HD 126053

A Spectroscopic Study

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Abstract

This research paper presents a spectrographic study on the star HD 126053. This study will reflect an analysis of the target HD 126053's stellar class, effective temperature, radial velocity, surface gravity and metallicity. High-resolution spectroscopy techniques were employed to determine the stellar class, providing insights into the star's evolutionary stage and intrinsic properties. The effective temperature of HD 126053 was derived, aiding in characterizing the star's physical properties and evolutionary stage. Doppler shifts in the spectral lines were utilized to measure the star's radial velocity, offering information on its stellar motion. The metallicity was determined by relating the target to stars of similar class, providing insights into the star's chemical composition and surface gravity. Overall, this study enhances our understanding of HD 126053's properties and contributes to the broader field of stellar astrophysics, facilitating future investigations into the diverse population of stars in the universe.

Introduction

1.1 - Spectroscopy

Spectroscopy, a discipline studying the interaction between matter and electromagnetic radiation, has a fascinating historical background that spans centuries. Its origins can be traced back to the early 17th century when Isaac Newton conducted experiments with prisms and discovered the separation of white light into a continuous spectrum of colors. This groundbreaking work laid the foundation for understanding the fundamental principles of spectroscopy.

Spectroscopy took a significant leap forward with the development of spectrographs and spectrometers, which together enabled the precise analysis of spectral lines. In the late 19th and early 20th centuries, scientists such as Joseph von Fraunhofer, Anders Jonas Ångström, and Sir William Huggins made notable contributions to the field. Fraunhofer's observation of dark absorption lines in the solar spectrum, known as Fraunhofer lines, provided insights into the composition of celestial objects. Ångström's work on measuring and classifying spectral lines laid the foundation for the study of atomic spectra. Huggins, often regarded as the father of astrophysics, pioneered the use of spectroscopy to examine the spectra of stars and nebulae, revealing crucial information about their composition and physical properties.

Since this pioneering, applications of spectroscopy in the study of stars has revolutionized our understanding of the universe. In modern astrophysics, spectroscopy plays a crucial role in unraveling the mysteries of stellar properties and evolution. By analyzing the spectra of stars, astronomers can determine their temperature, chemical composition, surface gravity, as well as many other physical parameters. Spectroscopic classification enables the categorization of stars into different spectral types, aiding in the characterization of their physical properties and evolutionary stages. Moreover, spectroscopy allows for the measurement of stellar motions and velocities. The Doppler shift of spectral lines provides insights into the radial velocity of stars, which can indicate the presence of binary systems or other gravitational interactions. Spectroscopic observations also enable the study of stellar atmospheres, including phenomena such as stellar winds and chromospheric activity. By analyzing emission and absorption features in the spectra, astronomers can investigate magnetic fields, stellar activity cycles, and the properties of exoplanetary systems.[1]

1.2 - Applying Spectroscopy to HD 126053

In this paper, the target star HD 126053 will be studied within the world of spectroscopy. Using the techniques developed in this field as briefly mentioned above, this paper will discuss the star classification of HD 126053 and return data that reflects various physical properties of the target, such as effective temperature, metallicity, chemical composition, surface gravity, and radial velocity in order to gain a deeper understanding of the target star HD 126053.

[1] *The era of classical spectroscopy*. MIT Spectroscopy Lab - History.

Methods

2.1 - The LCO's Data

The Las Cumbres Observatory (LCO) is a global network of telescopes dedicated to astronomical research and education. The LCO operates a network of robotic telescopes positioned around the world, enabling continuous observations of celestial objects and phenomena. In addition to its observational capabilities, LCO also provides access to spectroscopy data. This spectroscopic data offers valuable insights into the composition, temperature, and other properties of stars, galaxies, and other celestial objects. With its commitment to open data sharing, the LCO offers stored data to researchers in both raw and processed formats. Despite this commitment to open data sharing, direct access to the LCO's spectroscopes/spectrographs was unavailable at the time this paper was produced. However, the LCO maintains a library of stellar observations that can be synthesized to process a spectroscopy of studied stars. These stellar observations can be synthesized through the use of the LCO's BANZAI-NRES data reduction pipeline. This pipeline is capable of processing any LCO Network of Robotic Echelle Spectrographs, and using the contained data to produce files (namely a PDF format for visualization) containing information regarding the following: wavelength calibrated spectra along the wavelength range {3800,8600}Å, and stellar classification parameters.

This returned data also contains the best-fit Phoenix template. Phoenix templates are synthetic stellar spectra that are generated using the PHOENIX model atmosphere code. These templates are important tools in studying and analyzing stellar spectra. They are created by simulating the physical conditions and processes occurring in stellar atmospheres, such as temperature, surface gravity, metallicity, and chemical composition. The PHOENIX model atmosphere code employs sophisticated theoretical calculations and complex physics to compute the emergent radiation from imagined stars. It takes into account factors such as radiative transfer, molecular opacities, and line broadening mechanisms. By solving these intricate equations, the code predicts the intensities and wavelengths of the spectral lines and continuum flux at various wavelengths.

To generate the Phoenix templates, the code utilizes a grid of model atmospheres spanning a range of stellar parameters. Each model atmosphere represents a specific combination of temperature, surface gravity, and chemical composition. Through iterative calculations, the code adjusts these parameters to achieve the best match between the synthetic spectrum and observations of reference stars with known properties. The resulting Phoenix templates provide theoretical spectra that enable the comparison and interpretation of observed stellar spectra.

