepsilon$ | 0.5 |
| $F_{\beta}$ | $0.1 \frac{\text { photon }}{s \cdot c m^{2}}$ |

TABLE I: The various constants used to calculate $A_{\varepsilon}, N_{T}$, and the signal to noise ratio. These values are based on the telescope used for the observations.

The point source signal flux depends on both the magnitude of the object and the filter it was observed in and can be found using Equation 2.

$$
\begin{align*}
F=10^{-0.4 M} & F_{o}\left(\frac{d \lambda}{\lambda}\right) \\
& \cdot\left(1.51 x 10^{7} \frac{\text { photons }}{\mathrm{m}^{2} \cdot \mathrm{~s} \cdot J y}\right)\left(\frac{m^{2}}{100^{2} \mathrm{~cm}^{2}}\right) \tag{2}
\end{align*}
$$

In the first part of Equation 2, $M$ is the magnitude of the observed object, $F_{o}$ is the flux of an object with magnitude 0 for a given filter, and $\frac{d \lambda}{\lambda}$ is the filter bandpass; this part of the equation calculates the desired flux for a given $M$, while the rest of the equation converts it to the desired untis, $\frac{p h o t o n s}{c m^{2} \cdot s}$. In our calculations of $F$, the magnitude of the fainter star was used so as to ensure that the fainter star would be visible in our images ( 9.05 for STF 42 AB and 8.64 for STF 1169 AB). The values for $F$ can be found in Table II [15].

The effective area, $A_{\varepsilon}$, and time-dependent noise per unit time, $N_{T}$, are calculated using properties of the telescope, with $N_{T}$ also being dependent on $A_{\varepsilon}$ and $F$. The expressions for these values are as follows:

$$
\begin{gather*}
A_{\varepsilon}=A \varepsilon Q_{e}  \tag{3}\\
N_{T}=F A_{\varepsilon}+i_{D C}+F_{\beta} A_{\varepsilon} \Omega \tag{4}
\end{gather*}
$$

In Equation 3, $\varepsilon$ and $Q_{e}$ are the telescope and quantum efficiencies respectively. The variables in Equation 4 are the dark current $\left(i_{D C}\right)$, the background flux $\left(F_{\beta}\right)$, and the pixel size $(\Omega)$. These values can be found in Table I.

After $A_{\varepsilon}$ and $N_{T}$ were calculated, we used Equation 1 to calculate the signal to noise ratios for both stars; the signal to noise ratio for each filter can be found in Table II. Since all of them were above or near 150, we decided that $1 s$ would be an appropriate exposure time for our observations.

Once the images had been received, they were processed in AstroImageJ, a common program used in the

| Signal to Noise Ratios for STF 1169 and STF 42 |  |  |  |
| :--- | :--- | :--- | :--- |
| Star | Filter | Flux $\left(\frac{\text { photons }}{c m^{2} \cdot \mathrm{~s}}\right)$ | S/N |
| 1169 | V | 307.75 | 191.6 |
| 1169 | B | 495.23 | 243.4 |
| 1169 | i | 402.44 | 219.3 |
| 1169 | r | 332.16 | 199.1 |
| 42 | V | 210.96 | 158.5 |
| 42 | B | 339.48 | 201.3 |
| 42 | i | 275.87 | 144.9 |
| 42 | r | 227.69 | 164.7 |

TABLE II: This table shows the fluxes of STF 1169 AB and STF 42 AB in each filter used to take our images. The signal to noise ratios for these images were calculated from these values.
photographic approach to measuring the separation and position angle. Two of the images' properties were checked before any measurements were made in AstroImageJ: first, the image files were not labelled by filter, so the filter of each image was checked through the image header in FITS Liberator; then the image was uploaded into AstroImageJ where we could check if the image had been plate solved or not. The LCO automatically plate solved each image so we did not have to do the plate solving ourselves.

