Computing Properties of Binary and Double Stars

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I. INTRODUCTION

Double and Binary star systems make up a large portion of the sky, with over one third of stars in the milky way being part of a double or binary star system. These systems are determined by their constituents' apparent proximity to each other. While double stars need only appear close to each other, binary stars are classified as being gravitationally bound to each other. This interaction allows for the determination of physical properties, such as the mass of each star, that would be difficult to determine otherwise. Even if the double stars aren't gravitationally bound, their measurement can serve as a way to test a telescope's capabilities.
Due to their prominence, there are many binary and double star systems whose reported properties are dubious or incomplete. We selected a set of previously reported binary stars with varying reported properties. For each system, we used a 0.4m telescope with the B V and g' filters to corroborate previous findings.

A. Determining binary systems

Binary systems can generally be determined through three methods: visual, eclipsing, and spectroscopic.
Theoretically our telescope can detect eclipsing and visual binaries, which the systems that we chose to examine were. Choosing the angular separation to look at is challenging - while a larger angular separation is easy to measure, the time it would take these farther-away systems would be very large, making it difficult to measure the period. This led to us looking at systems with a wide range of angular separations.

II. METHODS

A. System Selection

There are many easily searchable databases that contain already found binary and double stars. When looking for potential systems to further study, we specified that each star in the system must have a magnitude $5 < M < 12$ with a difference in their magnitudes no greater than 3. We required that the angular separation of the systems was greater than 0.5" , which is the natural limit of the 0.4m telescope we had access to. We first searched the Binary Star Database[4] for these systems, where we found system HD153756, STF2128, and STF2398. Significant observations had been performed on STF2128 prior to our observation[6]
The Washington Double Star Catalog (WDS)[5] is a large double star database that began collecting data in 1963. Because of its advanced age, and the sheer number of observations that have been reported to this catalogue, there are many double star candidates with incomplete or outdated measurements. A list of these systems is published by the WDS, called the "Neglected List". A python script was developed to search this list for systems that fit our previously stated requirements. From this search, we chose to more thoroughly study the systems HEI9004 and HJ4640. I is a table of prior information that we got from the aforementioned databases of all the systems we observed.

1. Instrument Specification

Measurements of the systems we chose to study were taken with a 0.4m STL 6303 telescope using the Las Cumbres Observatory Telescope Network[1]. Information relevant to calculation of the S/N ratio of measurements from this system are found in table II

B. Signal To Noise Ratio

It's important to calculate the signal to noise ratio for a given observation.
We can write the signal of an observation as

$$S = F\tau A\varepsilon Q_e \tag{1}$$

Where F is the point source flux, τ is the integration time, A is the telescope area, ε is the telescope efficiency, and Q_e is the quantum efficiency of the telescope.

There are three main sources of noise in this measurements. The first is dark current, due to excess heat in the system exciting electrons. The signal generated from this can be written as

$$S_{DC} = i_{DC}\tau \tag{2}$$

where i_{DC} is the dark current. Readout noise is due to electrons not being fully removed images, leading to incorrect detections. The noise from this effect can be written as N_R. Background noise, due to extraneous light from other sources. The signal from this can be written as

$$S_\beta = F_\beta A\varepsilon Q_e \Omega\tau \tag{3}$$

where F_β is the background flux from the sky and Ω is the pixel size in arclength.

The signal to noise ratio is given by this equation

$$\frac{S}{N} = \frac{FA_\varepsilon\sqrt{\tau}}{\left(\frac{N_R^2}{\tau} + FA_\varepsilon + i_{DC} + F_\beta A_\varepsilon \Omega\right)^{1/2}} \tag{4}$$

System	HJ4640	HEI9004	HD153756	STF2128	STF2398
Right Ascension (hr:min:sec)	14 01 14.00	21 26 48.5	17 01 23.66	17 03 18.66	18 42 46.69
Declination (deg:amin:asec)	-10 22 38.37	+37 30 51.3	-0 47 09.3	59 35 07.3	59 37 49.4
Separation (arcseconds)	4"	109.8"	0.7"	12.17	11.4
Position Angle (deg)	134.3	335.0	17	Unknown	157.0
M_1	8.91	7.75	9	8.76	9.11
M_2	9	9.53	Unknown	10.34	9.96
B Magnitude	9.36	9.19	10.08	9.69	Unknown
V Magnitude	8.91	7.74	8.81	8.65	Unknown
$B-V$ Magnitude	0.48	1.45	0.6	1.04	1.55
Distance (ly)	651.01 ($\pm$5.71)	1309.86($\pm$194.93)	724.79 ($\pm$8.34)	82.13($\pm$0.05)	11.47 ($\pm$0.00)

