Period-luminosity Relation of the Cepheid Variable Stars and Its Importance in Measurement of Cosmic Distance

Physics Department, University of California, Santa Barbara, CA 93106

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This article aims to find and varify the period-luminosity relation of the cepheid variable stars. One can use this relationship to accurately measure cosmic distances. In order to achieve this purpose, this experiment selected targets from three different types of Cepheid variables and analyzed them through the pictures provided by LCO and the data provided by ASAS-SN. The final targets are BI CAS, RR Lyr and NSV 4148. Among them, BI CAS, which is a classical Cepheid variable star, has a more accurate cycle-light relationship, and can be used as a standard candle for cosmic ranging to accurately measure the Hubble constant.

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I. INTRODUCTION

When we were young, whenever we looked up at the starry sky, we would find that the stars were twinkling, as if they were blinking at us. At the time, we would naively think that it was the rapid changes in the brightness of the stars. As we grew up, we discovered that this is because any star (except the sun) is far, far away from the earth, and basically only one ray of light reaches the earth. When light enters the atmosphere, it will be scattered differently when passing through air of different temperatures. If that ray of light scatters away from us, the stars seem to disappear temporarily; when the light just hits the eye, the stars reappear, creating a flickering effect. Although the "flickering" of stars we usually see does not mean that their luminosity has changed, there is indeed a type of star in the universe: variable stars, whose luminosity show periodic changes. The most characteristic and important of these stars is the Cepheid variable star.

The members of Cepheid variables are very bright variable stars whose luminosity and pulsation period have a very strong direct correlation. They are divided into several subcategories that exhibit distinct masses, ages, and evolutionary histories: Classic Cepheids, Type II Cepheids, Anomalous Cepheids, and Dwarf Cepheids. The Cepheid variable gets its name from the Delta Cepheus star in the constellation Cepheus, which was discovered as a variable star by John Goodrick in 1784. Because it is the first confirmed variable star of this type, and its Chinese name is Cepheus One, hence the name. Cepheid One is also a Cepheid variable star that is particularly important when verifying the period-luminosity relation because its distance is the most accurate among Cepheid variable stars, thanks to the fact that its members are all in the star cluster and can be seen from Hubble Space Telescope or Hipparcos get reliable parallax.[1]

Cepheid variables play an extremely important role in determining the distance scale of extragalactic galaxies. The reason why they are widely used is that they have the following characteristics at the same time: (1) There is a definite period-luminosity relation, which can be used to calculate the luminosity distance of the target celestial body, and the accuracy is very high; (2) The unique optical variable properties make it easy to confirm this type of variable star, and there is no misjudgment; (3) These are some high-luminosity supergiant stars, which can be observed even in faraway places. The distance range can reach about 20 Mpc (Virgo cluster range); (4) They are common in the Milky Way (including the sky area near the sun) and some extragalactic galaxies.[10]

A. Historical Background

In 1908, the American female astronomer Leavitt noticed for the first time that in the Wheat Cloud (SMC), the longer the light change period of the Cepheid variable star, the greater the brightness of the variable star, and there were 16 sample variable stars. Four years later, observations of 25 Cepheid variables in the SMC are again This is confirmed: there is a simple linear relationship $m \propto \log_{10} P$ between the stellar light change period P and the apparent magnitude m, which is called the period-luminosity relation. Given the importance of the period-luminosity relation for the study of galaxies and cosmology, including the determination of the Hubble constant, at a symposium commemorating the 100th anniversary of the aforementioned discovery by Leavitt held at the Harvard-Smithsonian Center for Astrophysics (CFA) in November 2008, the conference organizer proposed to call the period-luminosity relation of Cepheid variables as Leavitt's law.[2]

In fact, the discovery of Leavitt at that time only gave the slope of the period-luminosity relation but did not make an absolute calibration for the apparent magnitude m. In 1913, Hertzsrung used the proper motions of 13 Cepheid variable stars in the Milky Way to determine the statistically apparent difference (i.e. the average distance), the absolute visual magnitude of a variable star with a light variation period of 6.6 $days$ is obtained as $M_V = -2.2\ mag$, and the following the period-luminosity relation is given by means of the slope obtained by Leavitt:

$$< M_V >= -2.11 * log_{10}P - 0.6 \tag{1}$$

$< M_V >$is the average absolute visual magnitude of the variable star, and Hertzsrung also calculated the distance of the SMC accordingly. Soon, Hubble determined the distances of several nearby galaxies by using the period-luminosity relation of Cepheid variables.[6]

Following Hertzsrung, until recently, people have done a lot of work on the absolute calibration of the period-luminosity relation (including slope and zero point) and explored and commented on some important issues related to the period-luminosity relation from many aspects, such as two period-luminosity relation of Cepheid variable stars of different stellar populations, cycle light-color (PLC) relation, the period-luminosity relation of non-linear form, the multiband period-luminosity relation, the period-luminosity relation of maximum luminosity (instead of average luminosity), the metalness effect of the period-luminosity relation, etc.[3]

As a class of extremely important scale-distance celestial bodies, Cepheid variable stars not only enjoy a high reputation in history, even today, they still play a very important role in distance calibration and the determination of the Hubble constant H_0. In recent years, Cepheid variable stars This is amply illustrated by extensive work on the period-luminosity relation. On the other hand, if comparing the distances of the same target celestial body measured by various methods, the luminosity distance obtained from the Cepheid variable star period-luminosity relation is often the most accurate, and this is the main reason why people attach importance to the study of the Cepheid variable star period-luminosity relation.

For a scientific problem, with the deepening of the research work, the solution to the problem often becomes more and more complicated. In the beginning, people's initial understanding of the problem can open up a research field, and with the continuous accumulation of observation data, this early understanding will often be replaced by a later understanding, the problem of Cepheid variables as standard distance objects, the situation is also in this way. It is pointed out by the way that besides Cepheids, the period-luminosity relation of some other variable stars (such as Mira variables) can also be used for photometric distance determination, but the scope of application is far less than that of Cepheids.[5]

B. Importance of the Research

Whenever we look up at the bright starry sky, we can't help but admire it. Looking at these twinkling lights, one can't help but wonder: how far are these stars from us? The closest star to us is the sun. Using the Venus transit method, we can easily measure the distance from the earth to the sun. If other celestial bodies are identical to the sun, then we can know the distance between the celestial bodies and us according to the brightness of the celestial bodies we observe on the earth. However, the actual situation is not so simple, we cannot simply treat other celestial bodies in the same way as the sun. What's more, this method is obviously ineffective in the face of distant galaxies, and Cepheid variable stars, as the first generation of "standard candles", have become a ruler for human beings to measure the universe. Leavitt discovered the "period-light relationship" of Cepheid variable stars, that is: the absolute magnitude of Cepheid variable stars is proportional to their light change period, which is also called "Leavitt's law". We can use Cepheid variable stars as "standard candles" and turn them into a ruler that can measure the universe. The principle is simple, imagine a candle and measure the brightness of the candle at a known distance. The farther the candle is from us, the less brightness we measure. The distance between the candle and us can be determined according to the simple relationship that "brightness is inversely proportional to the square of the distance". So as long as the absolute magnitude of the Cepheid variable star is known, then the distance measurement of the star will only need to measure the luminosity change period and apparent magnitude of the Cepheid variable star. Leavitt's discovery was the first standard candle found in human history.