To calculate the current separation and position angle for STF 1169 AB and STF 42 AB , we calculated these values for each image individually and then averaged them; this can be done using the aperture tool in AstroImageJ. After opening AstroImageJ, a toolbar will appear - selecting File->Open and choosing a file will open the chosen image in a separate window with its own toolbar. In order to use the aperture tool on this image, the aperture radius must be set. The aperture size can be changed by then hovering over the Edit menu on the image toolbar and selecting aperture settings. This will open a third window in which you can you can change the Radius of object aperture. In the same window, make sure the box for centroid apertures is checked; this makes sure that when you use the aperture tool, it will snap to the center of the selected star automatically. Hit $O K$ on the aperture window to save your settings; these settings should carry over to each new image you open so that you do not have to reset the aperture every time [5]. For our measurements, an aperture of 7 was used for every image of STF 1169 AB ; for STF 42 AB a miscommunication occurred and different apertures were chosen for different sets of images. For the Bessel B and SDSS r' images an aperture of 4 was chosen, where as for the Bessel V and SDSS i' images, an aperature of 7 and 5 was used for the A star and B star respectively. Despite the miscommunication, the results for the STF 42 AB images were consistent, so we did not make new measurements with

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- 
File Edit Image Process Analyze Plugins Window Help
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Magnifying glass (or "+" and "-" keys; alt or long click for menu)
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FIG. 4: The toolbar that appears when opening AstroImageJ. This toolbar can be used to open images and select the aperture tool used to measure the right ascension and declination of each star in a binary.
different apertures.
The aperture tool can be opened from the AstroImageJ toolbar seen in Figure 4; the icon for the tool is a blue dot surrounded by a red circle. Once the tool is selected drag the cursor over the A star in the image and click; this creates an automatically centered red circle around the star. After the star is selected, a pop-up window containing various measured values of the star will appear. Repeating the aperture tool process for the B star will cause its measurements to appear in the same pop-up as star A. Included in these measurements are the right ascension and declination of each star, the values we used to calculate the separation and position angle [5]. To make these calculations, we copy and pasted the measured values into an Excel sheet and use the following equations:

$$
\begin{gather*}
d R A=15 *(R A(B)-R A(A)) \\
* \operatorname{COS}(R A D I A N S(D E C(B)))  \tag{5}\\
d D E C=D E C(B)-D E C(A)  \tag{6}\\
S E P=S Q R T\left((d R A)^{2}+(d D E C)^{2}\right) * 3600  \tag{7}\\
P A=D E G R E E S\left(A T A N\left(\frac{d R A}{d D E C}\right)\right) . \tag{8}
\end{gather*}
$$

Equations 5 and 6 give the change in right ascension and declination. These values can then be plugged into Equations 7 and 8 to give the separation and position angle. All of these calculations can be made in Excel by replacing $R A, D E C, d R A$, and $d D E C$ with their appropriate Excel cell IDs [5].

After using the above process to calculate the separation and position angle for each image, we then took the average of these values for every filter using the Excel average function,

$$
\begin{equation*}
=A V E R A G E(\text { value } 1: \text { value } 2) \tag{9}
\end{equation*}
$$

To take the average, replace value 1 and value 2 with the Excel cell IDs for the first and last values in the columns for separation and position angle. Then, using the same equation, we took the average of the average values for each filter; this produced a single value for the separation and position angle of the binary star. This process was done for both STF 1169 AB and STF 42 AB individually.


FIG. 5: Images of STF 1169 AB as seen in AstroImageJ. The images coincide to the following filters: A) Bessel B; B) Bessel V ; C) SDSS r'; D) SDSS i'.


FIG. 6: Images of STF 42 AB as seen in AstroImageJ. The images coincide to the following filters: A) Bessel B; B) Bessel V; C) SDSS r'; D) SDSS i'.

## III. RESULTS

The images of STF 1169 AB and STF 42 AB taken through the LCO telescopes can be seen in Figure 5 and Figure 6. As shown in both figures, the images taken in Bessel B and SDSS i' were the least clear, especially for the STF 42 AB Bessel B image, in which the B star is barely visible. This would have created more error when measuring the right ascension and declination of the stars. The images for STF 1169 AB in Bessel B have a different orientation than those of the other filters; this did not appear to affect our measurements of the right ascension and declination and was most likely due to the LCO telescope orientation when they were taking the images.

| STF 1169 AB Separation \& Postion Angle (Bessel B) |  |  |
| :--- | :--- | :--- |
| Image \# | SEP (arcsec) | PA (deg) |
| 1 | 21.02962037 | 14.82907711 |
| 2 | 21.16747835 | 14.86780391 |
| 3 | 21.09634988 | 14.86412929 |
| 4 | 21.07153148 | 14.41235621 |
| 5 | 21.14349919 | 14.58233799 |
| 6 | 21.19271665 | 14.62985391 |
| 7 | 21.152438 | 14.52035633 |
| 8 | 20.89620536 | 15.06537853 |
| 9 | 21.06985297 | 14.74482697 |
| 10 | 21.02969856 | 14.44170222 |
| 11 | 21.10023257 | 14.7510048 |
| 12 | 20.99477581 | 14.85375239 |
| 13 | 20.97805028 | 14.00578514 |
| 14 | 21.14098829 | 14.74900861 |
| 15 | 21.12212508 | 14.23915789 |
| STDEV | 0.081273 | 0.276633 |
| STD Error | 0.020985 | 0.071426 |
| Average | 21.07904 | 14.6371 |