2.2 - Selecting a Target

After working to understand the BANZAI-NRES pipeline and figuring out how to get it to return data, our target had to be selected. Within the BANZAI-NRES pipeline, recent data regarding the star HD 126053 had been made available. Aside from availability, HD 126053 is a photometrically analogous star to our sun, also possessing similar size and mass.

$$Mass_{HD\ 126053} = 0.89\, M_{\odot} \qquad Radius_{HD\ 126053} = 0.93 R_{\odot}$$

Understanding the similarities of the Sun and HD 126053 enables us to be better able to interpret the data acquired from running the BANZAI-NRES pipeline, as the Sun is a well studied star with vast amounts of information present about its effective temperature, chemical makeup, surface gravity, metalicity, and star type classification.

2.3 - Retrieving HD 126053 Data

With our target acquired, we ran the BANZAI-NRES pipeline using the data provided for HD 126053. To process this data, we duplicated the Github repository using Homebrew (an open source package manager that simplifies the process of installing and managing packages by automatically handling dependencies, ensuring that all necessary components are installed correctly) onto our computer, allowing us to begin to interpret the data retrieved from the LCO archive. The data we retrieved was produced by the NRES Unit 01 (located in Chile) with the fa09 camera. The date and time at which the data was collected is May 30th, 2023, 00:23:10. At this time, the atmospheric conditions around Chile (specifically around the Cerro Tololo Observatory) were favorable for viewing observations on HD 126053 to be recorded (the air mass was 1.4 below the maximum threshold), allowing for the produced data at this time to be acceptable for studying. After downloading the raw data to a directory (which functioned as an input for the pipeline), the processed files were accumulated and exported, resulting in the creation of the aforementioned PDF file visualizing the spectroscopy data required to study HD 126053. Beyond the scope of the PDF file, two FITS files were also produced, one of which was one-dimensional, and the other two-dimensional. The one-dimensional file (when interpreted with the Astropy library), however, provided the same information that was present within the PDF. The two-dimensional file proved more difficult to make sense of, resulting in this paper's heavy reliance on the one-dimensional data (which is fully reflected in the PDF visual overview of the data).

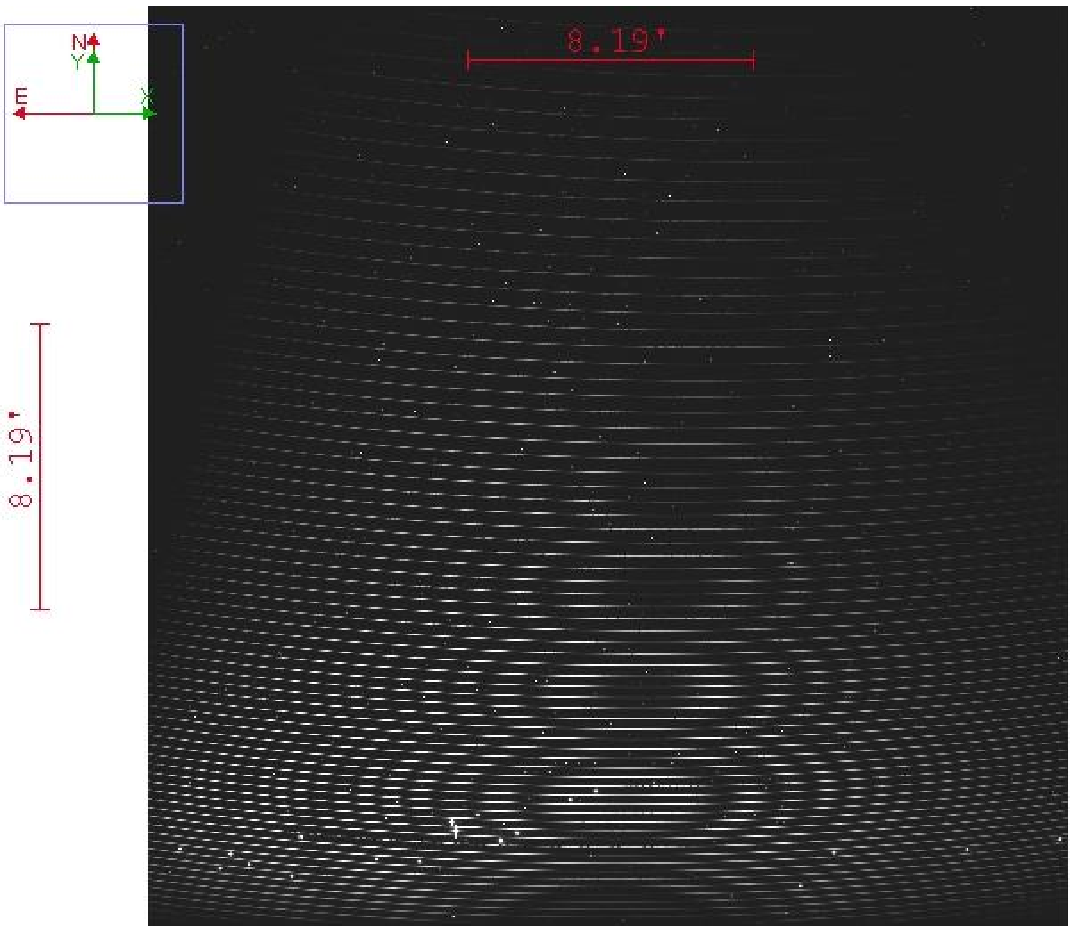

Figure 1.) This is an example of a two-dimensional FITS file we were able to open. While it is not our target star, it serves as an example of how our two-dimensional file would look. Standing alone, the depiction relates minimal usable information. When zoomed in, absorption lines are visible, however, they are non-reflective of useful information without being compiled into a singular spectrum, a process that exists outside the scope of this project.