Moreover, Hubble's law says that galaxies are moving away from Earth at speeds proportional to their distance. In other words, the farther they are, the faster they are moving away from Earth. The velocity of the galaxies has been determined by their redshift, a shift of the light they emit toward the red end of the visible spectrum. Hubble's law is considered the first observational basis for the expansion of the universe, and today it serves as one of the pieces of evidence most often cited in support of the Big Bang model. The motion of astronomical objects due solely to this expansion is known as the Hubble flow. It is described by the equation $v = H_0 * D$, with H_0 the constant of proportionality,the Hubble constant, between the "proper distance" D to a galaxy, which can change over time, unlike the comoving distance, and its speed of separation v, i.e. the derivative of proper distance with respect to the cosmological time coordinate. The value of Hubble's constant is usually measured via the redshift of distant galaxies, which is a different method than Hubble's law for measuring the distance to the same galaxy. However, uncertainties in the physical assumptions used to measure these distances have led to varying results for the value of the Hubble constant. During the second half of the 20th century, most values of the Hubble constant H_0 were estimated to be between 50 and 90 $(km/s)/Mpc$. On May 7, 2009, NASA released the latest measurement of the Hubble constant. According to the latest measurement results of Ia supernovae in distant galaxies, the constant was determined to be 74.2 ± 3.6 $(km/s)/Mpc$, and the uncertainty was further reduced to within 5%. On October 3, 2012, astronomers used NASA's Spitzer Infrared Space Telescope to accurately calculate the Hubble constant, and the numerical result was 74.3 ± 2.1 $(km/s)/Mpc$. On December 20, 2012, the Wilkinson Microwave Anisotropy Probe experimental team of NASA announced that the Hubble constant is 69.32 ± 0.80 $(km/s)/Mpc$. On March 21, 2013, according to the data obtained from the Planck satellite observation, the Hubble constant is 67.80 ± 0.77 $(km/s)/Mpc$. In July 2018, using the Hubble telescope and the Gaia mission, the measured value of the Hubble constant is 73.52 ± 1.62 $(km/s)/Mpc$. It can be seen that there is no precise conclusion on the related determination of the Hubble constant so far. Observations and measurements are continuously updated. According to the "standard candle" mentioned above, one can calculate the "proper distance" D of a cepheid variable star. With further observation of the speed of separation of this star, one can get a precise Hubble constant H_0. [5]

II. METHODS

The main purpose of this research is to continuously observe cepheid variable stars to explore and varify their period-luminosity relation. Secondly, during the observation process, one can also measure the apparent magnitude of Cepheid variable stars to calculate and verify their distance.

The basic principle of this experiment is also very simple. According to Leavitt's Law, the period-luminosity relation of cepheid variable stars can usually be expressed by the following formula:

$$< M_V >= a * log_{10} P + c \tag{2}$$

In the formula, $< M_V >$ is the mean absolute magnitude of the variable star, P is the light variation period, and a and c are the slope and zero point of the period-luminosity relation, respectively. It is often believed that for classical Cepheids, $-6\ mag < (< M_V >) < -2\ mag$ and $2\ days < P < 100\ days$.[11]

Moreover, almost all astronomical objects used as physical distance indicators belong to a class that has a known brightness. By comparing this known luminosity to an object's observed brightness, the distance to the object can be computed using the inverse-square law. These objects of known brightness are termed standard candles, coined by Leavitt. In our research the standard candles are the cepheid variable stars due to their precise period-luminosity relation.

The brightness of an object can be expressed in terms of its absolute magnitude. This quantity is derived from the logarithm of its luminosity as seen from a distance of 10 parsecs. The apparent magnitude m, the magnitude as seen by the observer (an instrument called a bolometer is used), can be measured and used with the absolute magnitude M to calculate the proper distance D to the object in parsecs as follows:

$$5 * log_{10} D = m - M + 5 \tag{3}$$

or,

$$D = 10^{(m-M+5)/5} \tag{4}$$

With Equation 2 and Equation 4, one can easily find the period-luminosity relation and calculate the proper distance D.

A. Choice of Targets

In 1944, Baade proposed that stars can be divided into two categories, population I and population II, according to their observed characteristics and physical properties. 1956 Baade confirmed that Cepheid variable stars should be classified into star population I (type I) Cepheid variable stars (also known as classical Cepheid variable stars)and Population II (Type II) Cepheids, they have different zero points of period-luminosity relation, and the zero point difference is $c_{II} - c_I = 1.5\ mag$, that is, for Population I and Population II Cepheids with the same optical cycle, the average absolute magnitude of the former is about 1.5 mag smaller (that is, brighter) than the latter (actually Baade reported his result at the IAU Congress in Rome in 1952). [3]This finding (known as the zero point correction of the period-luminosity relation) is crucial, as it shows that different period-luminosity relation must be used for Cepheids of different stellar populations. Otherwise, it will bring an error of 1.5 mag to the distance modulus of the target celestial body so that the measured distance value of the celestial body will be higher than the actual value. This difference will further affect the accuracy of the Hubble constant H_0.[4]

Therefore, the expressions of the period-luminosity relation should be discussed according to the class of cepheid variable stars. According to what was mentioned before, they are mainly divided into: (Here, Dwarf Cepheids are not considered in this research because they are rare and there is no specific classification of this type of cepheid variable stars in ASAS-SN Variable Stars Database.)

1. Classical Cepheids: (also known as Type I Cepheids, or Delta Cephei) variable stars of the first population, 4-20 times more massive than the Sun, and can be as much as 100,000 times as luminous as the Sun. They pulsate very regularly with cycles ranging from days to months. Classical Cepheids are used to measure the distances of galaxies within and outside the Local Group and are used to estimate the Hubble constant. They have also been used to elucidate many properties of our galaxy, such as the position of the Sun above the galactic disk and the local spiral structure of the Milky Way. For this type of cepheid variable stars, BI CAS was chosen to be the observation object.[9]

2. Type II Cepheids: (also known as Population II Cepheids) are Population II variable

stars with pulsation periods between 1-50 days. Type II Cepheids are usually metal-poor, old (10 billion years), and low mass (0.5 solar mass). Type II Cepheid variables are divided into several subgroups, the BL Her subgroup with a period of 1-4 days, the W Vir subgroup with a period of 10-20 days, and the RV Tau subgroup with a period of more than 20 days. Type II Cepheids can be used to establish distances to galactic centers, globular clusters, and galaxies. For this type of cepheid variable stars, NSV 4148 was chosen to be the observation object.[9]

3. Anomalous Cepheids: They are a group of pulsating variable stars with periods of less than 2 days in the unstable belt, similar to RR Lyrae variable stars, but with higher luminosity. Anomalous Cepheids are more massive than Type II Cepheids, RR Lyrae, and higher than our Sun. It is unclear whether they are young stars that are turning back to the horizontal branch, blue stragglers undergoing mass transfer in binary galaxies, or even a mix of the two. For this type of cepheid variable stars, RR Lyrae was chosen to be the observation object. (Although they are not exactly the same, they share the most similar properties.)[9]