TABLE III: Table of the separation and position angle of STF 1169 AB calculated from 15 images taken in the Bessel B filter. Included in the table are the standard deviation, standard error, and average values for the separation and position angle.

| STF 1169 AB Separation \& Postion Angle (Bessel V) |  |  |
| :--- | :--- | :--- |
| Image $\#$ | SEP $($ arcsec $)$ | PA $(\mathrm{deg})$ |
| 1 | 20.75074 | 14.94954 |
| 2 | 20.68814 | 14.9958 |
| 3 | 20.72076 | 15.25346 |
| 4 | 20.74568 | 14.89704 |
| 5 | 20.74244 | 15.32166 |
| 6 | 20.75809 | 15.25363 |
| 7 | 20.85388 | 15.01385 |
| 8 | 20.76118 | 14.94185 |
| 9 | 20.66288 | 15.04285 |
| 10 | 20.69387 | 15.24559 |
| 11 | 20.7564 | 15.19861 |
| 12 | 20.81125 | 15.07345 |
| 13 | 20.81657 | 15.01338 |
| 14 | 21.01225 | 14.84191 |
| 15 | 20.82167 | 15.06563 |
| STDEV | 0.084787 | 0.147201 |
| STD Error | 0.021892 | 0.038007 |
| Average | 20.77305 | 15.07388 |

TABLE IV: Table of the separation and position angle of STF 1169 AB calculated from 15 images taken in the Bessel V filter. Included in the table are the standard deviation, standard error, and average values for the separation and position angle.

| STF 1169 AB Separation \& Postion Angle (SDSS r') |  |  |
| :--- | :--- | :--- |
| Image $\#$ | SEP (arcsec) | PA (deg) |
| 1 | 20.75959 | 14.88684 |
| 2 | 20.63625 | 14.92135 |
| 3 | 21.16053 | 16.39225 |
| 4 | 21.09665 | 16.66636 |
| 5 | 21.0722 | 16.5463 |
| 6 | 20.97782 | 16.67934 |
| 7 | 20.56713 | 15.28496 |
| 8 | 20.70235 | 14.87256 |
| 9 | 21.17709 | 16.26804 |
| 10 | 21.03772 | 16.57447 |
| 11 | 20.69255 | 14.96437 |
| 12 | 20.95934 | 16.75062 |
| 13 | 20.71183 | 14.89372 |
| 14 | 20.70901 | 14.98038 |
| 15 | 20.66543 | 15.06919 |
| STDEV | 0.211898 | 0.82405 |
| STD Error | 0.054712 | 0.212769 |
| Average | 20.8617 | 15.71672 |

TABLE V: Table of the separation and position angle of STF 1169 AB calculated from 15 images taken in the SDSS r' filter. Included in the table are the standard deviation, standard error, and average values for the separation and position angle.

| STF 1169 AB Separation \& Postion Angle (SDSS i') |  |  |
| :--- | :--- | :--- |
| Image \# | SEP (arcsec) | PA (deg) |
| 1 | 21.11097 | 16.38504 |
| 2 | 21.13246 | 16.47914 |
| 3 | 21.17046 | 16.44897 |
| 4 | 20.9803 | 16.77026 |
| 5 | 21.02071 | 16.62502 |
| 6 | 21.01301 | 16.52001 |
| 7 | 21.1055 | 16.47293 |
| 8 | 21.21532 | 16.27476 |
| 9 | 21.08332 | 16.40694 |
| 10 | 21.06625 | 16.56053 |
| 11 | 21.02762 | 16.61949 |
| 12 | 21.03517 | 16.58551 |
| 13 | 21.17179 | 16.39225 |
| 14 | 21.09583 | 16.45303 |
| 15 | 20.69625 | 14.92542 |
| STDEV | 0.120903 | 0.424033 |
| STD Error | 0.031217 | 0.109485 |
| Average | 21.06166 | 16.39462 |