Figure 1. shows how the two dimensional FITS file is not helpful given our abilities. Instead, the one-dimensional representation and PDF visualized representation are used across the following results section, portrayed in the various figures below.

Results

The following compilation of figures and tables are the results of the processed data (compiled into the aforementioned PDF summary) and display the normalized flux in electromagnetic radiation along several orders. The orders presented are regions of general interest, and the figures allow for visual inspection to accompany the numerical data acquired from the FITS files.

3.1 - First Look at Data

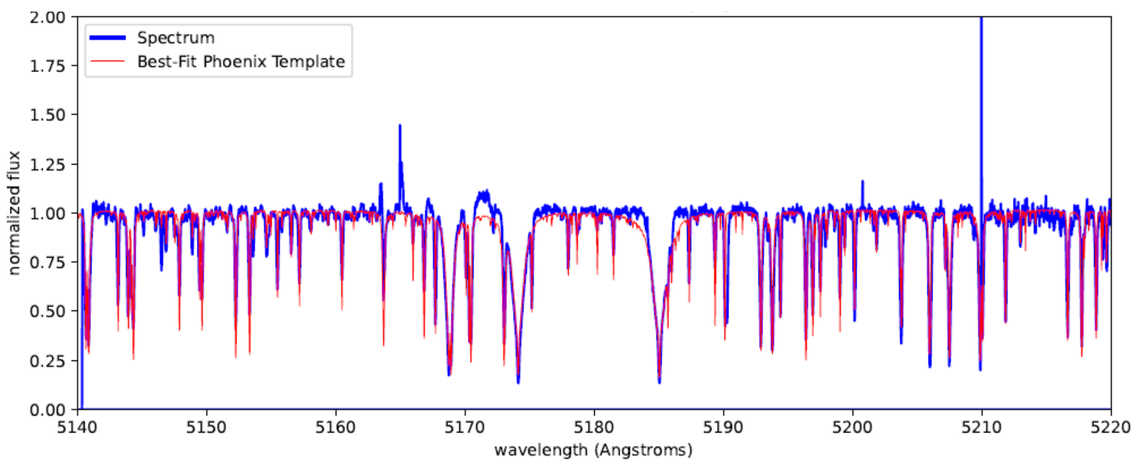

Figure 2.) Continuum-normalized spectrum for target HD 126053. The spectrum is accompanied by the best-fit Phoenix model template for the order (wavelength range 5140Å-5220Å) that contains the Mg b triplet.

From this figure, we can observe the general shape of our spectrum along the wavelength range {5140,5220}Å. Looking at the major emission and absorption lines, we can relate the Phoenix template to our spectra well. The template follows the general trends in the spectrum, enabling generalizations to be made about the target HD 126053. These generalizations are based on the imagined atmosphere and physical qualities of this spectrum's best-fit phoenix template's star.

While the synthetic data from the Phoenix template follows the natural data well (as seen in *figure 2)*, there are areas of disagreement between the two data sets. These disagreements result in the best-fit Phoenix template's lack of accuracy when describing the environment/physical properties of the target star HD 126053.

When addressing the discrepancies between the model and the produced spectrum, it is important to consider the signal to noise ratio (SNR) and radial velocity along the areas of disagreement. The following figure portrays the radial velocity of HD 126053, as well as the SNR along the entirety of the measured wavelengths {3800,8600}Å. Recalling the fair conditions present on May 30th, 2023, it is sensible that the SNR values present in the right plot of *figure 3.* are as high as they are. The SNR values in the range {5140,5220}Å are all around 200, meaning that the natural data values for HD 126053 in *figure 2.* are well recorded, and are relatively accurate. The plot on the left of *figure 3.* represents the CCF values of all orders (gray), and the combined CCF of all orders (black), which are used to calculate the radial velocity (RV) of the target HD 126053. This plot will be helpful when investigating discrepancies between the Phoenix template and the actual data from HD 126053.