TABLE I. Previously calculated information about our initial systems

Variable	Value
	0.5
N_R	14.5
i_{dc}	0.03
Q_e	0.2
F_b	0.1
Ω	0.571
A	1260

TABLE II. Variables relevant to the signal to the signal to noise calculations of our systems. This information was obtained from the LCO website.

The time-dependent noise per unit time can be written as

$$N_T = FA_\varepsilon + i_{DC} + F_\beta A_\varepsilon \Omega \tag{5}$$

by substituting N_T into equation 4, and multiplying the numerator and denominator by τ, we can write the signal to noise ratio as

$$\frac{S}{N} = \frac{FA_\varepsilon \tau}{\left(N_R^2 + \tau N_T\right)^{1/2}} \tag{6}$$

We can approximate the flux that will be given off from the selected stars from their previously calculated magnitudes using the equation

$$\frac{b_1}{b_2} = 10^{0.4(m_2 - m_1)} \tag{7}$$

Where b_1 is the flux of the star, and b_2 is the flux of an reference object. In class we learned that a magnitude 10 object would have a flux of roughly 300. Using this, we can calculate the flux for all of the stars we plan to observe, based on past magnitude measurements.

By calculating the S/N ratio for all stars we are interested in observing, we can determine the correct exposure time to select. Figure 1 is a plot of the S/N ratio of each system as a function of exposure time. The S/N ratios for these systems at $t = 1$ are: HJ4640 = 649, HEI9004 = 1888, HD153756 = 597, STF2128 = 745, STF2398 = 539.4, and an object with a magnitude 10 would have an S/R = 238. We chose an exposure time of 1s for all systems based off of the large-enough S/N.

From this calculation, we chose to take 5 pictures per filter with an exposure time of 1 second, for a total of 15 photos per system. All photos from the same filter were combined using the AstroArt preprocessing tool. These combined images can be seen in III

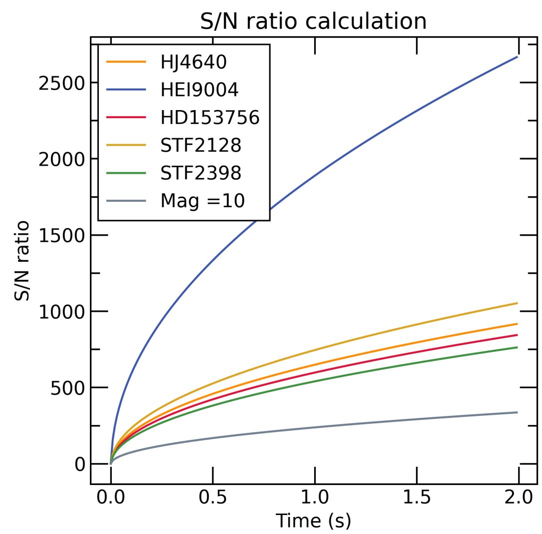

FIG. 1. The signal to noise ratio was plotted for all stars, and for an object with Magnitude 10.

III. RESULTS

A. Undifferentiable Systems

Two of the systems that we chose to look at, HD153756 and HJ4640, appeared so close together in our observations that we were unable to meaningfully measure their separation and position angle. Although these stars were technically farther apart from each other than the theoretical limit for distinction would necessitate, their bright magnitudes and other experimental factors, such as the airmass during the observation, led to these measurements being unclear.
In the future, this problem can potentially be solved for systems with short periods by plotting the magnitude of the system over time - the measured magnitude will drop if one of the stars eclipses the other.