TABLE VI: Table of the separation and position angle of STF 1169 AB calculated from 15 images taken in the SDSS i' filter. Included in the table are the standard deviation, standard error, and average values for the separation and position angle.

| STF 42 AB Separation \& Postion Angle (Bessel B) |  |  |
| :--- | :--- | :--- |
| Image \# | SEP (arcsec) | PA (deg) |
| 1 | 6.134937 | 20.99629 |
| 2 | 6.341314 | 19.83252 |
| 3 | 6.306774 | 21.7645 |
| 4 | 6.243299 | 21.99722 |
| 5 | 6.511425 | 20.60703 |
| 6 | 6.203269 | 22.14675 |
| 7 | 6.443348 | 23.5298 |
| 8 | 6.367634 | 21.99923 |
| 9 | 6.015635 | 22.39328 |
| 10 | 6.383747 | 21.48898 |
| 11 | 6.393876 | 20.55534 |
| 12 | 6.461935 | 22.10868 |
| 13 | 6.423966 | 21.34789 |
| 14 | 6.29427 | 22.26928 |
| 15 | 6.360303 | 21.57208 |
| STDEV | 0.13186145 | 0.89924497 |
| STD Error | 0.03404648 | 0.23218405 |
| Average | 6.32571547 | 21.6405913 |

TABLE VII: Table of the separation and position angle of STF 42 AB calculated from 15 images taken in the Bessel B filter. Included in the table are the standard deviation, standard error, and average values for the separation and position angle.

| STF 42 AB Separation \& Postion Angle (Bessel V) |  |  |
| :--- | :--- | :--- |
| Image \# | SEP (arcsec) | PA (deg) |
| 1 | 6.22628881 | 20.2140826 |
| 2 | 6.18239252 | 20.3639481 |
| 3 | 6.25332283 | 20.1229036 |
| 4 | 6.24994269 | 20.1342573 |
| 5 | 6.23980379 | 20.1683929 |
| 6 | 6.20264702 | 20.2945146 |
| 7 | 6.21953271 | 20.2369996 |
| 8 | 6.3032942 | 20.4094935 |
| 9 | 6.26789681 | 19.619581 |
| 10 | 6.18239249 | 20.3639472 |
| 11 | 6.19589453 | 20.3176091 |
| 12 | 6.22628888 | 20.2140845 |
| 13 | 6.24318322 | 20.1570033 |
| 14 | 6.22628875 | 20.2140813 |
| 15 | 6.29163947 | 19.5425253 |
| STDEV | 0.0360237 | 0.25049577 |
| STD Error | 0.00930128 | 0.06467773 |
| Average | 6.23405391 | 20.1582282 |

TABLE VIII: Table of the separation and position angle of STF 42 AB calculated from 15 images taken in the Bessel V filter. Included in the table are the standard deviation, standard error, and average values for the separation and position angle.

| STF 42 AB Separation \& Postion Angle (SDSS r') |  |  |
| :--- | :--- | :--- |
| Image $\#$ | SEP (arcsec) | PA (deg) |
| 1 | 6.266845 | 20.0776 |
| 2 | 6.367445 | 20.19485 |
| 3 | 6.254334 | 19.66387 |
| 4 | 6.270227 | 20.06631 |
| 5 | 6.213671 | 19.79791 |
| 6 | 6.256703 | 20.11156 |
| 7 | 6.208175 | 19.35803 |
| 8 | 6.260084 | 20.10023 |
| 9 | 6.253323 | 20.1229 |
| 10 | 6.244165 | 19.69722 |
| 11 | 6.266846 | 20.0776 |
| 12 | 6.334518 | 19.85404 |
| 13 | 6.279682 | 20.48967 |
| 14 | 6.267897 | 19.61958 |
| 15 | 6.239804 | 20.16839 |
| STDEV | 0.04040558 | 0.28781995 |
| STD Error | 0.01043268 | 0.07431479 |
| Average | 6.26558127 | 19.959984 |

TABLE IX: Table of the separation and position angle of STF 42 AB calculated from 15 images taken in the SDSS r' filter. Included in the table are the standard deviation, standard error, and average values for the separation and position angle.