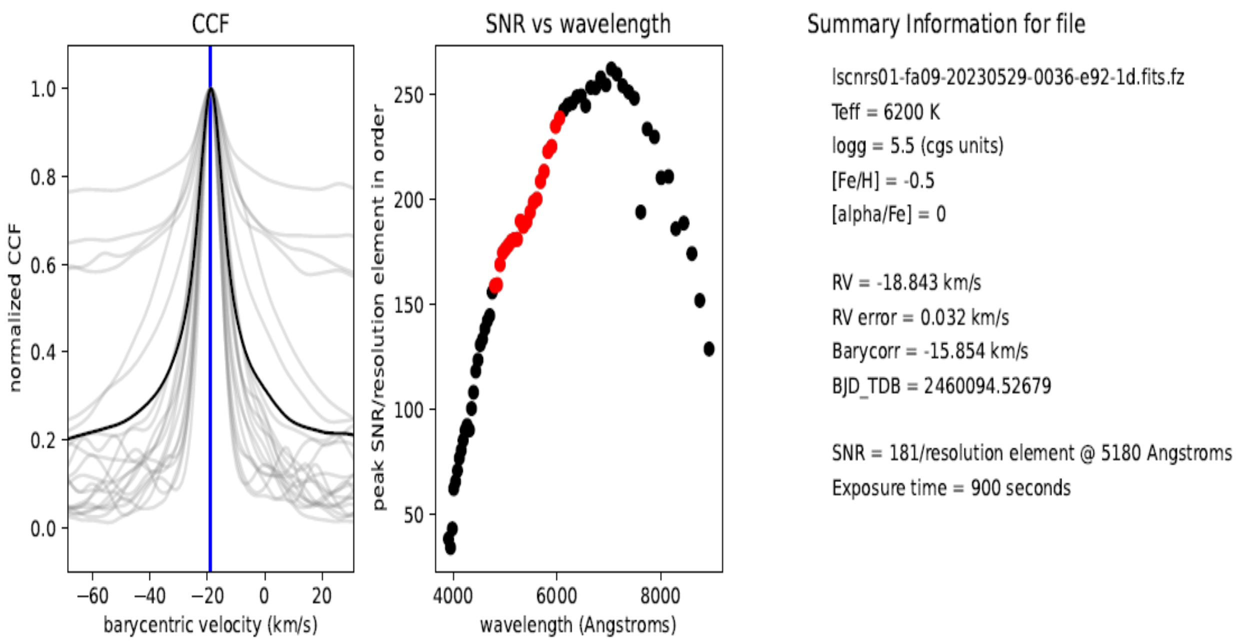

Figure 3.) It is important to consider radial velocity, as well as SNR ratios when looking at the data produced. Radial velocity, if not accounted for, will produce inaccurate data due to doppler shift. The SNR chart also highlights the presence of noise in our measurements, and describes where it may/may not present problems. The information on the right displays some of the numerical values of physical parameters calculated from the data.

3.2 - Stellar Classification

Using the numerical value for effective temperature present in *figure 3.,* HD 126053's stellar classification can be found. Using the effective temperature of the target to calculate its luminosity (L), a Hertsprung-Russell diagram (HR diagram) can be used to identify the stellar classification.

Recalling the radius of HD 126053 to be $Radius_{HD\ 126053} = 0.93R_{\odot}$, the following equation can be filled in to calculate the star's luminosity.[2]

$$L = Area \times Flux = 4\pi R^2_{HD\ 126053}\sigma_{SB}T_{eff}^4 \quad (1)$$

In equation (1), $4\pi R^2_{HD\ 126053}$ represents the surface area of the target, and $\sigma_{SB}T_{eff}^4$ represents the flux of the star's energy/second/area.

The equation relies heavily on the assumptions that the target star HD 126053 has a relatively spherical shape, and that it behaves as a blackbody in terms of its radiation. For stellar classification, exact values are not required, as the magnitude of L is the guiding factor in following this paper's used HR diagram. With this in mind, these assumptions should be fine to use, and equation (1) can be followed to find luminosity.

Calculating:

$$L_{HD} = 4\pi R^2_{HD}\sigma_{SB}T_{eff}^4 = 4\pi(0.93R_{\odot})^2 \times 5.67 * 10^{-8}Wm^{-2}K^{-4} * (6200K)^4 = 5 * 10^{26}W \quad (2)$$

**recall that $R_{\odot} = 6.957 * 10^8\ meters$, which is the value substituted in the above equation to fulfill unit requirements.

To follow the sampled HR diagram, the luminosity of HD 126053 must be placed relative to our own Sun's in order to establish relative magnitude. This is done as follows:

$$\frac{L_{HD}}{L_{Sun}} = \frac{5*10^{26}}{3.8*10^{26}} = \frac{5}{3.8} = 1.3,\ relative\ magnitude\ of\ 1.$$

[2] Astronomy 1144: Lecture 9.

