g Measured With a Simple Pendulum

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Abstract

We present the results of the measurement of g with a simple pendulum of length $L \approx 1.5\,\mathrm{m}$. The period of the pendulum was measured with a handheld stopwatch. We find that $g = 9.59 \pm 0.18\,\mathrm{m/s^2}$, and we compare this value with that expected, and discuss sources of error.

INTRODUCTION

The simple pendulum, consisting of a weight or "bob" suspended at the end of a string, is a classic problem in physics. In the idealized approximation, the time for a round-trip of the bob, called the period T, is independent of θ_0, the release angle from vertical from of the bob.[1] This property of the simple harmonic oscillator approximation is a surprise, because the larger θ_0, the longer the distance the bob must travel to complete a period.

We have measured the period of a simple pendulum. The period is related to the acceleration of of gravity, g, in our laboratory. We present a measurement with error analysis of g, and we discuss possible sources of systematic error in our result.

METHOD

We constructed the simple pendulum shown in Fig. 1. We draw the bob back to the initial angle θ_0, release it from rest, and measure the time (the period) T for the bob to return to its original position for the first time. We then compute g from Eqn. 1:

$$g = 4\pi^2 \frac{L}{T^2} \tag{1}$$

Construction Details

The length L was measured by G. Gaucho to be 153.1 cm, and by T.M. Storke to be 153.3 cm. We take $L = 153.20 \pm 0.06\,\mathrm{cm}$, where we estimate the error to be the full difference, 0.2 , divided by $\sqrt{12}$, which is the standard deviation of a distribution that is uniform between two limits.

The bob is made of lead, and is a sphere of diameter 2.54 cm, mass 97 gm. The mass of the string that suspends the bob is 0.8 gm. We neglect the contribution of the rotational energy of these components of the pendulum to the determination of g, and estimate that their inclusion would alter our the reported value of g by $< 0.3\,\%$.

Measurement Details

One lab partner drew back the bob to an angle $\theta_0 = 15.0 \pm 0.5°$, measured with a protractor. They released the bob and the second partner used a handheld stopwatch to measure the time of one period of the bob. The roles of the partners were swapped after each measurement.

A total of ten runs were timed by G. Gaucho and fourteen runs were timed by T.M. Storke. The results of these runs are shown in Table I.

RESULTS

The combined mean period from Table I is $2.51 \pm 0.02\,\mathrm{s}$. The central value of g determined from Eq. 1 is $9.59\,\mathrm{m/s^2}$.

Errors in our determination of g arise from the sources summarized in Tab. II. Our final result is $g = 9.59 \pm 0.18\,\mathrm{m/s^2}$.

The local value of g at the latitude of UCSB, 34.4140° N and an altitude of 100 ft is $9.79675\,\mathrm{m/s^2}$.[2] Our value 2.2 % lower than expectation, which is -1.2 standard deviations low. Assuming a normal distribution, the probability of a value less than -1.2 standard deviations from the mean is 12 %. This value is just small enough to cause us concern.

We had checked whether decay of the oscillation influenced our measurements. While running, we followed up to five periods, and saw no change in the period duration within our errors.

We investigated the impact of the known, non-ideal behavior of the simple pendulum, summarized in Fig. 2.[3] At our release angle of $15 \pm 0.5°$, the non-ideal behavior should reduce the reported value of g by 3.7%, and could account for a portion of the deviation we see.

In a future experiment, it would be useful to change the release angle to 30°, where according to Fig. 2, the impact of non-ideal behavior on the reported value of g would be $-13\,\%$, which is much larger than the 1.9 % error we achieve here, and would be observable.

[1] H. Crew and A. de Salvio, *Two New Sciences by Galileo Galilei (translation)* (The Macmillan Company, 1914).

[2] https://www.sensorsone.com/local-gravity-calculator/.

[3] R. B. Kidd and S. L. Fogg, The Physics Teacher **40**, 81 (2002).