| STF 42 AB Separation \& Postion Angle (SDSS i') |  |  |
| :--- | :--- | :--- |
| Image \# | SEP (arcsec) | PA (deg) |
| 1 | 6.21497934 | 19.3363231 |
| 2 | 6.24756652 | 19.6863888 |
| 3 | 6.29729849 | 19.9767081 |
| 4 | 6.23401009 | 19.7309734 |
| 5 | 6.28038451 | 20.032809 |
| 6 | 6.18320746 | 19.8999064 |
| 7 | 6.1899779 | 19.8772188 |
| 8 | 6.20013542 | 19.8432816 |
| 9 | 6.22292261 | 20.2258342 |
| 10 | 6.20690858 | 19.8207254 |
| 11 | 6.21368258 | 19.7982155 |
| 12 | 6.20013563 | 19.8432868 |
| 13 | 6.20352194 | 19.8319989 |
| 14 | 6.22630073 | 20.2143806 |
| 15 | 6.20013564 | 19.8432871 |
| STDEV | 0.03222992 | 0.21256097 |
| STD Error | 0.00832173 | 0.05488301 |
| Average | 6.22141116 | 19.8640892 |

TABLE X: Table of the separation and position angle of STF 42 AB calculated from 15 images taken in the SDSS i' filter. Included in the table are the standard deviation, standard error, and average values for the separation and position angle.

All 15 calculations of separation and position angle for each filter and binary star can be seen in Tables III through X. As well as the measurements, the standard deviation and standard error were calculated and
included in the tables. The standard deviation was calculated with the Excel function listed below.

$$
\begin{equation*}
=S T D E V(\text { value } 1: \text { value } 2) \tag{10}
\end{equation*}
$$

The standard error can be found by dividing the standard deviation by the square-root of the number of measurements made and was calculated using the following Excel function:

$$
\begin{align*}
& =S T D E V(\text { value } 1: \text { value } 2) / \\
& \quad \operatorname{SQRT}(\operatorname{COU} N T(\text { value } 1: \text { value } 2)) \tag{11}
\end{align*}
$$

As with Equation 9, Equations 10 and 11 can be performed in Excel by replacing value 1 and value 2 with the first and last cell IDs of the Column of data under analysis.

For the separation calculations of STF 1169 AB, the standard deviation was less than 0.3 arcsec in all filters; the highest standard deviation was 0.21 arcsec from the SDSS r' image, while the smallest standard deviation occurred in the Bessel B image at a value of 0.08 arcsec. The standard error for the separation did not go higher than 0.06 arcsec, with the largest error being 0.055 arcsec (SDSS r') and the smallest error being 0.022 arcsec. Given these values, we determined that all of the calculated values of STF 1169 AB's separation were within a reasonable error.

In terms of STF 1169 AB's position angle, the standard deviations and errors were higher in all filters than for the separation. The standard deviation reached a maximum of $0.82^{\circ}$ in the SDSS r' filter and a minimum of $0.15^{\circ}$ in the Bessel V filter. On the other hand, the standard error ranged from $0.21^{\circ}$ in the SDSS r' image to $0.038^{\circ}$ in the Bessel V image. Although they were larger than the position angle measurements, these values indicated that the measurements had a small enough error.

For both the separation and position angle of STF 1169 AB , the SDSS r' filter's image were the greatest source of error. In both measurements, the Bessel filters produced the smallest error.

| Final Measurement of STF 1169 AB |  |  |
| :--- | :--- | :--- |
| Filter | SEP $(\operatorname{arcsec})$ | PA $(\mathrm{deg})$ |
| B | 21.07904 | 14.6371 |
| V | 20.77305 | 15.07388 |
| r | 20.8617 | 15.71672 |
| i | 21.06166 | 16.39462 |
| STDEV | 0.15063952 | 0.767158 |
| STD Error | 0.07531976 | 0.383579 |
| Average | 20.9438625 | 15.45558 |

TABLE XI: This table includes the averaged values of the separation and position angle of STF 1169 AB for each filter used. The average of these values is at the bottom of this table, along with its standard deviation and position angle.