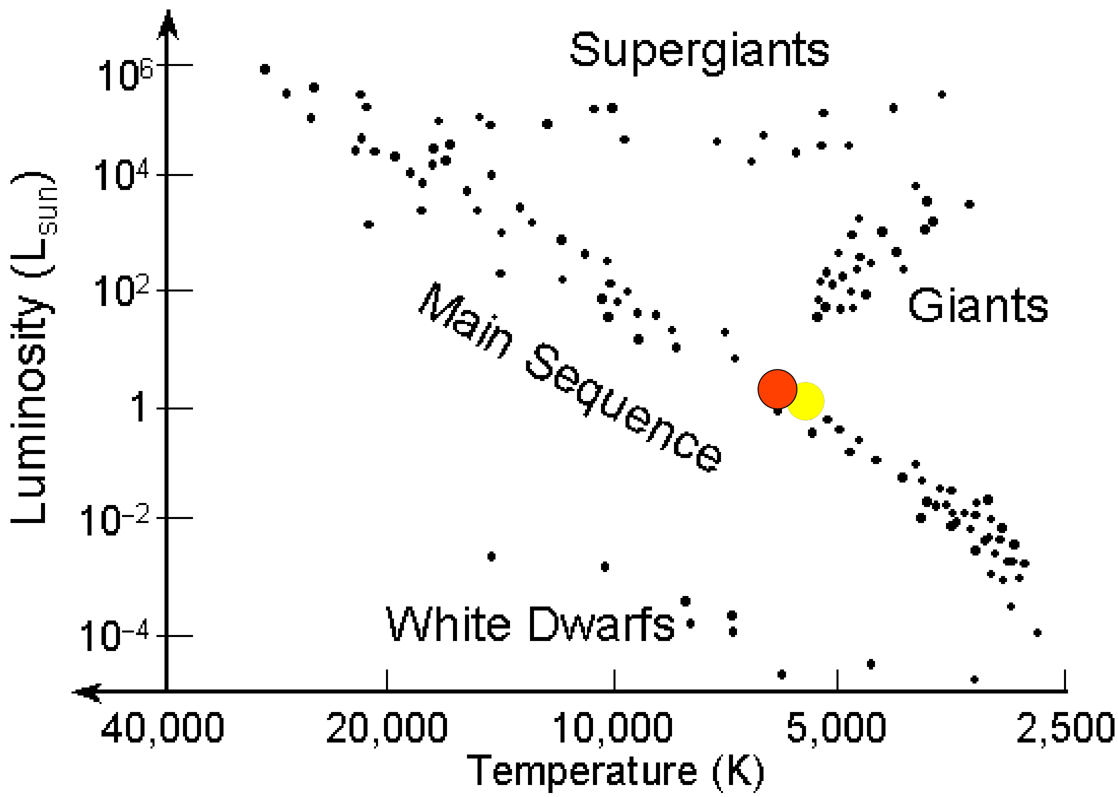

Figure 4.) This Hertzsprung-Russell (HR) Diagram was initially created separately in 1911 by Danish astronomer Ejnar Hertzsprung and in 1913 by American astronomer Henry Norris Russell. Hertzsprung constructed the diagram for nearby star clusters like the Hyades and Pleiades, while Russell focused on nearby individual stars. The version of the diagram commonly used today is attributed to Russell. Observing this figure, it is clear that stars do not occur in random positions, but rather group themselves into distinct regions. The red dot signifies the location of HD 126053 on the diagram (large sized to account for lack of placement accuracy), and the yellow dot represents our Sun's location on the HR diagram (sized similarly for the same reason). (Taken from:Astronomy 1144: Lecture 9).

After placing the general location of HD 126053 on the HR diagram (as seen by the red dot), it is clear that the star is a main sequence star. It is also clear that along the Main Sequence diagonal, it is very similar to the Sun in its stellar classification. The Sun has stellar classification of G2V, meaning it is a G2 main sequence star. In the labeling G2V, G is the letter used to depict main sequence stars, and the 2 represents its relative position within the G stellar class, and v represents the stage of life the star is in. The range exists {0,9}, where numbers closer to 0 are hotter than the larger numbers towards the end of the range. The range also splits up into sub integer intervals, producing a broad range that describes the position of the star within the class. The V (roman numeral for 5) represents the star's phase of its lifetime, which is the main sequence in this case. Comparing the effective temperature of the Sun (5780K) to the effective temperature of HD 126053 (6200K), we can assume that the stellar classification for HD 126053 is of main sequence, but has a F classification. This is because its temperature is in the 6000-7000K range. Because it is on the cooler end (9-7=2), its numerical value must be 7. and from the HR diagram we can assume it to be the main sequence (V). Compiling this, we can classify the star as F7V.

3.3 - Physical Properties assumed from Stellar Classification and Produced Data

Knowing the stellar classification of HD 126053 results in the ability to compare this target star to other stars of similar stellar class. We can also compare HD 123056 to the Sun, which as mentioned has stellar class of G2V, but is photometrically analogous to HD 126053. Using this information, we can infer that HD 126053 likely has the following physical parameters:

- Color index (B-V): 0.62-0.65. This means that HD 126053 likely has a yellow-white color.[3]
- Chemical composition: Consists mostly of hydrogen and helium. Hydrogen is likely the most abundant element, with helium coming in at second, as hydrogen is converted to helium at the star's core via nuclear fission. [4]
- Metallicity: Based on the stellar classification, it is likely that the metallicity of HD 126053 is lower than that of the Sun. HD 126053 has a [Fe/H] of -0.5. By definition, the sun has [Fe/H] of 0, meaning that HD 126053 has a lower metallicity than the Sun.[5]
- Surface Gravity: Given the V characterization of both stars, it is a safe assumption that HD 126053 maintains relatively normal surface gravity, balancing the inward gravitational forces with internal pressure forces resulting from nuclear fission. Given the difference between F and G class, it stands to reason that the surface gravity of HD 126053 is lower than that of the suns, which is sensible when considering the lower mass of HD 126053. This is also displayable via the logg value given by the data: $logg_{HD\ 126053} = 5.5cgs$. This is 12.3 times stronger than the surface gravity of Earth, whereas the sun has a surface gravity 24 times stronger than Earth, highlighting the smaller surface gravity of HD 126053 when compared to the sun.[6]

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[3] Encyclopædia Britannica, inc. (n.d.). *Stellar classification*.
[4] Physics, D. of, College, A. at D., & OpenStax. (n.d.). *15.1 the structure and composition of the sun*.
[5] Metallicity. (n.d.).
[6] Astronomy 1144: Lecture 9.

Discussions

4.1 - Phoenix Model Disagreements

As mentioned in section 3.1, there are discrepancies between the best-fit Phoenix model and the spectra produced from HD 126053. These discrepancies result in some of the values produced being inaccurate. The Phoenix template placed onto the spectra is labeled as best-fit, and the value acquired from the Phoenix template mirrors this best fit theme. Some physical parameters were calculated without the usage of the Phoenix template, however, some of them relied on the Phoenix template to make assumptions that resulted in the given values. For example, effective temperature could have been calculated two ways: through finding the best fit Phoenix template and referencing the effective temperature used in the synthetic atmosphere the Phoenix model is based off, or through the utilization of Wien's Law, which relates the maximum wavelengths of the given spectra to the effective temperature. If the latter method was used, the value can be trusted as reflectant of the actual value of HD 126053's effective temperature. If the first method was used, then the produced value will have larger margins of error, as the Phoenix model doesn't mirror the natural data identically. The following figure begins to explain this.