The observation images and data of the three selected observation objects come from Las Cumbres Observatory (LCO). Here, due to the limitation from LCO (they cannot observe too bright cepheid variable stars), the apparent magnitude of the target should be around 10 and the period should be about 0-3 days. (The longer the period, the more difficult it is to observe the change of luminosity, which makes it difficult to obtain the period.) When the research began, many observation requests for other Cepheids were submitted, but none of them met the requirements. Finally, by entering the aforementioned requirements in the ASAS-SN Variable Stars Database: VMag from 10-13, and Period from 0-3, BI CAS and NSV 4148 were found to be the most appropriate for (DCEP, The ASAS-SN Catalog of Variable Stars: II) and (CWB, The ASAS-SN Catalog of Variable Stars: II). [(1,2), 1 represent the Variable Type and 2 represent the References in ASAS-SN Variable Stars Database]. (RR Lyrae is not found in ASAS-SN Variable Stars Database since its original observation request are approved.) The basic data for these targets are given by FIG. 1, FIG. 2, and FIG. 3. [8]

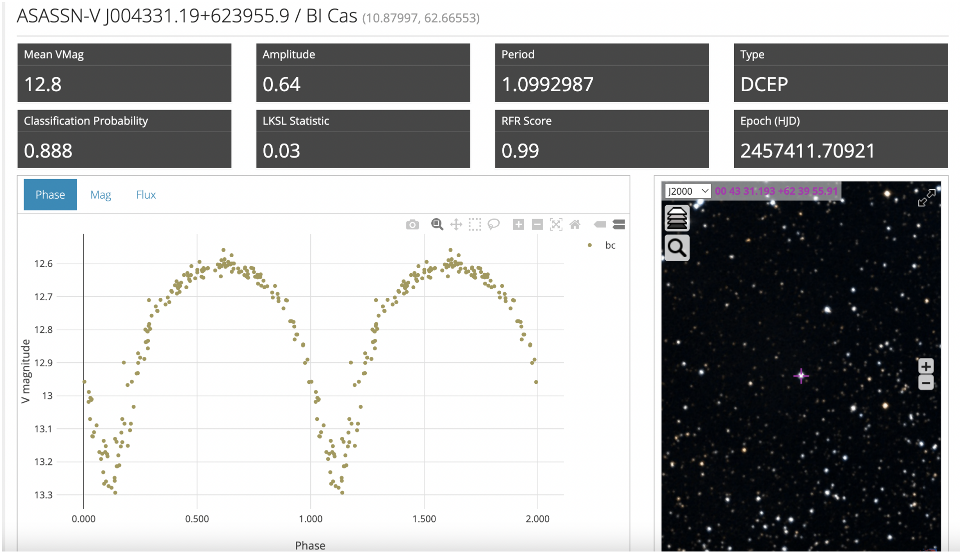

FIG. 1: This is the basic data for target BI CAS in ASAS-SN Variable Stars Database.

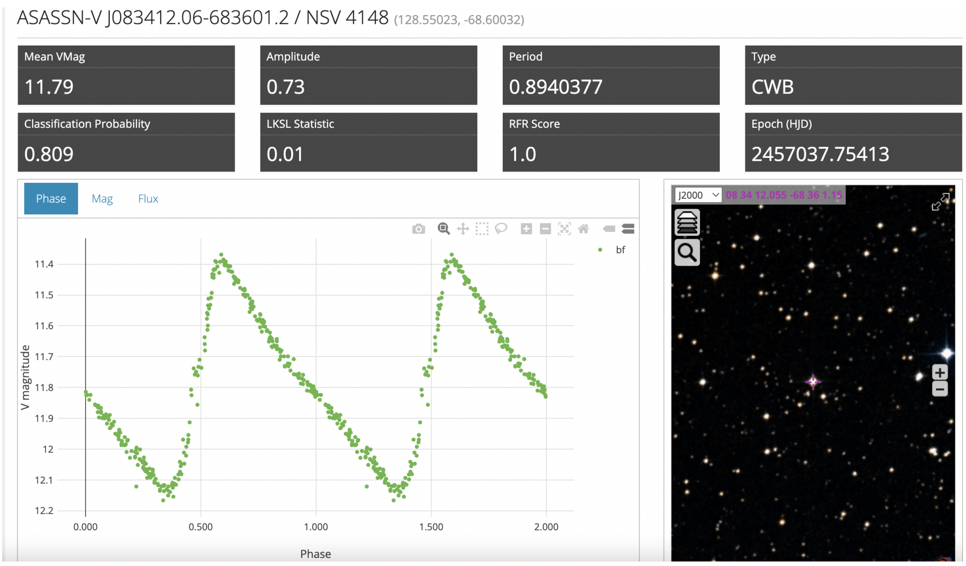

FIG. 2: This is the basic data for target NSV 4148 in ASAS-SN Variable Stars Database.

B. Observation Request Form to LCO

In FIG. 4, the things that can be easily determined are RA, DA, Exposures, and Integration Time. RA and DE shows the coordinate of targets and one can easily find them in the basic data provided in this paper. Exposures and Integration time are chosen to be approved by the Las Cumbres Observatory Exposure Time Calculator.

One hard thing to be determine is Filters. An introduction to each filter can be found

FIG. 3: This is the basic data for target RR Lyr in Stellarium. The basic data for target RR Lyr in ASAS-SN Variable Stars Database has Saturation warning. The mean magnitude suggests that it could be saturated in their V-band data. The website told people to use this data with caution.

Table 1: Final Observation Request Form

Target Name	RA (J2000)	DE (J2000)	Filter	# Exposures	Integration Time (s)	Observational Windows
BI CAS	10.880	62.666	SDSS u’	3	120	UT 3-8 at ELP; 22 observations in 2 day
BI CAS	10.880	62.666	SDSS g’	3	120	UT 3-8 at ELP; 22 observations in 2 day
BI CAS	10.880	62.666	SDSS r’	3	120	UT 3-8 at ELP; 22 observations in 2day
BI CAS	10.880	62.666	SDSS i’	3	120	UT 3-8 at ELP; 22 observations in 2 day
NSV 4148	128.550	-68.600	SDSS u’	3	50	UT 23-3 at CPT; 10 observations in a day
NSV 4148	128.550	-68.600	SDSS g’	3	50	UT 23-3 at CPT; 10 observations in a day
NSV 4148	128.550	-68.600	SDSS r’	3	50	UT 23-3 at CPT; 10 observations in a day
NSV 4148	128.550	-68.600	SDSS i’	3	50	UT 23-3 at CPT; 10 observations in a day
RR Lyr	19 25 27.913	+42 47 03.694	SDSS u’	3	1	UT 19-22 at TFN; 10 observations in a day
RR Lyr	19 25 27.913	+42 47 03.694	SDSS g’	3	1	UT 19-22 at TFN; 10 observations in a day
RR Lyr	19 25 27.913	+42 47 03.694	SDSS r’	3	1	UT 19-22 at TFN; 10 observations in a day
RR Lyr	19 25 27.913	+42 47 03.694	SDSS i’	3	1	UT 19-22 at TFN; 10 observations in a day

