Blackbody Colors
bbColors

Introduction

Often times astronomers will want more information about an object than just its luminosity. Indeed, the color of an object can be even more important. To determine colors, astronomers often use **color indices**, or the differences in magnitudes as measured in different filters. Of particular interest is the theoretical value of the **effective temperature**, often denoted as T_{eff}. This temperature is the temperature that the object *would* have if it were a perfect blackbody. That is,

$$T_{eff} = \left[\frac{L}{4\pi R^2 \sigma} \right]^{1/4} \qquad (1)$$

Where L is the bolometric luminosity of the object, R is its radius, and σ is the Stefan-Boltzmann constant. With stars, we can usually measure the overall luminosity L (or at least get close and then use a bolometric correction, which assumes a known color… sort of a catch 22 there), but the radius is typically not known. Additionally, stars are *not* perfect blackbodies, so this temperature doesn't necessarily mean that the star has that temperature, though the temperature of the photosphere (the surface from which the photons are emanating) is likely close to the effective temperature.

Nonetheless, we often will use effective temperatures to talk about stars. From introductory physics (and personal experience), you probably know that the color of a blackbody is related to its temperature. For instance, a yellow coal in a grill is hotter than a red coal. We can use this effect to gain a temperature from what we measure: color indices. In this project, we will come up with a simplified way of doing so.

Instructions

We will be assuming that all stars are perfect blackbodies, so that we know their spectral energy distribution (SED); it will be that of a blackbody (the Planck function, essentially). Recall that the magnitude of an object in a particular filter is given by

$$m = -2.5\log\left[\frac{\int S(\lambda) F_\lambda d\lambda}{\int S(\lambda) F_{\lambda,0} d\lambda} \right] = -2.5\log \int S(\lambda) F_\lambda d\lambda + m_0 \qquad (2)$$

Where here, $S(\lambda)$ is the **sensitivity function** of the filter (varies from filter to filter), F_λ is the SED of the object whose magnitude you are measuring, $F_{\lambda,0}$ is the SED of the zero-point object (object whose magnitude is *defined* to be zero), and m_0 is the zero-point magnitude (obviously $F_{\lambda,0}$ and m_0 are directly related.) Note that the first expression in (2) is independent of units, whereas the second expression assumes you have calibrated m_0 correctly for the units you use in F_λ.

For a blackbody, we know F_λ, so long as we know the temperature. It is simply the Planck function:

$$F_\lambda(T) \propto \pi B_\lambda(T) = \frac{2\pi hc^2}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda kT}}-1} \quad (3)$$

The factor of π is present so that the outgoing flux has already been integrated over all solid angle. It makes little difference, though, since the *ratio* of fluxes is all that matters. Additionally, we have only stated that the SED is *proportional* to $\pi B_\lambda(T)$. This is because we also need to take into account the size of and distance to the star. So in truth, we are missing a term of R^2/r^2, where R is the radius of the star, and r is the distance to the star (the 4π's cancel out, obviously). Since we are only measuring magnitude differences in each filter, this term can be separated (by logarithm rules), and will always cancel out, so we will be ignoring it.

In addition to the filter information, we still need to know what our calibration star is. Tradition- ally, Vega has been considered the standard star, where $m = 0$ for *any* filter. Nowadays the system is a bit more complex (for one reason, Vega is mildly variable), but we'll use this as a guideline. Vega is a type A0 star, with effective temperature of $T_{\text{eff},0} = 10,800\,\text{K}$. Thus, we will let that define our zero-point.

Finally, we need to specify what filters we are using. We would like to use the Johnson-Cousins *UBV RI* filters. However, the sensitivity functions cannot be written in a clean, analytic form. As an approximation, we will assume that they are box functions. That is, they have 100% transmission within their band (centered at their peak wavelength with a width of their full width at half max- imum), and 0% transmission outside. As a reminder, Table 1 gives the relevant data on these filters.

Filter	Central Wavelength	FWHM
U	360 nm	70 nm
B	440 nm	100 nm
V	550 nm	90 nm
R	650 nm	100 nm
I	800 nm	150 nm

Table 1: Rough specifications of the Johnson-Cousins photometric system.

Your assignment is to produce tables that relate effective temperatures with color indices. In 200 K increments, from 3,000 K to 12,000 K, give values for $U-B$, $B-V$, $V-R$, and $R-I$ color indices.

This is not a task you can do by hand, even with our simplified sensitivity functions. The Planck function, when not integrated over its entire domain, is not integrable by hand without significant approximations. So, use your favorite numerical software (I'd probably use *Mathematica*, but whatever you prefer is fine). The goal is then to have a consistent table from which one could look up an effective temperature after measuring one or two color indices. Please submit both your completed tables (in LATEX, Excel, or some other format) as well as the code that generated them as well as plots.