	V filter	B filter	g' filter
HD153756			
HEI9004			
HJ4640			
STF2128			
STF2398AB			

B. Position Angle, Separation measurements

The position angle of a binary star system is the angle the secondary star makes with the north celestial pole when taking the primary star to be the center of your angular measurement. We calculated these values using AstroImageJ[2]. We used the aperture photometry tool to get relevant information about these systems. Figure 2 demonstrates this tool. First, we find the difference in right ascensions of the double stars in the systems. dRa:

$$dRA = 15(RA_1 - RA_2)\cos(DEC_2) \tag{8}$$

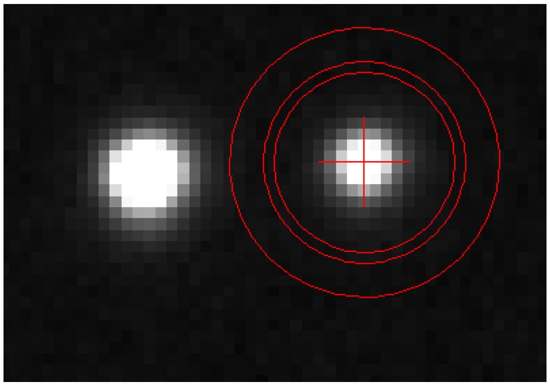

FIG. 2. An example of the aperture photometry tool in AstroImage J. Choosing the correct sizes for the rings of this tool dramatically impact the accuracy of the calculations. For some systems, such as this one, some precision might have been lost due to the light from the second binary star affecting the values of the outer ring.

System	HEI 9004	STF2128	STF2398
Separation (arcseconds)	109.77	11.66	11.2
Position Angle (deg)	334.93	43.92	181.09
Separation Angle Standard Deviation	0.06	0.4	0.1
Position Angle Standard Deviation	0.04	0.42	0.03
$M1_V$	7.74	8.55	8.81
$M2_V$	9.47	10.37	9.67

TABLE III. System observations

We can then find the differences in declination. $dDec$

$$dDec = DEC_2 - DEC_1 \tag{9}$$

From this information, the position angle of the system can then be calculated.

$$PA = \operatorname{atan}\left(\frac{dRa}{dDec}\right) \tag{10}$$

Additionally, we can calculate the separation between the two stars in the system - the distance between the center of both stars in arcminutes using the Pythagorean theorem.

$$\text{sep} = \sqrt{dRa^2 + dDec^2} * 3600 \tag{11}$$

We calculated these values for the remaining systems we observed. We performed this calculation on each filter we took an image from, then averaged their values to get out final measurement.

C. Periods of Systems

1. *Color Flux*

The frequency of the light coming out from stellar objects can allow us to calculate important information about the object. One such important property is "temperature" - this can

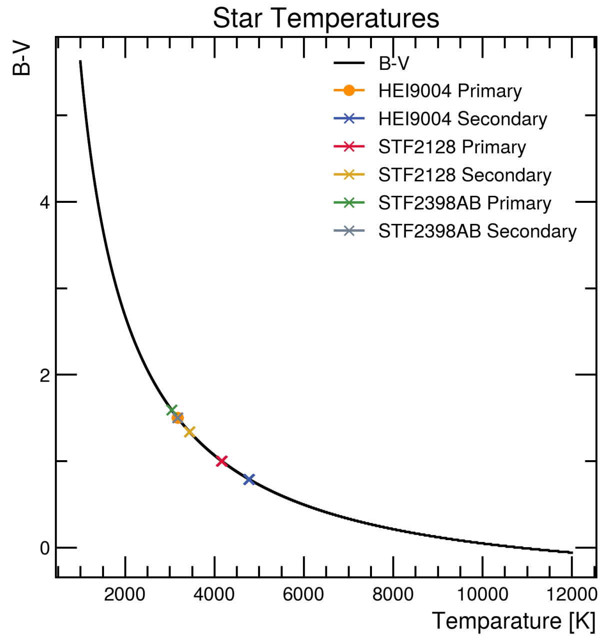

FIG. 3. The approximate temperatures of the stars were calculated. the temperatures range from 3000 - 5000K, making them M and K class stars. Their corresponding colors would be orange and red

be easily calculated from the "B-V" value of the star by assuming the star can be approximated as a black body. We have calculated the magnitudes of the flux from each lens for every stellar system we have analyzed. The flux was obtained using the "Aperture Photometry" tool in AstroImageJ. For all measurements, the inner aperture was set to at least 1.5 times the FWHM of the stellar object we were attempting to measure. From the flux of the system, the magnitude can be computed