FIG. 4: This is the final observation request form. The table is so wide that cannot be placed in one page so one figure was used here.

on the LCO website. (https://lco.global/observatory/instruments/filters/) During this research, all the $0.4m$ filters are available. For better measure the apparent magnitude, the set of filters with the largest width of transmitted wavelength were used: SDSS u’, g’, r’, i’. Their transmitted wavelengths are shown in FIG. 5. The detail of each filter can be found

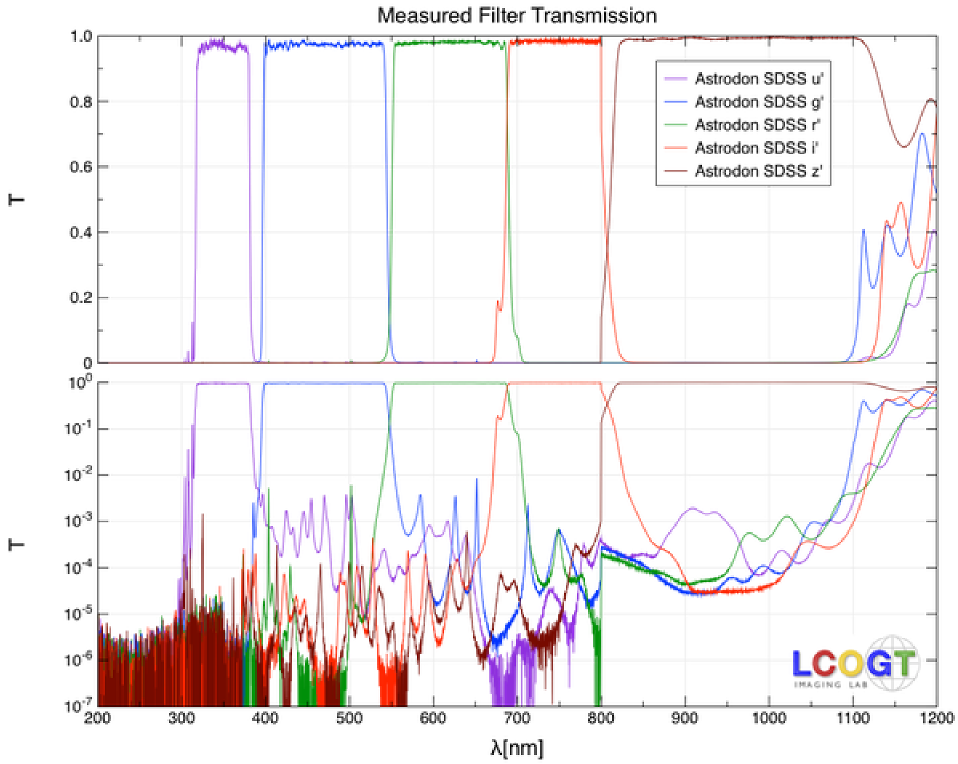

FIG. 5: These are the transmitted wavelengths for each filter in set SDSS. This figure can be found on the LCO website. (https://lco.global/observatory/instruments/filters/sdss-u/)

in the website with the above given link by clicking the name of each filter. The comparison process were not shown in this paper to save space.

Another hard thing to determine is the Observation Windows. The relative information can be found on the LCO website, as well. (https://lco.global/observatory/tools/visibility/) By filling in RA and DE of the target in target coordinates filters and start date and end date in seasonal visibility, an observation window like FIG. 6 will be found. The number of observations should be as much as possible. It is limited by the width of the observation window the most. Since different observation platforms have different observation windows, in order to maximize the number of observations, an observation platform with a longer observation window should be selected. For example, one should choose UTC 1-7 at ELP as the observation window in FIG. 6. [8]

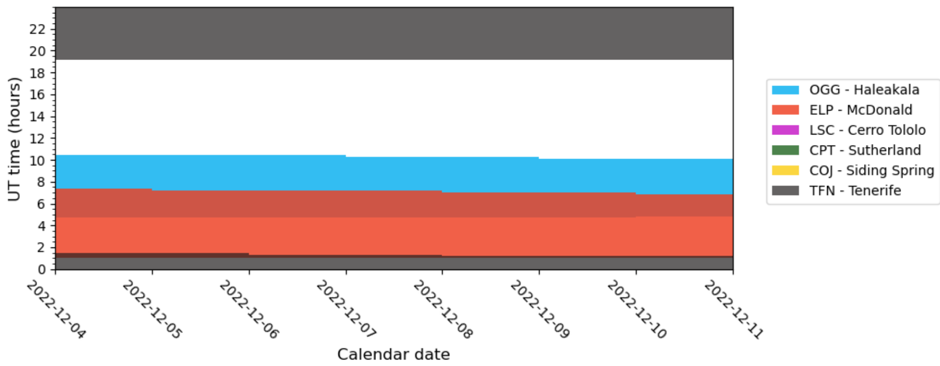

FIG. 6: This is one example observation window from 2022-12-04 to 2022-12-11 of BI CAS. The observation window for other targets can be found in the same way.

C. Period-Luminosity Relation

The period-luminosity relation of a certain class of cepheid variable stars should be explored by measuring a large sample of cepheids of different periods belonging to this class and drawing a linear fit. Due to the limited observation time, LCO does not support measuring so much data. In order to better measure the period of a single cepheid variable star, the number of observations has been increased as much as possible, but the number of cepheid variable stars of the same type measured had to be reduced. In this experiment, only one sample of the same type of cepheid variable star was taken.

In order to obtain a more precise period-luminosity relation, the data in the ASAS-SN Variable Stars Database was quoted. By filling in the corresponding variable type and the references, VMag from 10-13, and Period from 0-3, many cepheid variable stars that meet the requirements will appear. 8 samples for each type mentioned above were chosen to find the fit line with $log_{10}P$ as the independent variable and M as the dependent variable. Absolute magnitude can be derive by an equation equivalent to Equation. 3:

$$M = m + 5 - 5 * log_{10}D \tag{5}$$

This line relation represent the period-luminosity relation. (The period-luminosity relation of RR Lyr were decided not to be considered due to two reasons: firstly, RR Lyr is only similar to anomalous cepheids but not exactly one cepheids. Instead, it is just a common variable star, which is not exactly follow the Leaviit's Law. Secondly, according to ASAS-SN

Variable Stars Database, the data of the RR Lyr has some error mentioned previously.)