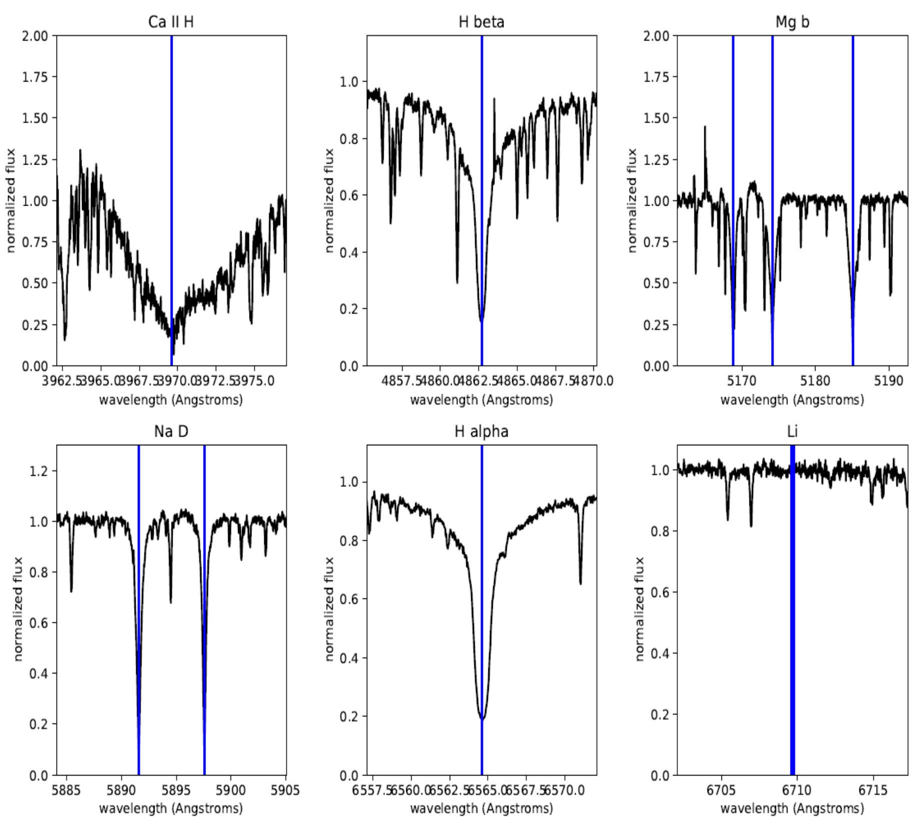

Figure 5.) This figure shows 6 regions of interest: Ca II H, H beta, Mg b, Na D, H alpha, and the Li line. Each region has vertical blue lines that show the line center position with radial velocity considered, enabling the true center of the absorption lines for visual inspection of equivalent widths (EWs) of these lines.

Looking at these regions, we can calculate the EW of interesting absorption lines. Specifically, the Mg b triplet (seen top right of *figure 5*) contains information regarding the target's stellar atmosphere. The EW of these three lines, which are labeled as $Mg\ b_1,\ Mg\ b_2,\ Mg\ \ b_3$ respectively, indicate the strength of the respective absorption line. The strength of the line relates stellar information according to several physical relationships. Namely, temperature is inversely proportional to the strength of certain Mg b lines, while surface gravity and metallicity are directly proportional to other Mg b line strengths. By comparing the strength of the Phoenix template's Mg b lines to the strength of the ones present in our target's data, we can discuss the differences between the template and HD 126053. Through comparing the relative strengths of Mg b lines (and performing similar comparisons for the other ranges of interest), we can infer the ways in which the target is either different, or the same as the best-fit Phoenix template. This will give us an idea of how much data that sampled the use of the Phoenix template could be.