$$M = -2.5 \log\left(\frac{\text{Flux}}{t_{exp}}\right) \tag{12}$$

The magnitude of any system is based off a reference point. To get sensible values for these magnitudes, we must find the "Zero-Point" magnitude for each filter for each system by comparing the magnitudes of reference stars we observed with their previously observed values. The zero point was found by averaging through these values for all reference stars we chose. A table of this work is presented HERE Assuming that the stars are black bodies, we can approximate their temperatures by calculating the temperature a blackbody would have to be to have the $B-V$ value that we calculated for each star. The results are plotted in Figure 3 We will call the magnitude obtained through the V filter the "aparent visual magnitude" of the system, and take it to be the aparent magnitude of the system for the following calculations.

		HEI9004		
Star	Ideal B magnitude	Ideal V Magnitude	Measured B Magnitude	Measured V Magnitude
2Mass J21271211 +3731153	12.25	10.48	-8.09	-10.06
BD + 364561	10.01	9.56	-10.19	-11.43
	B_{zp} = -20.27		V_{zp} = -20.76	
Calculating magnitude using zero-point values			7	
Primary Star	9.19	7.74	9.13	7.63
Secondary Star	Unknown	Unknown	10.27	9.47
		STF2128		
Star	Ideal B magnitude	Ideal V Magnitude	Measured B Magnitude	Measured V Magnitude
HD 238653	10.62	10.20	-9.00	-10.15
TYC 3898-64-1	10.27	9.73	-9.36	-10.65
	B_{zp} = -19.63		V_{zp} = -20.38	
Calculating magnitude using zero-point values				
Primary Star	9.69	8.65	9.55	8.55
Secondary Star	Unknown	Unknown	11.71	10.37
		STF2398AB		
Star	Ideal B magnitude	Ideal V Magnitude	Measured B Magnitude	Measured V Magnitude
HD 238928	11.06	9.66	-8.81	-10.84
	B_{zp} = -19.87		V_{zp} = -20.27	
Calculating magnitude using zero-point values				
Primary Star	Unknown	Unknown	10.402	8.81
Secondary Star	Unknown	Unknown	11.53	9.67

TABLE IV. The magnitudes of each system matched well with previously recorded magnitudes of these systems.

D. Calculating Period of Systems

For these calculations, we assume that the system is tangent to our observation. The distance between us and the given system has previously been calculated. To measure distance, we would have had to measure the parallax angles of the system, which is not possible in 10 weeks.We can calculate the absolute magnitude of each star from this distance.

$$M_{abs} = m_{apparent} - 5\log\left(\frac{D_1}{10}\right) \tag{13}$$

From this absolute magnitude, we can calculate the luminosity of each star using the equation:

$$\frac{L_{star}}{L_{sun}} = 10^{0.4(4.85-M_{abs})} \tag{14}$$

And from the luminosity of the star, we can calculate its mass using the mass-luminosity relationship for main sequence stars with small masses compared to the sun's mass.[3]

$$m = m_{sun}\left(\frac{L_{star}}{L_{sun}}\right)^{1/2.3}\frac{1}{0.23} \tag{15}$$

With the distance from us to the system, we can also find the distance between the two stars using the law of cosines.

$$D_s = \sqrt{2}D_1\sqrt{1-\cos(\theta)} \tag{16}$$

We can then find the period of this system using Kepler's third law.

$$T = \sqrt{\frac{4\pi D_s^3}{G(m+m_{sun})}} \tag{17}$$

We performed these calculations for the systems HEI9004, STF2128, and STF2398. The results are in the table V

IV. CONCLUSION

We measured properties of three double star systems. We corroborated past findings, such as the separation angle and position angle of the system. We analyzed the fluxes of the system in different filters, and found the apparent magnitudes of the systems as well as their approximate temperature. The periods of these systems were calculated using previously created distance measurements.
While we were able to compare the data we gathered with prior data, the long orbital periods of these systems prohibited us from directly measuring the period of binary star systems. In the future I would opt to study stars with higher magnitudes, so that they might be distinguished more easily from each other. I also would choose to study stars with smaller separation angles, as that might lead to systems whose periods could be directly calculated.