D. Apparent Magnitude m

One only needs to consider the apparent magnitude of the target during this research. This magnitude can be found by analysing the feedback images from LCO in AstroImageJ as FIG. 7 shown. The images are in .fz format and one needs to change it into .fits format and open it by AstroImageJ. This program can tell people the relative apparent magnitude difference Δm between the targets and a reference star. Reference star is a star with known apparent magnitude m_r and near to the targets. It can be found in Stellarium or Centre de Données astronomiques de Strasbourg (https://cds.u-strasbg.fr/), and then one should determine which star it represent in the feedback image. After linking the target and reference stars with the right mouse button in AstroImageJ, one can read Δm and the apparent magnitude of the target for certain filter m_f can be calculated by:

$$m_f = m_r + \Delta m \tag{6}$$

The filter use for certain image can be see from the image header if one open this image in FITS Liberator as FIG. 8. Then, by averaging the apparent magnitude of the target for certain filter with its width of transimitted wavelength w_f, one can calculate the apparent magnitude of the target:

$$m = \frac{\sum_{f=u'}^{i'} m_f * w_f}{\sum_{f=u'}^{i'} w_f} \tag{7}$$

E. Absolute Magnitude M

There are two places where absolute magnitude needs to be used. Firstly, when finding the period-luminosity relation, one needs to find absolute magnitude for each sample. This absolute magnitude can be calculated from Equation. 5. Secondly, once the period-luminosity relation is found, one needs to use this relation and the period of the target to calculate the absolute magnitude of the target and further calculate its distance.

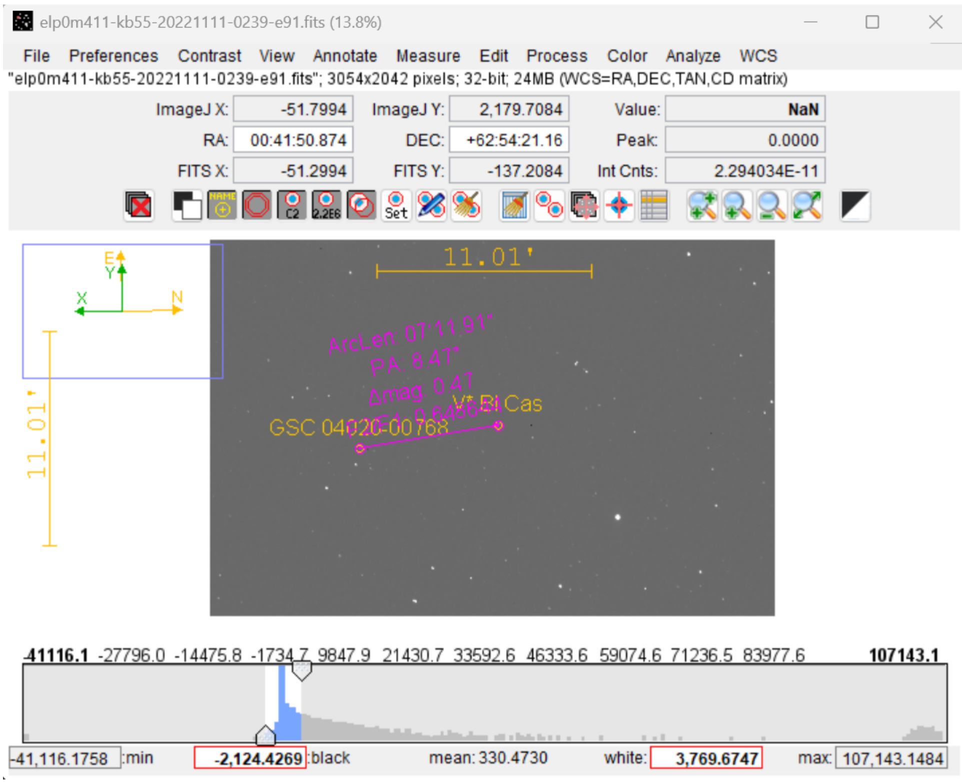

FIG. 7: This is one example of the measurement in AstroImageJ.

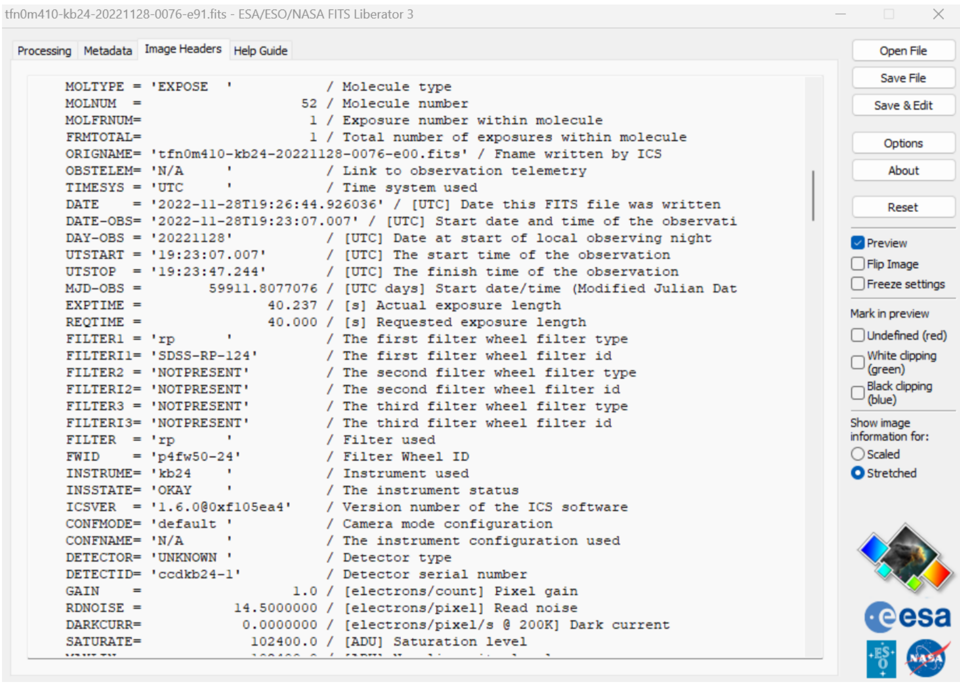

FIG. 8: This is one example in finding the image header.

F. Period P

When finding the period, one can draw a line chart with time t as the independent variable and the apparent magnitude m as the dependent variable. In image header, one can also find the observation start time and the observation end time. The average of these two time minus the reference time was set to be t. The difference of these two time can be used to calculate its uncertainty. Since our observation applications often exceed the target period, the period can be calculated by connecting two points with the same apparent magnitude on the line chart and calculating the time difference between them. This time difference is the period of the target.

G. Uncertainty σ

Unless otherwise specified, the uncertainties of the measurements in this research are given by the following formula:

$$\sigma = \frac{d}{\sqrt{12}} \tag{8}$$

Here, d can be considered as the precision of one measurement or the observation time period for P.

For the uncertainies of the calculated physics quantities, one can calculate them by the following fomula:

$$y = f(x_1, ..., x_n), \qquad \sigma_y = \sqrt{\sum_{i=1}^{n} (\frac{df}{dx_i} * \sigma_{xi})^2} \tag{9}$$

Here, n represent the number of relevant measurements and σ_{xi} is the corresponding uncertainty of measurement x_i.

III. RESULTS

A. BI CAS

As the representative of the classical Cepheid variables studied in this study, BI CAS has the most observation data, and the observation results are relatively good. All the raw data and the results obtained according to the approach mentioned in "Method" section are placed in Table 2. The line chart is presented by FIG. 10.