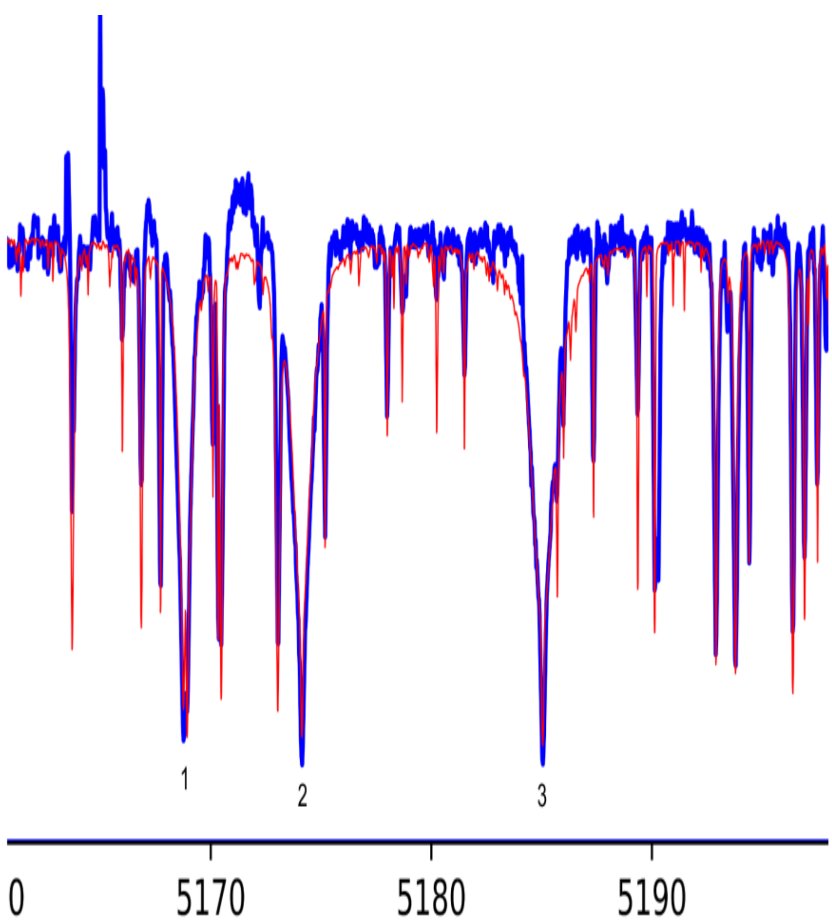

Figure 6.) This figure is zoomed in on a section of figure 2. This zoomed-in section highlights the three Mg b lines. They are marked as 1, 2, and 3, representing each Mg b line respectively. Not only does this section show the target's Mg b lines, but it also shows the Mg b lines for the best-fit Phoenix template for visual comparison. The horizontal axis still reflects wavelength measured in Å, and the vertical dimension still reflects normalized flux.

The strength of an EW is found out via integrating the function about the entirety of the absorption line. Calculating these values numerically would imply a more accurate data set that could be used to help explain the difference in the synthetic data vs natural data. However, as aforementioned, the numerical data within the two-dimensional FITS file remains un-usable, so visual analysis will be used in the comparison of the EWs.

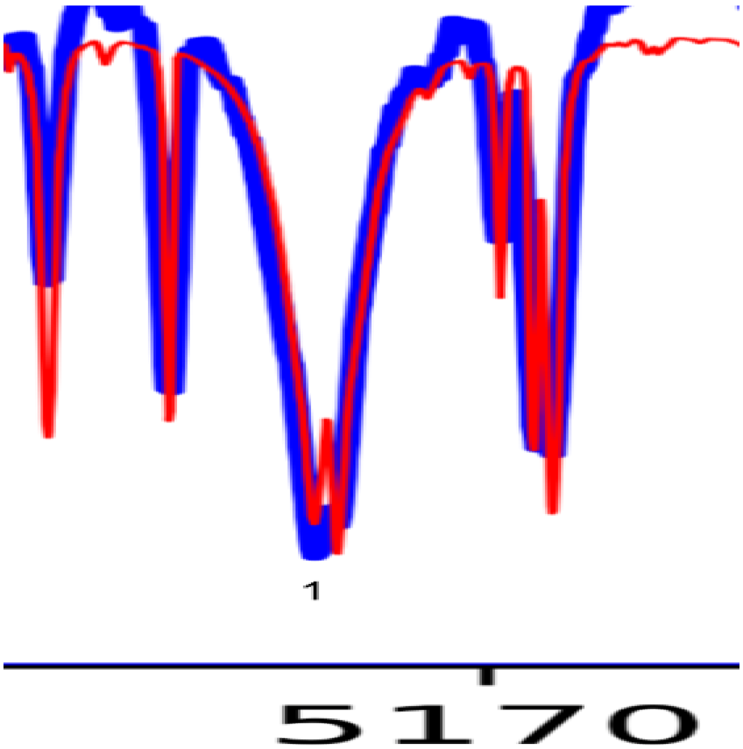

Figure 7.) A further zoomed in image of the $Mg\ b_1$ *line.*

Using *figure 7.*, and recalling that the EW of an absorption line is proportional to the strength and calculable through the integration, we can assume the Mg b line with a larger area will be the stronger line. Visually inspecting, the first Mg b line (depicted in *figure 7.*) has a larger area in the natural data (HD 126053) when compared to the synthetic data (best-fit Phoenix model). This implies that the first Mg b line of our recorded data has a EW greater than that of the Phoenix model.

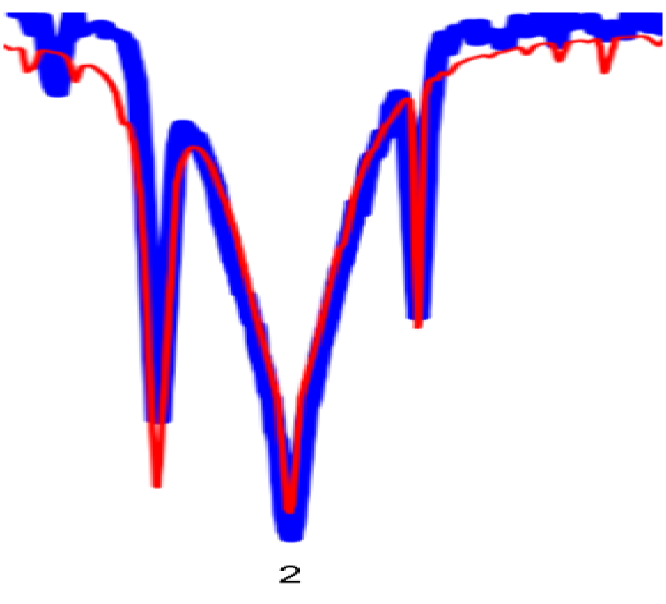

Figure 8.) A further zoomed in image of the $Mg\ b_2$ *line.*

Investigating *figure 8.,* we can see a similar result to the investigation of *figure 7.* The best-fit Phoenix model again has a smaller EW than the data for HD 126053.