Table XI shows the averaged values of the separation and position angle from each filter combined for STF 1169 AB , which came out to be 20.94 arcsec and $15.45^{\circ}$. The standard deviation and standard error for both values were low enough for the measurements to be deemed reasonable, but the errors in the position angle were higher than that of the separation. This could be due to a miscommunication in which star is the A or B star; in Equations 5 and 6, it does not matter which values for the right ascension and declination are subtracted first as long as it is consistent between both equations, but Equation 5 has a dependency on the $\cos$ of the declination of the B star. This factor changes depending on which star is selected as the B star and could change the values of $d R A$ calculated. It was not discussed which star should be considered the A and B star, so it is likely that some measurements were made with the A and B stars backwards. The factor of $d R A$ is used in both the separation and position angle expressions but it is possible that this error affected the position angle more.

| STF 1169 AB Historical Data |  |  |
| :--- | :--- | :--- |
| Year | SEP $(\operatorname{arcsec})$ | PA $(\mathrm{deg})$ |
| 1832.35 | 20.74 | 10.1 |
| 1900.35 | 20.82 | 11.5 |
| 1930.19 | 20.913 | 12.8 |
| 1947.27 | 20.8 | 13.8 |
| 1962.227 | 20.692 | 14.962 |
| 2018.973 | 20.56 | 15.13 |
| 2022 | 20.94 | 15.5 |

TABLE XII: The first six values listed are historical measurements of STF 1169 AB's separation and position angle (provided by the WDS) and the last measurement included is the one made in this project. The historical data set included dozens of observations; these six were chosen as they represented the beginning of the time period in which the each particular position angle was most common. For example, starting in 1930 a position angle of about $14^{\circ}$ was most common.

Our measurements for STF 1169 AB also fit the orbital trend established by past observations. As seen in Table XII, the binary system's separation has remained relatively constant since its first measurement in 1832; this is likely due to the stars large orbital period. On the other hand, STF 1169 AB's position angle has increased steadily since 1832, with its last measurement - prior to ours - being in 2018 and having a position angle of $15.13^{\circ}$. The measured position angle from this project, $15.5^{\circ}$, agrees with the prior measurement.

The standard deviations and errors for STF 42 AB's measurements are similar to that of STF 1169 AB. STF 42 AB's separation had standard deviations no greater than 2 arcsec; the highest standard deviation was in the

Bessel B filter at 0.13 arcsec, while the lowest was at 0.032 arcsec in the SDSS i' filter. Its standard error for the separation had a maximum of 0.034 arcsec in the Bessel B filter and a minimum of 0.0083 arcsec in the SDSS i' filter. The higher degree of error in the B filter is likely due to how faint the second star is as seen in Figure 6; this would have made it difficult for AstroImageJ to select the same or near-the-same centroid for the second star in each image. Despite this, we considered the error to be small enough.

STF 42 AB's position angle calculations also had a higher standard deviation and error compared to its separation, which is again considered to be a consequence of not choosing the same A and B star for each measurement. The highest standard deviation came from the Bessel B images with a value of $0.90^{\circ}$, whereas the smallest standard deviation was produced by the SDSS i' filter and had a value of $0.21^{\circ}$. Bessel B and SDSS i' gave the highest and lowest standard error respectively with values of $0.23^{\circ}$ and $0.054^{\circ}$. Again these errors were determined to be low enough.

The Bessel B filters produced the worst error in the data set for both the separation and position angle of STF 42 AB, while SDSS i' gave the best results in each case. Bessel B's poor performance is, again, most likely due to the poor visibility of the second star in its images.

| Final Measurement of STF 42 AB |  |  |
| :--- | :--- | :--- |
| Filter | SEP (arcsec) | PA (deg) |
| B | 6.32571547 | 21.6405913 |
| V | 6.23405391 | 20.1582282 |
| r | 6.26558127 | 19.959984 |
| i | 6.22141116 | 19.8640892 |
| STDEV | 0.0465494 | 0.83230682 |
| STD Error | 0.0232747 | 0.41615341 |
| Average | 6.26169045 | 20.4057232 |

TABLE XIII: This table includes the averaged values of the separation and position angle of STF 42 AB for each filter used. The average of these values is at the bottom of this table, along with its standard deviation and position angle.

The finalized measurements of STF 42 AB's separation and position angle can be found in Table XIII, which are 6.26 arcsec and $20.41^{\circ}$. As expected, the standard deviation and error for the position angle are greater than for the separation, but both sets of errors are small enough for the data to be considered reasonable.

| STF 42 AB Historical Data |  |  |
| :--- | :--- | :--- |
| Year | SEP $(\operatorname{arcsec})$ | PA $(\mathrm{deg})$ |
| 1824.88 | 6.55 | 33.7 |
| 1895.92 | 5.56 | 28.3 |
| 1948.71 | 5.76 | 25.6 |
| 1984.877 | 6.120 | 22.879 |
| 2015.632 | 6.2838 | 20.945 |
| 2020.74 | 6.037 | 20.82 |
| 2022 | 6.26 | 20.41 |

TABLE XIV: The first six values listed are historical measurements of STF 42 AB's separation and position angle (provided by the WDS) and the last measurement included is the one made in this project. The historical data set included dozens of observations; these six were chosen as they represented the beginning of the time period in which the each particular position angle was most common. For example, starting in 1948 a position angle of about $22^{\circ}$ was most common.