From Table 2, the uncertainty of the time and the apparent magnitude are 0.00013 and 0.003 respectively. They were calculated strictly following the instruction in "Uncertainty" subsection. However, they are two small relative to their central values. Although the smaller uncertainty indicates that the experimental data is more precise, I think it should be a statistical error. The equipment accuracy of the entire observation, the sample size, and the data fitting do not support this accuracy. Here I think that the reason for this statistical error is the lack of error-related information. Due to limited observation resources, we cannot observe multiple times at the same time. This leads to the fact that the moment can only be measured once, and its uncertainty is hidden by the precision of the experimental instrument. The apparent magnitude is mainly obtained through AstroImageJ and Stellarium. But how the data in Stellarium is obtained and how much their error is, these information are beyond my access. So the uncertainty here is not credible. In fact, not only here, but also in the experimental data of the next two targets. But in the following sections, this issue will not be mentioned again.

At the bottom of Table 2, each filter period is displayed. By comparing with the data in the ASAS-SN database, I found that they are all on the small side. Here I think the reason for the data deviation is a system problem. Due to the limitation of observation conditions, the number of samples is limited. The images we received are not enough to fill a complete cycle so we cannot get an accurate image like Vmag-phase in ASAS-SN. A line chart connects two sample points with a straight line. But we can easily imagine that the functional relationship between these two points must not be linear. Therefore, using other points other than the sample points to measure the period will cause huge errors. Another reason for the small period may be the lack of SDSS u' data. Since objects in SDSS u' images are difficult to find, I discarded the data during the analysis. But this also leads to incomplete data and errors. It is not difficult to find that the apparent magnitude is also smaller than the data in the database. The gap between them is similar to the gap between cycles and is far greater than the uncertainty. This becomes a statistical error. I think the reason for this is also the absence of SDSS u' data.

According to the method described in the "Method" section, the period-luminosity relation of the category to which BI CAS belongs is shown in FIG. 11. The expression for this fitted

Table 2: Raw Data Extract From LCO Images of BI CAS

Filter	Start Time (s)	End Time (s)	Time t (days)	Uncertainty of t (days)	Delta Apparent Magnitude	Apparent Magnitude	Uncertainty of AM
SDSS g'	248.981	289.215	0.00311	0.00013	5.33	13.12	0.003
SDSS i'	309.27	349.503	0.00381	0.00013	3.99	11.78	0.003
SDSS r'	367.23	407.467	0.00448	0.00013	4.43	12.22	0.003
SDSS g'	1267.806	1308.039	0.01491	0.00013	5.51	13.3	0.003
SDSS i'	1328.776	1369.007	0.01561	0.00013	4.07	11.86	0.003
SDSS r'	1387.007	1427.244	0.01629	0.00013	4.42	12.21	0.003
SDSS g'	16870.356	16910.588	0.19549	0.00013	5.34	13.13	0.003
SDSS i'	16930.146	16970.384	0.19618	0.00013	3.92	11.71	0.003
SDSS r'	16988.515	17028.744	0.19686	0.00013	4.35	12.14	0.003
SDSS g'	20343.156	20383.386	0.23569	0.00013	5.18	12.97	0.003
SDSS i'	20403.263	20443.496	0.23638	0.00013	3.96	11.75	0.003
SDSS r'	20460.989	20501.223	0.23705	0.00013	4.26	12.05	0.003
SDSS g'	23120.875	23161.111	0.26784	0.00013	5.25	13.04	0.003
SDSS i'	23180.813	23221.049	0.26853	0.00013	4.01	11.8	0.003
SDSS r'	23239.928	23280.159	0.26921	0.00013	4.36	12.15	0.003
SDSS g'	90225.762	90265.996	1.04451	0.00013	5.82	13.61	0.003
SDSS i'	90285.593	90325.825	1.04520	0.00013	4.39	12.18	0.003
SDSS r'	90344.708	90384.941	1.04589	0.00013	4.92	12.71	0.003
SDSS g'	98599.477	98639.712	1.14143	0.00013	5.48	13.27	0.003
SDSS i'	98659.207	98699.443	1.14212	0.00013	4.12	11.91	0.003
SDSS r'	98717.256	98757.493	1.14279	0.00013	4.46	12.25	0.003
Zero Point of Time:	2022-11-28T19:00						
AM_V of Ref Star:	7.79	HD 3949			Apparent Magnitude m:	12.56	
Coordinates (J2000):	RA: 00 42 48.51	DE: +62 45 57.09			Uncertainty:	0.003	
	SDSS g'	SDSS r'	SDSS I'	ASAS-SN Data			
Period in Days:	1.067	1.025	1.047	1.099			

FIG. 9: This the Table for the raw data and result of BI CAS.

line is:

$$M = (-2.1962 \pm 1.0533) \log_{10} P + (-0.5982 \pm 0.1523) \tag{10}$$

This fitting line is basically consistent with the cycle-light relationship obtained by Leavitt through a large amount of experimental data (Equation. 1). This fitted line was surprisingly accurate. Due to the tight observation time, the observation target can only choose the one with a smaller period. Therefore, when looking for samples used to obtain the period-luminosity relation, the samples whose period is closer to BI CAS are widely selected. I think that due to the lack of data for samples with large periods, the deviation of each sample from the fitted line relative to the gap between their periods is greatly increased. This will cause great uncertainty in the slope of the fitted line. But the relationship after processing is still accurate. This just proves that the classical Cepheid variable star has the most typical and precise period-luminosity relation, which is more suitable for distance measurement. Using this relation, one can play the role of cepheid variable star as "standard candle" and easily calculate the distance D of BI CAS: 4985.4 pc. With this distance and the speed of separation, one can further calculate the Hubble constant H_0.

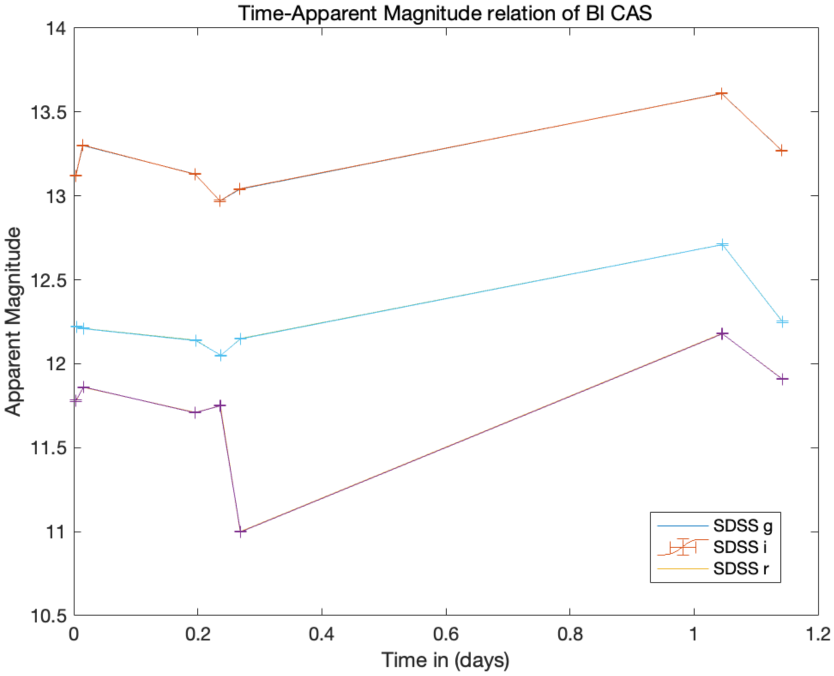

FIG. 10: This the line chart of the time-apparent magnitude relation of BI CAS. This figure was used to find the period.