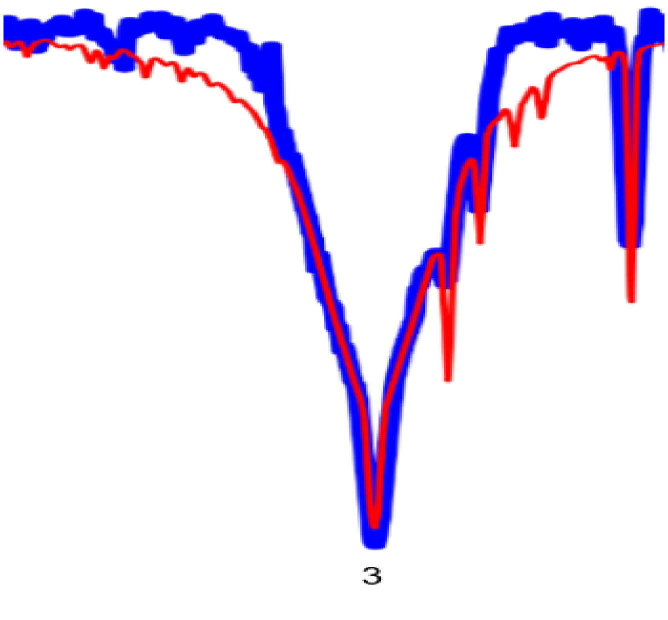

Figure 9.) A further zoomed in image of the $Mg\ b_3$ line.

Looking at *figure 9.,* we see a different result than the ones found in *figures 7/8* : The EW of the best-fit Phoenix model seems to be greater than the EW of the natural data from HD 126053.

Using these three inequalities, the following table can be used to summarize the results:

Data Set	$Mg\ b_1$ EW	$Mg\ b_2$ EW	$Mg\ b_3$ EW
Phoenix Template	Relatively Smaller	Relatively Smaller	Relatively Larger
HD 126053	Relatively Larger	Relatively Larger	Relatively Smaller

Table 1.) Mg b EW values for Phoenix template and target HD 126053 related to each other via relative size.

In looking at *table a.*, it is apparent that there are discrepancies between the strength of each of the Mg b lines across both data sets. Given that the SNR of the data in this range was high, the discrepancies should not be blamed on this. The discrepancies should also not be blamed on problems regarding the radial velocity of HD 126053, as this was accounted for via the lining up of absorption line centers (recall the vertical blue lines in *figure 5*. This means that values assumed from the Phoenix template are not fully accurate, even though they depict a similar stellar atmosphere to that of HD 126053.[7]

[7] *Sasso, C., Andretta, V., Terranegra, L., & Gomez, M. T.*

4.2 - Unopenable Data

Unfortunately, the two dimensional FITS file produced from the BANZAI-NRES pipeline Has remained unhelpful. The file format proved difficult to open, and when opened, could only be viewed in the style of *figure 1.* This led to the unfortunate lack of specific numerical data that would have resulted in a more complete and accurate analysis of the target HD 126053. For example, in calculating the discrepancies between the Phoenix template and HD 126053's spectra, numerical integration could have been used to return specific values of the EW's along Mg b lines, further improving the relationship between the Phoenix model's stellar properties and HD 126053's stellar properties. Other orders of interest could have also been better examined (such as the other depicted absorption lines in *figure 5.*).

Beyond the comparison of the Phoenix template to the raw data produced spectra, the numerical data could have been used to find more appropriate and accurate values shown in section 3.3. Being able to calculate the values within the rightmost section *figure 3.* would also enable us to understand how these values were acquired (either template based or calculated from recorded data). Understanding which method was used would better highlight which data points are generalizations about the stellar atmosphere and properties of HD 126053, and which are calculated directly from the data acquired through the pipeline.

4.3 - Onto the Next

To further this experiment, finding a way to properly open the data tables hidden within the two-dimensional FITS file would be essential in producing a more complete understanding of HD 126053. While the data available in this paper is capable of properly identifying the stellar classification of the target, the results were mostly based off of generalizations developed from the comparison to similarly classed, priorly studied stars.

Gaining numerical data would also provide insights towards the chemical composition of the star, relating a proper metallicity value and describe the various elements contained within the star aside from hydrogen and helium. The relative abundance of each of these elements could also be determined and studied, furthering a comprehensive look at HD 126053.

Conclusion

This paper utilized the spectroscopic data for the target HD 126053 made available from the LCO's BANZAI-NRES pipeline. The data, however, proved cumbersome and difficult to both acquire and work with. However, the produced data files, specifically the one-dimensional FITS file and PDF visualized data enabled conclusions to be drawn about the stellar classification of HD 126053. The produced data also returned information regarding the effective temperature, surface gravity, color of the star, metallicity, and chemical composition of the star. While the accuracy of this data remains hidden within the two-dimensional FITS file, the comparison of HD 126053's spectra to the best-fit Phoenix model implies that it is at least generally accurate.

Resulting from this paper, HD 126053 remains a star with further research required to fully understand its stellar atmosphere and physical properties.

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