FIGURES

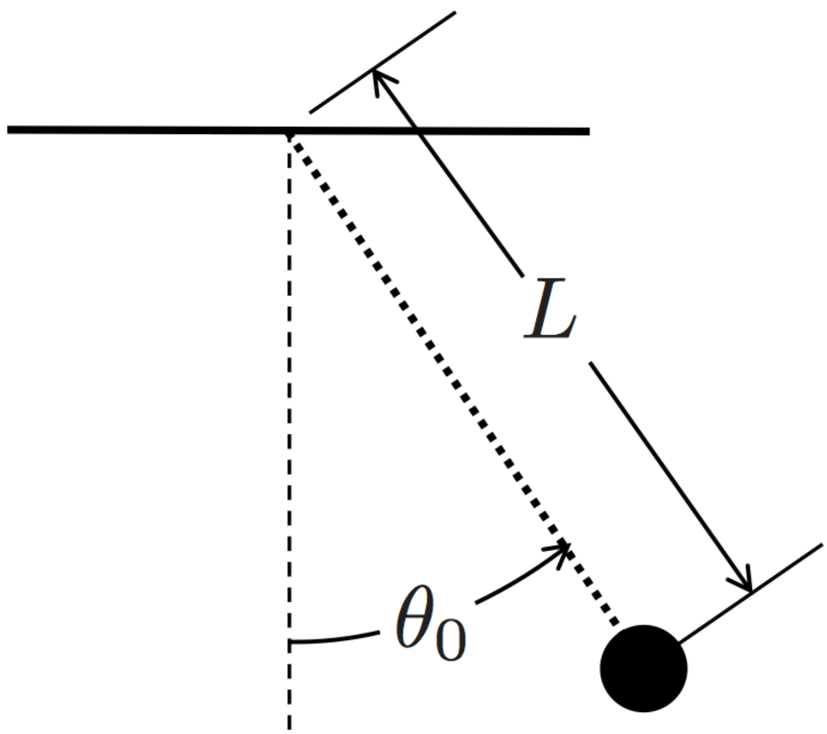

FIG. 1. The simple pendulum used in our experiment. The distance L is measured from the point of suspension to the center of mass of the bob, shown as the filled circle. The initial angle, θ_0, is measured from the vertical.

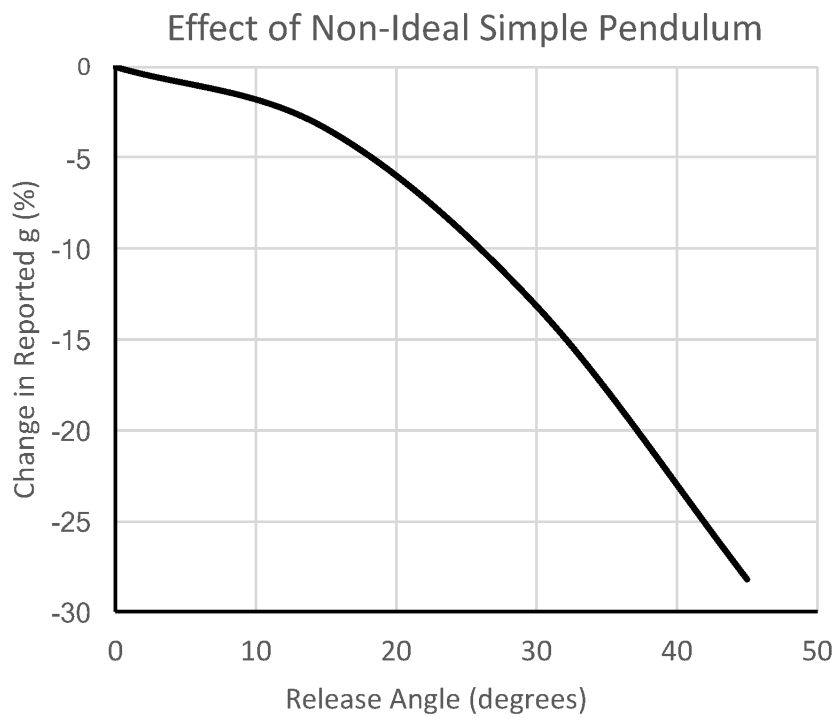

FIG. 2. Change in measured g caused by the simple pendulum not following the motion of a simple harmonic oscillator.

TABLES

TABLE I. Summary of data runs. "Standard Deviation" is abbreviated as SD. The means of the two data sets agreed to 0.7 SD in the means, so were combined into a weighted mean.

Timer	# Runs	Mean (s)	SD (s)	SD, Mean (s)
G. Gaucho	10	2.52	0.08	0.03
T.M. Storke	14	2.47	0.22	0.07
Combined	24	2.51		0.02

TABLE II. Errors in the quantities in Eq. 1 and from the moment of inertia, discussed in Sec. . Here, SD stands for Standard Deviation. The percentage error in g is the quadrature sum of all the errors in the final column.

Quantity	SD	SD(%)	SD(%) in g
$L = 1.53\,\mathrm{m}$	$0.06\,\mathrm{m}$	0.04	0.04
$T = 2.51\,\mathrm{s}$	$0.02\,\mathrm{s}$	0.95	1.9
Moment of Inertia			$< 0.3\,\%$
For $g = 9.59\,\mathrm{m/s^2}$	$0.18\,\mathrm{m/s^2}$	1.9	