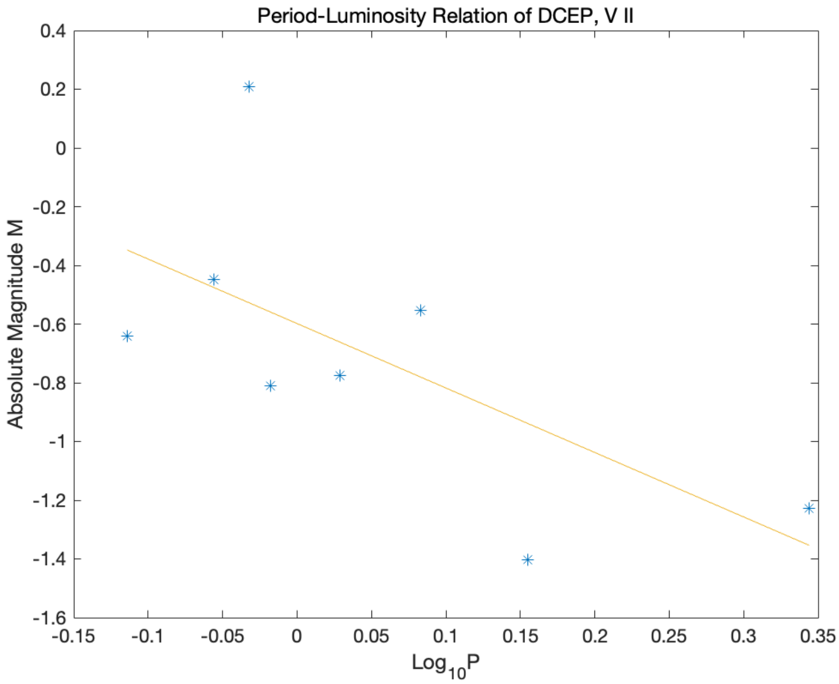

FIG. 11: This is the period-luminosity relation of DCEP, Variable II.

Table 3: Raw Data Extract From LCO Images of RR Lyr

Filter	Start Time (s)	End Time (s)	Time t (days)	Uncertainty of t (days)	Delta Apparent Magnitude	Apparent Magnitude	Uncertainty of AM
SDSS g’	527.392	528.736	0.006112	0.000004	-4.26	7.36	0.003
SDSS r’	563.022	564.366	0.006524	0.000004	-4	7.62	0.003
SDSS i’	594.832	596.178	0.006892	0.000004	-4.04	7.58	0.003
SDSS g’	1290.729	1292.072	0.014947	0.000004	-3.96	7.66	0.003
SDSS r’	1324.068	1325.413	0.015333	0.000004	-3.91	7.71	0.003
SDSS i’	1356.269	1357.613	0.015705	0.000004	-3.89	7.73	0.003
SDSS g’	13243.96	13244.897	0.153292	0.000003	-2.42	7.64	0.003
SDSS r’	13294.491	13295.432	0.153877	0.000003	-2.47	7.59	0.003
SDSS i’	13347.09	13348.031	0.154486	0.000003	-2.68	7.38	0.003
SDSS g’	66108.086	66109.362	0.765147	0.000004	-2.44	7.62	0.003
SDSS r’	66158.287	66158.297	0.765721	0.000000	-2.49	7.57	0.003
SDSS i’	66208.041	66209.309	0.766304	0.000004	-2.94	7.12	0.003
SDSS g’	65736.067	65737.326	0.760841	0.000004	-2.38	7.68	0.003
SDSS r’	65786.66	65787.939	0.761427	0.000004	-2.64	7.42	0.003
SDSS i’	69435.694	69436.967	0.803661	0.000004	-2.88	7.18	0.003
SDSS g’	69712.727	69714.002	0.806868	0.000004	-2.32	7.74	0.003
SDSS r’	69764.218	69765.493	0.807464	0.000004	-2.47	7.59	0.003
SDSS i’	69814.109	69815.383	0.808041	0.000004	-2.75	7.31	0.003
Zero Point of Time:	2022-11-10T01:00					Apparent Magnitude m:	7.47
AM_V of Ref Star:	11.62	V* V589 Lyr	10.06	BD+43 3334		Uncertainty:	0.003
Coordinates (J2000):	RA: 19 25 31.83	DE: +42 51 12.88	RA: 19 25 04.76	DE: +42 41 31.26			

FIG. 12: This the Table for the raw data and result of RR Lyr.

B. RR Lyr

According to the reasons mentioned before, RR Lyr’s period-luminosity relationship cannot be obtained accurately. So I won’t focus too much on it here. All relevant data and processed results are presented in Table 3 and FIG. 13. Instead, I will foucus more on the period since the feedback images from LCO of RR Lyr is abundant. But the line chart meanders and has no cyclical trend at all. Although this is surprising given that a variable star has a very ambiguous period, it does not mean that there is a systematic error in the observation of the target. In the ASAS-SN database, the period of RR Lyr is not given. This is the only one I found in my search for various suitable observation targets. At the same time, the website also suggested that there may be problems with the data about RR Lyr and urged users to use it with caution. This just shows that RR Lyr does have some particularities, otherwise he would not be specially classified as a variable star. So not being able to find the cycle of RR Lyr is not so strange. But this means that our analysis of RR Lyr can only focus on its apparent magnitude. The apparent magnitudes here are the same as those mentioned above, but smaller than those in the database. They may have the same systematic error since SDSS u’ also cannot observe RR Lyr. Another difference from before is that I changed the reference star midway here, because the initial reference star cannot be observed for a certain period of time.

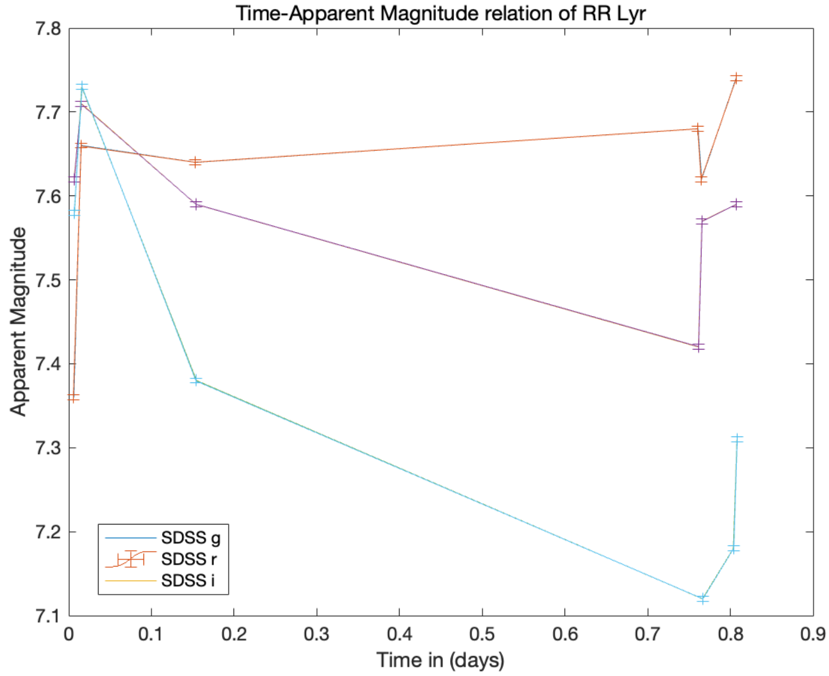

FIG. 13: This is the line chart of the time-apparent magnitude relation of BI CAS. This figure was used to find the period. However, the luminosity of RR Lyr does not show a periodic trend.