V. REFERENCES

[1]T. M. Brown, N. Baliber, F. B. Bianco, M. Bowman, B. Burleson, P. Conway, M. Crellin, É. Depagne, J. De Vera, B. Dilday, D. Dragomir, M. Dubberley, J. D. Eastman, M. Elphick, M. Falarski, S. Foale, M. Ford, B. J. Fulton, J. Garza, E. L. Gomez, M. Graham, R. Greene, B. Haldeman, E. Hawkins, B. Haworth, R. Haynes, M. Hidas, A. E. Hjelstrom, D. A. Howell, J. Hygelund, T. A. Lister, R. Lobdill, J. Martinez, D. S. Mullins, M. Norbury, J. Parrent, R. Paulson, D. L. Petry, A. Pickles, V. Posner, W. E. Rosing, R. Ross, D. J. Sand, E. S. Saunders, J. Shobbrook, A. Shporer, R. A. Street, D. Thomas, Y. Tsapras, J. R. Tufts, S. Valenti, K. Vander Horst, Z. Walker,

System	HEI 9004	STF2128	STF2398
D(ly)	1309.86	78.42	11.47
M_{ap}primary)	7.62	8.81	8.81
M_{ap}secondary	9.47	9.67	9.67
Separation (arcseconds)	109.77	11.66	11.2
M_{abs}primary	-0.4	6.90	11.08
M_{abs}secondary	1.45	7.76	11.94
$\frac{L_{\text{star 1}}}{L_{\text{sun}}}$	125.90	0.151	0.0032
$\frac{L_{\text{star 2}}}{L_{\text{sun}}}$	22.87	0.068	0.0015
$m_{\text{primary}}(kg)$	7.08×10^{31}	3.799×10^{30}	7.141×10^{29}
$m_{\text{secondary}}(kg)$	3.37×10^{31}	2.693×10^{30}	5.896×10^{29}
$D_s(meters)$	6.599×10^{15}	4.196×10^{13}	5.896×10^{12}
$T(years)$	1278712	2602	316

TABLE V. The periods of these systems have been calculated using the method mentioned above. The long period for HEI9004 and the very large distance between the stars in the system makes it unlikely that these stars are a physical binary system. The period calculated for STF2128, 2602 years, compares well with the previously reported value, 2294.4944 ± 871.8519 years. The period calculated for STF2398AB, 316 years, is close to the reported value of the period, 408 years. We believe that this discrepancy is caused by our inaccurate separation angle measurements and brightness measurements.

G. White, and M. Willis. Las Cumbres Observatory Global Telescope Network. *Publications of the Astronomical Society of the Pacific*, 125:1031, September 2013. ADS Bibcode: 2013PASP..125.1031B.

[2]Karen A. Collins, John F. Kielkopf, Keivan G. Stassun, and Frederic V. Hessman. AstroImageJ: Image Processing and Photometric Extraction for Ultra-Precise Astronomical Light Curves (Expanded Edition). *The Astronomical Journal*, 153(2):77, January 2017. arXiv:1701.04817 [astro-ph].

[3]Neb Duric. *Advanced Astrophysics*. Cambridge University Press, 2004.

[4]D. Kovaleva, P. Kaygorodov, O. Malkov, B. Debray, and E. Oblak. Binary star DataBase BDB development: Structure, algorithms, and VO standards implementation. *Astronomy and Computing*, 11:119–125, June 2015. ADS Bibcode: 2015A&C....11..119K.

[5]Brian D. Mason, Gary L. Wycoff, William I. Hartkopf, Geoffrey G. Douglass, and Charles E. Worley. The 2001 US Naval Observatory Double Star CD-ROM. I. The Washington Double Star Catalog. *The Astronomical Journal*, 122(6):3466, December 2001. Publisher: IOP Publishing.

[6]Holly Minarik, Victoria Helm, Ezra Asquith, Andrew Hoffman, Alec Fielding, Humberto Gaytan, Kevin Marvier, Walter Warneke, James Howell, David Rowe, Rachel Freed, and Russell Genet. Exploring the Binary Nature of STF 2128 Using Separation and Position Angle Measurements. *Journal of Double Star Observations*, 14:54–58, January 2018. ADS Bibcode: 2018JDSO...14...54M.