We can again compare our data to the historical data by referring to Table XIV. As with STF 1169 AB, STF 42 AB's separation has held constant since its initial calculation in 1824 . The position angle has increased periodically over time and has consistently been at around $20^{\circ}$ for the last seven years. STF 42 AB's last measurement was made two years ago in 2020; our measurements of the separation and position angle are approximately equal to the prior measurements made, suggesting that STF 42 AB has not progressed much along its orbit. This is again likely due to STF 42 AB's large orbital period.

## IV. DISCUSSION

Since our values of separation and position angle for both STF 1169 AB and STF 42 AB were consistent with both the historical trend and the latest measurement prior to our own, we can - for now - say that both binary stars are following the estimated orbital plots. Despite the accuracy of our data though, due to the long orbital periods of STF 1169 AB and STF 42 AB, it will take many more years before these estimated orbits can be considered accurate. As seen in Figures 1 and 2, the historical measurements span relatively short arcs of the estimated orbits despite including nearly two centuries worth of observations. For STF 1169 AB, the B star orbits in a counter-clockwise direction; if we had been able to insert our data point into this plot, it would appear on the outer edge of the pink, photographic data points. On both plots for STF 42 AB , the B star orbits in a clockwise direction and our data point would have appeared just after the blue, interferometric data points going in the same direction.

It was our intention going into this project to include our data points in a plot along with the historical data.

We attempted to do this using the Excel Plot Tool 3.19 developed by Richard Harshaw, but were unable to do so due to missing information; the plotting tool required information of the double stars that we did not have and failed to retrieve from the Gaia DR2 database suggested by Harshaw [8]. Another attempt was made to make a simple scatter plot of the data in Excel, but the plots generated provided no meaningful information on the double stars' orbits. This method most likely also required the information mentioned by Harshaw that we could not access. Therefore, since we could not graphically compare our data to the historical data, we chose to present a small portion of the historical data as a table so that the binary stars' progression could be seen visually. Gaining access to the missing Gaia DR2 data, though, would have helped us to analyze our data more effectively.

A potential way to make our data more accurate would be to directly measure the separation and position angle in AstroImageJ. In our haste to process our images, we used Equations 5 through 8 from a tutorial website with the intent of finding a scientific paper for them at a later date. While searching for this scientific source, we discovered that there was a way to make our measurements directly in the AstroImageJ program, but we had already calculated all of our data by that point and did not have time to redo it. The process of measuring the separation and position angle in AstroImageJ is simple; using the same aperture tool and settings, click on the A star and drag the mouse to the B star. Once the click is released, the aperture is automatically centered for both stars and a new pop-up window appears with the separation and position angle already calculated [2]. It is likely that this process uses the same equations as listed in this paper, but it would have been much more efficient to let AstroImageJ do the calculations for us.

Other changes that would be beneficial to our procedure would be selecting a standard aperture to use in every measurement for a given double and designating which star is the B star before making any measurements. Although it did not seem to affect the output of right ascension and declination measurements, a potential source of error could have been our irregular use of aperture size, specifically for STF 42 AB. Standardizing a specific aperture size across all filters could have created more accurate results for us to work with.

As discussed above, we did not state beforehand which stars were the A and B stars, causing our calculation of the change in right ascension, $d R A$, to have some deviation between filters. The differences in the declinations of the A and B star were small enough that these deviations were not noticeable and could potentially be considered negligible. Still it would be good practice to make sure we use the same star as the B star in future measure-
ments so as to decrease any error it may have caused. Of course, if we were to use AstroImageJ to directly measure the separation and position angle, we would be able to skip this step entirely.

With the Gaia DR2 data and us now knowing how to measure the separation and position angle in AstroImageJ with proper aperture settings, we should be able to produce more accurate and usable results for future observations of STF 1169 AB and STF 42 AB or any other
binary stars we may observe in the future

## V. ACKNOWLEDGMENTS

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