C. NSV 4148

Unlike the other two targets, NSV 4148 has the worst observations. Due to some problems when submitting the observation application, there are only 3 feedback pictures for each filter and their apparent magnitudes hardly change. So period calculations are completely unworkable for NSV 4148. The only feasible analysis is its period-luminous relationship as a Cepheid variable star. According to the method described in the "Method" section, the period-luminosity relation of the category to which NSV 4148 belongs is shown in FIG. 14. The expression for this fitted line is:

$$M = (-2.4429 \pm 1.1396) \log_{10} P + (0.6380 \pm 0.2960) \tag{11}$$

The period-luminous relationship of CWB is far worse than the result of Leavitt in classical Cepheid variables. This is mainly reflected in the fact that the longitudinal intercept of CWB is positive, while the normal period-luminosity relation is negative. This also reflects the unique value of classical Cepheid variables.

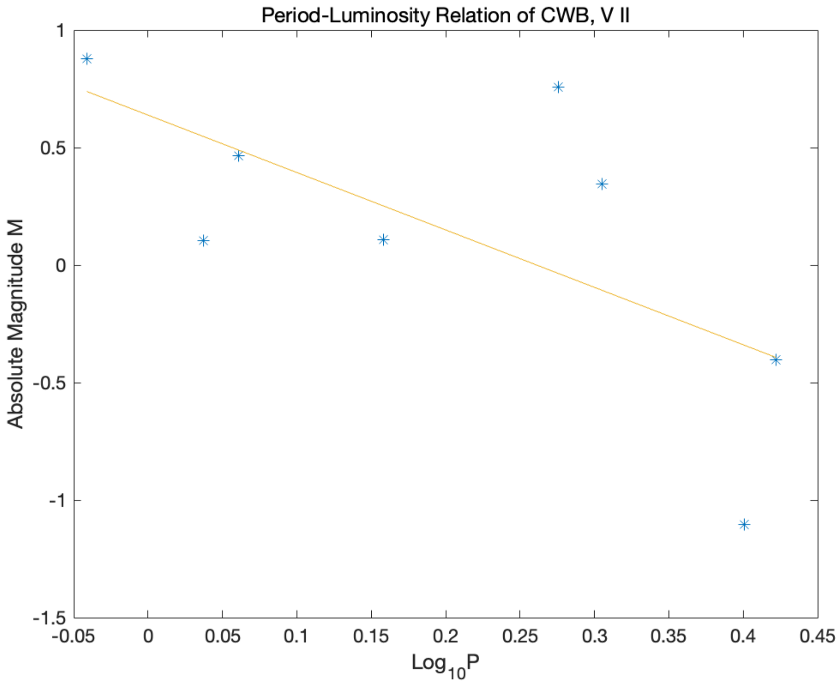

FIG. 14: This is the period-luminosity relation of CWB, Variable II.

IV. DISCUSSION

A. Implications

The main purpose of this article is to explore the period-luminosity relation of Cepheid variable stars and measure their distances. Through the above data analysis, we found that the period-luminosity relation of the classical Cepheid is closer to Leavitt's calculation. Due to the accurate period-luminosity relation, the measurement of the distance of the classical Cepheid variable star itself will become more and more accurate. This reflects the important role of classical Cepheids in modern universe ranging. As I mentioned in the introduction of this article, the specific value of the Hubble constant has not been determined so far. There are huge differences in the measured values at different times, different places, and different institutions. But the Hubble constant is a cosmic parameter that plays a very important role in all kinds of cosmological calculations. It determines the absolute scale, size and age of the universe, and is one of the most straightforward tools we have to quantify the evolution of the universe. In addition, it is also related to the properties of dark matter and dark energy, and the latter two are two of the great mysteries of the universe that we have not yet revealed. Hubble constant is determined by Hubble's Law: $v = H_0 * D$, with H_0 the constant of proportionality,the Hubble constant, between the "proper distance" D to a galaxy, which can change over time, unlike the comoving distance, and its speed of separation v, i.e. the derivative of proper distance with respect to the cosmological time coordinate. So when the distance is accurate, the exact value of the Hubble constant should also be calculated. Usually the speed of separation can be queried through CDS. Unfortunately, the speed of separation or redshift data of BI CAS has not been supplemented yet. Therefore, the calculation of the Hubble constant is left to future research.

B. Limitations

1. It can be seen from the figure drawn by this experiment that due to the influence of the observation window, we are always observing the same phase in the period of the Cepheid variable star. This prevents us from ever getting a full cycle. If there are enough observation resources in the future, I hope to use telescopes in different regions to observe simultaneously. This makes use of the limited observation window as much as possible. At the same time, this method can also observe a target at the same time, so as to facilitate the search for uncertainty. Since the cycle is not a whole day number, after long-term observation, we must be able to make up multiple complete cycles.

2. In the process of finding the cycle, the method I use is to find two points on the horizontal line and then make a difference between them. But this method has too much error for the line chart. When we have enough data to represent a full cycle, we can find the cycle by fitting a periodic function and then computing the period of the fitted function.

3. There are too few targets in each type of Cepheid variable star in this experiment, only one. I can't directly find the period-luminosity relation with this goal, because at least three points are needed to get a linear fit. When the observation time is sufficient, we can also observe long-period Cepheid variables, thereby expanding the period range of linear fitting, making the fitted period-luminosity relation more realistic.

4. When calculating the apparent magnitude, the method I use is the average value

obtained by weighting the transmitted wavelength of the filter. When calculating the apparent magnitude, the method I use is the average value obtained by weighting the wavelength of the filter. But the problem is that we have no way to determine whether the distribution of apparent magnitudes in each band is uniform. They may have certain special density functions. Therefore, this algorithm has a large error. Given enough time, I would look for this particular density function to synthesize accurate magnitudes.

5. In this experiment, most of the objects observed by SDSS u' cannot be effectively identified, thus affecting the calculation of apparent magnitude and period. In the future, if the function of the experimental equipment is strengthened, I hope to measure brighter Cepheid variables. This not only expands the range of absolute magnitude and apparent magnitude to make the period-luminosity relation more general, but also enables the observation objects to be identified under SDSS u' observation.

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4. This work is coorperated by groupmates Shihao Zhou and Jiayu Wang.

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